

Worked Solutions

Edexcel C4 Paper E

1. (a) when $y = 1$, $4x^2 + 3 = 12$

$$x^2 = \frac{9}{4}$$

$$x = \pm \frac{3}{2}$$

(2)

(b) differentiating, $8x + 6y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{8x}{6y} = -\frac{4x}{3y}$

at $\left(\frac{3}{2}, 1\right)$ gradient $= -\frac{4 \times \frac{3}{2}}{3} = -2$

at $\left(-\frac{3}{2}, 1\right)$ gradient $= 2$ (4)

2. $(8+x)^{\frac{1}{3}} = \left[8\left(1+\frac{x}{8}\right)\right]^{\frac{1}{3}} = 2\left(1+\frac{x}{8}\right)^{\frac{1}{3}} = 2\left[1 + \frac{1}{3}\left(\frac{x}{8}\right) + \frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{x}{8}\right)^2 + \dots\right]$

$$= 2 + \frac{x}{12} - \frac{1}{288}x^2 + \dots$$
 (4)

(b) for $(8+3m+m^2)^{\frac{1}{3}}$, let $3m+m^2 = x$

$$\begin{aligned} (8+3m+m^2)^{\frac{1}{3}} &= 2 + \left(\frac{3m+m^2}{12}\right) - \frac{1}{288}(3m+m^2)^2 \\ &= 2 + \frac{1}{4}m + \frac{1}{12}m^2 - \frac{1}{288} \cdot 9m^2 + \dots \\ &= 2 + \frac{1}{4}m + \frac{1}{12}m^2 - \frac{1}{32}m^2 = 2 + \frac{1}{4}m + \frac{5}{96}m^2. \end{aligned}$$
 (3)

3. (a) $\frac{dy}{dx} = \frac{\frac{1}{2} \cdot 2 \cos 2\theta}{-\sin \theta} = -\frac{\cos 2\theta}{\sin \theta}$

at $\theta = \frac{\pi}{6}$, gradient $= -\frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{6}} = -\frac{\frac{1}{2}}{\frac{1}{2}} = -1$ (2)

(b) at $\theta = \frac{\pi}{6}$, $x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2} \sin \frac{\pi}{3} = \frac{1}{4}\sqrt{3}$.

equation of tangent is $y - \frac{\sqrt{3}}{4} = -1\left(x - \frac{\sqrt{3}}{2}\right)$

$$4y - \sqrt{3} = -4x + 2\sqrt{3}$$

$$4y + 4y = 3\sqrt{3}$$
 (3)

(c) $y^2 = \frac{1}{4} \sin^2 2\theta = \frac{1}{4} (2 \sin \theta \cos \theta)^2 = \frac{1}{4} \cdot 4 \sin^2 \theta \cos^2 \theta = (1 - \cos^2 \theta) \cos^2 \theta$.

$$\therefore y^2 = (1 - x^2)(x^2)$$
 (3)

4. (a) (0, 10) (1)

(b) $\frac{dy}{dx} = -10(-k)e^{-kx}$

at $x = 0$, gradient $= 10k$

$$10k = 5 \Rightarrow k = \frac{1}{2}$$
 (3)

(c) area $= \int_0^4 \left(20 - 10e^{-\frac{1}{2}x}\right) dx = \left[20x + 20e^{-\frac{1}{2}x}\right]_0^4$

$$= 80 + 20e^{-2} - (0 + 20) = 60 + \frac{20}{e^2}$$
 (5)

5. (a) when $t = 0, \theta = 300 - 270e^\circ$

$$\theta = 30$$

(b) as $t \rightarrow \infty, \theta \rightarrow 300$

(c) $200 = 300 - 270e^{-0.05t}$

$$270e^{-0.05t} = 100$$

$$\ln e^{-0.05t} = \ln \left(\frac{100}{270} \right)$$

$$-0.05t = \ln \left(\frac{100}{270} \right)$$

$$t = 19.9 \text{ minutes}$$

(d) $\frac{d\theta}{dt} = -270(-0.05)e^{-0.05t}$

when $t = 2, \frac{d\theta}{dt} = 270 \times 0.05 \times e^{-0.1}$

$$= 12.2^\circ \text{ C/min}$$

6. (a) $\frac{2}{1-x} - \frac{2}{2-x}$ ('cover up' rule)

