

Worked Solutions

Edexcel C4 Paper J

1. (a) We are given that $\frac{dA}{dt} = 40 \text{ cm}^2 \text{ s}^{-1}$
 after 10 seconds area of circle = 400 cm^2
 so $\pi r^2 = 400$

$$r = \sqrt{\frac{400}{\pi}} \text{ cm} \quad (2)$$

(b) $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

when $r = \sqrt{\frac{400}{\pi}}$ we have, $40 = 2\pi \times \frac{20}{\sqrt{\pi}} \times \frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{1}{\sqrt{\pi}} = 0.56 \text{ cm s}^{-1} \quad (2 \text{ sig. fig.}) \quad (4)$$

2. (a) $A = \int_4^9 y \, dx = \int_4^9 y \frac{dx}{dt} \cdot dt$ $x = 2t^{\frac{1}{2}}$
 $\frac{dx}{dt} = t^{-\frac{1}{2}}$

$$= \int_4^9 (t^2 - 2)t^{-\frac{1}{2}} dt$$

\uparrow y , \uparrow $\frac{dx}{dt}$

$$= \int_4^9 (t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}) dt \quad (4)$$

(b) $A = \left[\frac{2}{5} t^{\frac{5}{2}} - 4t^{\frac{1}{2}} \right]_4^9 = \frac{2}{5} \times 243 - 4 \times 3 - \left(\frac{2}{5} \times 32 - 8 \right) = 80.4 \quad (3)$

3. (a) $\frac{2}{2x-1} + \frac{3}{x+2}$ (using 'cover up' rule) (3)

(b) $\int \frac{1}{y} dy = \int \left(\frac{2}{2x-1} + \frac{3}{x+2} \right) dx$

$$\ln y = \ln(2x-1) + 3 \ln(x+2) + c$$

$y = 1, x = 1: \quad \ln 1 = \ln 1 + 3 \ln 3 + c$

$$c = -3 \ln 3$$

$$\therefore \ln y = \ln \frac{(2x-1)(x+2)^3}{27}$$

$$y = \frac{(2x-1)(x+2)^3}{27}$$

4. (a) $f(x) = \left[1 + \frac{1}{2}x + \frac{\frac{1}{2} \left(-\frac{1}{2} \right)}{2} x^2 \right] \left[1 - \frac{1}{2}(-x) + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right)}{2} (-x)^2 \right]$
 $= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 \right) \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \right)$
 $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2$
 $= 1 + x + \frac{1}{2}x^2 \quad (4)$

(b) valid for $|x| < 1$ (1)

(c) put $x = \frac{1}{10}, \quad f\left(\frac{1}{10}\right) = \sqrt{\frac{1 + \frac{1}{10}}{1 - \frac{1}{10}}} = \sqrt{\frac{11/10}{9/10}} = \frac{1}{3} \sqrt{11}$

$$\therefore \frac{1}{3} \sqrt{11} \approx 1 + \frac{1}{10} + \left(\frac{1}{2} \times \frac{1}{100} \right) \quad \sqrt{11} \approx \frac{663}{200} \quad (3)$$

5. (a) (i) putting $B = A$, $\cos(A + A) = \cos A \cos A - \sin A \sin A$
 $= \cos^2 A - \sin^2 A$
 $= \cos^2 A - (1 - \cos^2 A)$
 $= 2 \cos^2 A - 1$

(ii) $\int \cos^2 x \, dx = \frac{1}{2} \int (1 + \cos 2x) \, dx$
 $= \frac{1}{2}x + \frac{1}{4} \sin 2x + c$

(b) let $I = \int_0^{\sqrt{3}} \sqrt{4 - x^2} \, dx$ $x = 2 \sin \theta$
 $\frac{dx}{d\theta} = 2 \cos \theta$
 $x = \sqrt{3}, \quad \theta = \frac{\pi}{3}$
 $x = 0, \quad \theta = 0$

