Core Mathematics C4 For Edexcel
Advanced Level

Paper D
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. The equation of a curve is

$$x^2 + y^2 = 9.$$

(*a*) Sketch the curve. **(2)**

(*b*) The region enclosed by the curve and the lines $y = 0, x = 0$ and $x = 2$ is rotated through 360° about the x-axis. Find the volume of the solid formed. **(4)**

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2. (*a*) Show that $\cos^3 x = \cos x - \cos x \sin^2 x$. **(1)**

(*b*) Work out $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 x\right)$. **(1)**

(*c*) Use (a) and (b) to find $\int \cos^3 x \ \mathrm{d}x$. **(3)**

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3. The surface area of a sphere is increasing at the rate of 640 cm^2 s^{-1}. Find the rate of increase of the radius of the sphere when the radius is 5 cm. Give the answer in terms of π. $\left[\text{The surface area } S \text{ of a sphere is } S = 4\pi r^2.\right]$ **(4)**

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4. The equation of a curve is

$$x^2 + y^2 - 2x + 4y = 20.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(2)**

(*b*) Find the equation of the tangent to the curve at the point (4,2) **(4)**

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5. (*a*) Solve the equation $5^x = 11$, correct to 3 significant figures.

(2)

(*b*) Given $y = 3^x$, find $\dfrac{dy}{dx}$ in terms of x.

(3)

(*c*) Solve the equation $3\,e^{-0.4x} = 15$, correct to 3 significant figures.

(2)

6.

Figure 1

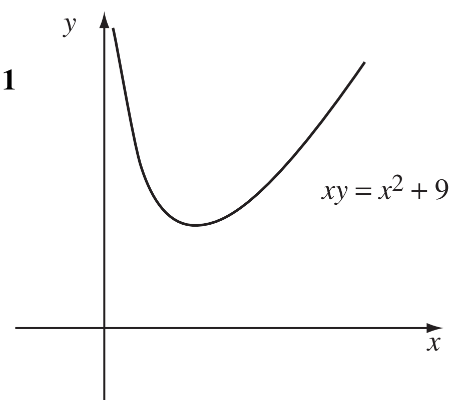

Figure 1 shows a sketch of the curve with equation

$$xy = x^2 + 9, \quad \text{for } x > 0.$$

(*a*) Show that for $x > 0$, $y \geq 6$.

(4)

The finite region R is bounded by the curve, the x-axis and the lines $x = 3$ and $x = 9$.

(*b*) Find the exact area of R.

(4)

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7. A curve has parametric equations

$$x = 2\,e^{2t} - t, \qquad y = e^{4t} - 3t.$$

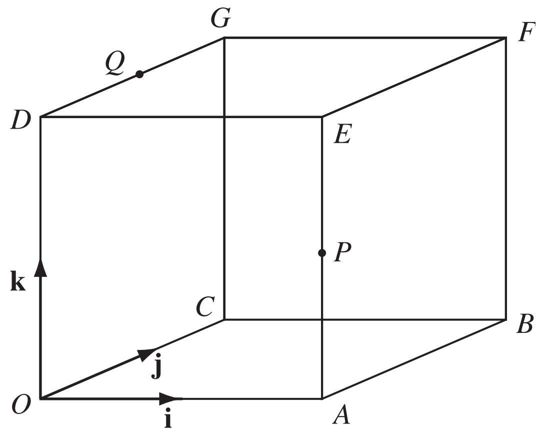
(*a*) Find $\dfrac{dy}{dx}$, in terms of t.

(3)

(*b*) The gradient of the tangent to the curve at the point P is 3. Find the value of t at P, giving your answer in the form $t = a \ln b$, where a and b are constants.

(5)

8.



The diagram shows a cube *OABCDEFG* with sides of length 2 units. Unit vectors **i, j, k** are directed along *OA, OC, OD* respectively. The mid-point of *AE* is *P* and the mid-point of *DG* is *Q*.

(*a*) Write down the position vectors of the points *P* and *Q*. **(2)**

(*b*) Find a vector equation of the line *QP*. **(2)**

(*c*) Calculate the angle between the lines *QP* and *OP*. **(3)**

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9. (*a*) (i) Expand $(\cos\theta + \sin\theta)^2$ and simplify the result. **(2)**

(ii) Show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos\theta + \sin\theta)^2 \, d\theta = \frac{\pi}{4} + \frac{1}{2}$ **(3)**

(*b*) Work out $\displaystyle\int_0^1 (2x+1)^4 \, dx$. **(4)**

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10. (*a*) Given that

$$f(x) \equiv \frac{3 + 5x - x^2}{(2-x)(1+x)^2} \equiv \frac{A}{2-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

find the values of A and B and show that $C = -1$. **(4)**

(*b*) Find $\displaystyle\int_0^1 f(x)dx$, expressing your answer in an exact form. **(6)**

(*c*) Express $f(x)$ as a sum of powers of x up to and including the term in x^3. **(4)**

(*d*) Determine the range of values of x for which this expansion of $f(x)$ is valid. **(1)**

END **TOTAL 75 MARKS**