(b) we have $\frac{2}{1-x} - \frac{2}{2(1-\frac{x}{2})} = 2(1-x)^{-1} - \left(1 - \frac{x}{2}\right)^{-1}$

$$= 2 \left[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \dots \right]$$

$$- \left[1 + (-1) \left(\frac{-x}{2} \right) + \frac{(-1)(-2)}{2} \left(\frac{-x}{2} \right)^2 + \dots \right]$$

$$= 2 + 2x + 2x^2 - 1 - \frac{x}{2} - \frac{x^2}{4}$$

$$= 1 + \frac{3}{2}x + \frac{7}{4}x^2$$

(1) $\int_0^{\frac{1}{2}} \left(\frac{2}{1-x} - \frac{2}{2-x} \right) dx = \left[-2 \ln(1-x) + 2 \ln(2-x) \right]_0^{\frac{1}{2}}$

(2) $= -2 \ln \frac{1}{2} + 2 \ln \frac{3}{2} - (-2 \ln 1 + 2 \ln 2)$

$= -2 \ln 2^{-1} + 2 \ln \frac{3}{2} - 0 - 2 \ln 2 = 2 \ln \frac{3}{2}$ (5)

7. (a) let $\angle ABC = \theta, \vec{BA} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix}, |\vec{BA}| = \sqrt{4^2 + 3^2} = 5$

$\vec{BC} = \begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix}, |\vec{BC}| = \sqrt{6^2 + 1^2 + 3^2} = \sqrt{46}$

$$\vec{BA} \cdot \vec{BC} = |\vec{BA}| \times |\vec{BC}| \cos \theta$$

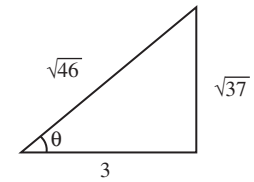
$\begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix} = 5\sqrt{46} \cos \theta \quad 24 - 9 = 5\sqrt{46} \cos \theta \quad \cos \theta = \frac{3}{\sqrt{46}}$ (4)

(b) area of $\triangle ABC = \frac{1}{2} |\vec{BA}| \times |\vec{BC}| \sin \theta$

$$\text{area} = \frac{1}{2} \cdot 5 \times \sqrt{46} \cdot \frac{\sqrt{37}}{\sqrt{46}}$$

$$= \frac{5}{2} \sqrt{37}$$

$\sin \theta = \frac{\sqrt{37}}{\sqrt{46}}$ (4)



(c) $\vec{AC} = \begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix} \quad \vec{OD} = \begin{pmatrix} 4 \\ -2 \\ -12 \end{pmatrix}$

$\vec{OD} = -2 \begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix}$, so AC is parallel to OD. (2)

8. (a) Let $I = \int_0^1 \frac{x}{(2x+1)^2} dx$

let $t = 2x + 1$
 $\frac{dt}{dx} = 2 \Rightarrow dx = \frac{1}{2} dt$

$x = \frac{1}{2}(t - 1)$

$\therefore I = \frac{1}{2} \int_1^3 \frac{1}{2} \frac{(t-1)}{t^2} dt$

when $x = 1, t = 3$
 $x = 0, t = 1$

$= \frac{1}{4} \int_1^3 \left(\frac{1}{t} - t^{-2} \right) dt = \frac{1}{4} \left[\ln t + \frac{1}{t} \right]_1^3$

$= \frac{1}{4} \left[\ln 3 + \frac{1}{3} - (\ln 1 + 1) \right]$

$= \frac{1}{4} \left[\ln 3 - \frac{2}{3} \right]$

(7)

(b) $\int_1^e x^2 \ln x \, dx = \int_1^e \ln x \frac{d}{dx} \left(\frac{x^3}{3} \right) dx$

$= \left[\frac{x^3}{3} \ln x \right]_1^e - \int_1^e \frac{x^3}{3} \cdot \frac{1}{x} dx$

$= \left(\frac{e^3}{3} \ln e - 0 \right) - \left[\frac{x^3}{9} \right]_1^e$

$= \frac{e^3}{3} - \left(\frac{e^3}{9} - \frac{1}{9} \right) = \frac{3e^3 - e^3 + 1}{9}$

$= \frac{2e^3 + 1}{9}$

(6)