Worked Solutions

Edexcel C4 Paper G

1. (*a*) $\dfrac{dy}{dx} = \dfrac{2\cos t}{-\sin t}$

when $t = \dfrac{\pi}{2}$, gradient $= 0$ **(2)**

(*b*) using $\cos^2 t + \sin^2 t = 1$,

$$x^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$x^2 + \frac{y^2}{4} = 1$$ **(3)**

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2. (*a*) $\dfrac{1}{x-2} + \dfrac{3}{2x+1} + \dfrac{1}{x+2}$ (using 'cover up' rule) **(4)**

(*b*) $\displaystyle\int_3^4 \left(\frac{1}{x-2} + \frac{3}{2x+1} + \frac{1}{x+2}\right) dx$

$$= \left[\ln(x-2) + \frac{3}{2}\ln(2x+1) + \ln(x+2)\right]_3^4$$

$$= \ln 2 + \frac{3}{2}\ln 9 + \ln 6$$

$$-\left(\ln 1 + \frac{3}{2}\ln 7 + \ln 5\right)$$

$$= \ln\left(\frac{2\times 6}{5}\right) + \frac{3}{2}\ln\left(\frac{9}{7}\right)$$

$$= \ln\left(\frac{12}{5}\right) + \frac{3}{2}\ln\frac{9}{7}$$ **(5)**

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3. (*a*) $(1+ax)^6 = 1 + 6ax + 15a^2x^2$ **(3)**

(*b*) $(1+bx)\left(1 + 6ax + 15a^2x^2\right) = 1 + 6ax + 15a^2x^2 + bx + 6abx^2$

we have $6a + b = -9$ ...[A]

$15a^2 + 6ab = 24$ ...[B]

from equation [A] $b = -9 - 6a$

substitute in [B] $15a^2 + 6a(-9-6a) = 24$

Hence $a = -2$, $b = 3$ **(7)**

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4. (*a*) $= \displaystyle\int x \frac{d}{dx}(\tan x)dx$ [By parts]

$$= x\tan x - \int \tan x \; dx = x\tan x + \ln\cos x + c$$ **(4)**

(*b*) $\displaystyle\int y^{-\frac{1}{2}} \; dy = \int x\sec^2 x \; dx$ $\quad 2y^{\frac{1}{2}} = x\tan x + \ln\cos x + c$

$y = 4, x = 0$: $2\sqrt{4} = 0 + \ln 1 + c$

$c = 4$

$2\sqrt{y} = x\tan x + \ln\cos x + 4$

when $x = \dfrac{\pi}{4}$, $2\sqrt{y} = \dfrac{\pi}{4}\cdot\tan\dfrac{\pi}{4} + \ln\dfrac{1}{\sqrt{2}} + 4$

$$2\sqrt{y} = \frac{\pi}{4} + \ln 2^{-\frac{1}{2}} + 4$$

$$2\sqrt{y} = \frac{\pi}{4} - \frac{1}{2}\ln 2 + 4$$

$$\sqrt{y} = \frac{\pi}{8} - \frac{1}{4}\ln 2 + 2$$

$$y = \left(\frac{\pi}{8} - \frac{1}{4}\ln 2 + 2\right)^2$$ **(6)**

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5. (*a*)

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$\frac{12}{x}$	12	8	6	4.8	4

(i) using two trapeziums, $I = \frac{1}{2}\left[12 + 4 + (2 \times 6)\right] = 14$ **(2)**

(ii) using four trapeziums, $I = \frac{1}{4}\left[12 + 4 + 2(8 + 6 + 4.8)\right] = 13.4$ **(2)**

(*b*) $\int_1^3 \frac{12}{x}\,dx = \left[12 \ln x\right]_1^3 = 12 \ln 3 = 13.1833\ldots$

in (i) % error $= \frac{14 - 13.1833}{13.1833} \times 100 = 6.2\%$

in (ii) % error $= \frac{13.4 - 13.1833}{13.1833} \times 100 = 1.6\%$ **(5)**

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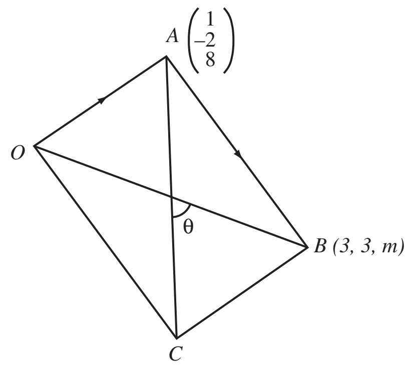
6. (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \\ m-8 \end{pmatrix}$



$\overrightarrow{AO} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ m-8 \end{pmatrix} = -2 + 10 - 8m + 64 = 0$

$m = 9$ **(4)**

(*b*) $\overrightarrow{OC} = \overrightarrow{AB}$ (*OABC* is a rectangle)

$= \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$ **(1)**

(*c*) $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix}$

equation of line *AC* is $r = \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix}$ **(2)**

(*d*) $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix}$

let angle between diagonals be θ

$\overrightarrow{AC} \cdot \overrightarrow{OB} = |\overrightarrow{AC}| \times |\overrightarrow{OB}| \cos\theta$

$\begin{pmatrix} 1 \\ 7 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix} = \sqrt{99}\sqrt{99}\cos\theta$

$3 + 21 - 63 = 99\cos\theta$

$\theta = 113.2°$

acute angle between diagonals $= 67°$ (nearest degree) **(3)**

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7. (*a*) $y = 12x - e^{\frac{1}{2}x}$

$\frac{dy}{dx} = 12 - \frac{1}{2}\ e^{\frac{1}{2}x}$

$\frac{dy}{dx} = 0, \quad 24 = e^{\frac{1}{2}x}$

$x = 2\ln 24$ **(3)**

(*b*) $y = 12 \times 2\ln 24 - 24 = 24\ln 24 - 24$ **(1)**

(*c*) $\frac{d^2y}{dx^2} = -\frac{1}{4}e^{\frac{1}{2}x}$

so $\frac{d^2y}{dx^2} < 0$ and the stationary point is a maximum. **(1)**

(*d*) area $= \int_2^4 \left(12x - e^{\frac{1}{2}x}\right)\ dx = \left[6x^2 - 2e^{\frac{1}{2}x}\right]_2^4$

$= 96 - 2\,e^2 - (24 - 2e) = 72 + 2e(1 - e)$ **(5)**

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8. (*a*) $4x - \left(x\frac{dy}{dx} + y \cdot 1\right) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - x) = y - 4x$

$\frac{dy}{dx} = \frac{y - 4x}{2y - x}$ **(4)**

(*b*) at $(2, -6)$ gradient of curve $= \frac{-6 - 8}{-12 - 2} = 1$

$\therefore$ gradient of normal $= -1$

equation of normal is $y + 6 = -1(x - 2)$

$x + y + 4 = 0$ **(3)**

(*c*) gradient $= 0$ where $y = 4x$

substitute $y = 4x$ into equation of curve.

$2x^2 - x \cdot 4x + 16x^2 = 56 \Rightarrow x = \pm 2$

gradient $= 0$ at $(2, 8)$ and $(-2, -8)$ **(5)**