$\therefore I = \int_0^{\frac{\pi}{3}} \sqrt{4 - 4 \sin^2 \theta} \cdot 2 \cos \theta \, d\theta$
 $= \int_0^{\frac{\pi}{3}} \sqrt{4(1 - \sin^2 \theta)} \cdot 2 \cos \theta \, d\theta = \int_0^{\frac{\pi}{3}} 2 \cos \theta \cdot 2 \cos \theta \, d\theta$
 $= 4 \int_0^{\frac{\pi}{3}} \cos^2 \theta \, d\theta$
 $= \left[2\theta + \sin 2\theta \right]_0^{\frac{\pi}{3}} \quad (\text{from (a) (ii)})$
 $= \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$ (5)

6. (a) l passes through A and C .

$$\vec{CA} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

(2)

$$l: \mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

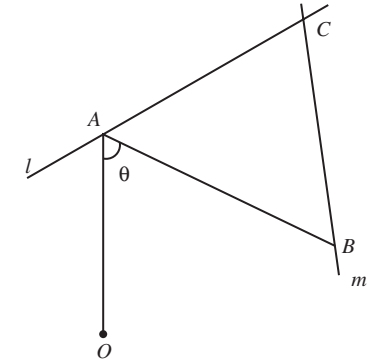
(3)

m passes through B and C

$$\vec{BC} = \begin{pmatrix} 2 \\ -5 \\ -7 \end{pmatrix}$$

$$\text{line } m: \mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -5 \\ -7 \end{pmatrix}$$

(5)



(b) l meets xz -plane where $y = 0$

i.e. where $1 - \lambda = 0$

$$\lambda = 1$$

point of intersection is $\begin{pmatrix} 9 \\ 0 \\ 3 \end{pmatrix}$

(c) $\vec{AO} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} -6 \\ 6 \\ 6 \end{pmatrix}$

$$|\vec{AO}| = \sqrt{30} \quad |\vec{AB}| = \sqrt{108}$$

$$\vec{AO} \cdot \vec{AB} = 30 - 6 - 12 = 12$$

$$\therefore 12 = \sqrt{30}\sqrt{108} \cos \theta$$

$$\theta = 78^\circ \quad (\text{nearest degree})$$

$$7. (a) \frac{dy}{dx} = x \cdot \frac{1}{2}(10 - x^2)^{-\frac{1}{2}} \cdot (-2x) + (10 - x^2)^{\frac{1}{2}}$$

$$= -\frac{x^2}{\sqrt{10 - x^2}} + \sqrt{10 - x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow x^2 = 10 - x^2$$

$$x^2 = 5$$

$$x = \sqrt{5}$$

$$\text{when } x = \sqrt{5}, y = \sqrt{5}\sqrt{10 - 5} = 5$$

turning point is $(\sqrt{5}, 5)$

$$(b) \text{ area} = \int_0^{\sqrt{10}} x\sqrt{10 - x^2} dx \quad [y = 0 \text{ at } x = 0, \sqrt{10}]$$

$$= \left[\frac{-2}{3} \cdot \frac{1}{2} (10 - x^2)^{\frac{3}{2}} \right]_0^{\sqrt{10}} \quad [\text{or let } u = 10 - x^2]$$

$$= -\frac{1}{3} \times 0 - \left(-\frac{1}{3} \cdot 10^{\frac{3}{2}} \right)$$

$$= \frac{10}{3} \sqrt{10} \quad (7)$$

$$8. (a) \frac{dN}{dt} = -kN \quad (2)$$

$$(b) \int \frac{1}{N} dN = -\int k dt$$

$$\ln N = -kt + \ln A$$

$$\ln \frac{N}{A} = -kt$$

$$\frac{N}{A} = e^{-kt}$$

$$N = Ae^{-kt} \quad (5)$$

$$(c) t = 0, N = 5 \times 10^{12}$$

$$\therefore A = 5 \times 10^{12}$$

$$t = 10, N = 2 \times 10^{11}$$

$$2 \times 10^{11} = 5 \times 10^{12} e^{-10k}$$

$$\frac{1}{25} = e^{-10k}$$

$$\ln \frac{1}{25} = -10k$$

$$k = -\frac{1}{10} \ln \frac{1}{25} = \frac{1}{10} \ln 25 = 0.32188 \dots \quad (3)$$

$$(d) \text{ when } t = 40, N = 5 \times 10^{12} \times e^{-0.32188 \times 40}$$

$$N = 1.28 \times 10^7 \quad (3 \text{ sig. fig.}) \quad (2)$$