Worked Solutions

Edexcel C4 Paper I

1. (*a*) $\dfrac{dy}{d\theta} = \dfrac{1}{(1+\cos\theta)}(-\sin\theta),\ \dfrac{dx}{d\theta} = 2\cos 2\theta$

$$\frac{dy}{dx} = \frac{-\sin\theta}{(1+\cos\theta)2\cos 2\theta}$$

where $\theta = \dfrac{\pi}{6}$, gradient $= \dfrac{-\frac{1}{2}}{\left(1+\frac{\sqrt{3}}{2}\right)\cdot 2\cdot\frac{1}{2}} = -\dfrac{1}{2\left(1+\frac{\sqrt{3}}{2}\right)}$

$$= -\frac{1}{2+\sqrt{3}} = -\frac{(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \sqrt{3}-2 \qquad \textbf{(5)}$$

(*b*) gradient $= 0$ where $\sin\theta = 0$

i.e. where $\theta = 0$

at $\theta = 0,\ x = 0,\ y = \ln 2$

gradient is zero at $(0, \ln 2)$ **(3)**

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2. $\displaystyle\int A\ dA = \int e^{\frac{1}{10}t}\,dt$

$$\frac{A^2}{2} = 10\,e^{\frac{1}{10}t} + c$$

$A = 20,\ t = 0$: $\dfrac{400}{2} = 10 + c,\ c = 190$

$$\therefore \quad \frac{A^2}{2} = 10\,e^{\frac{1}{10}t} + 190$$

when $t = 20$, $\dfrac{A^2}{2} = 10\,e^2 + 190$

$A = 23$ (2 sig. fig.) **(7)**

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3. (*a*) $\dfrac{1}{y}\dfrac{dy}{dx} + 3x^2 - 2 = 0$

$$\frac{dy}{dx} = y(2-3x^2) \qquad \textbf{(3)}$$

(*b*) (i) $e^x\dfrac{dy}{dx} + ye^x + 2y\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(e^x + 2y\right) = -ye^x$$

$$\frac{dy}{dx} = \frac{-ye^x}{e^x + 2y}$$

at $(0, 3)$ $\quad \dfrac{dy}{dx} = \dfrac{-3}{1+6} = -\dfrac{3}{7}$ **(3)**

(ii) equation of tangent at $(0, 3)$ is $y - 3 = -\dfrac{3}{7}x$

$3x + 7y = 21$ **(2)**

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4. (*a*) (i) $\cos 2x = 1 - 2\sin^2 x$ **(1)**

hence $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$

(ii) $\displaystyle\int \sin^2 x = \frac{1}{2}\int(1-\cos 2x)dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ **(2)**

(*b*) Integrating by parts,

$$\int_0^{\frac{\pi}{8}} x\frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)dx = \left[-\frac{x}{2}\cos 2x\right]_0^{\frac{\pi}{8}} + \int_0^{\frac{\pi}{8}} \frac{1}{2}\cos 2x\ \ dx$$

$$= \left[-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{8}}$$

$$= -\frac{\pi}{16}\cdot\frac{1}{\sqrt{2}} + \frac{1}{4}\cdot\frac{1}{\sqrt{2}} - (0+0) = \frac{4-\pi}{16\sqrt{2}} \qquad \textbf{(5)}$$

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5. (*a*) $\overrightarrow{OM} = \mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{ON} = \mathbf{i} + 2\mathbf{k}$ **(2)**

(*b*) line OM: $\quad \mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\0\end{pmatrix}$

line AB: $\quad \mathbf{r} = \begin{pmatrix}2\\0\\0\end{pmatrix} + \mu\begin{pmatrix}0\\1\\0\end{pmatrix}$

at intersection of OM and AB, $0 + \lambda = 2 + 0 \Rightarrow \lambda = 2$

$$0 + 2\lambda = 0 + \mu \Rightarrow \mu = 4$$

point of intersection is $\begin{pmatrix}2\\4\\0\end{pmatrix}$. **(4)**

(*c*) $\overrightarrow{MN} = \begin{pmatrix}0\\-2\\2\end{pmatrix}, |\overrightarrow{MN}| = \sqrt{8}, \overrightarrow{MO} = \begin{pmatrix}-1\\-2\\0\end{pmatrix}, |\overrightarrow{MO}| = \sqrt{5}$

$\overrightarrow{MN} \cdot \overrightarrow{MO} = \begin{pmatrix}0\\-2\\2\end{pmatrix} \cdot \begin{pmatrix}-1\\-2\\0\end{pmatrix} = 4$

$\therefore \quad 4 = \sqrt{8}\sqrt{5}\cos\theta$, where θ = angle required

$\theta = 50.8°$

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6. (*a*) $\int_1^2 \left(2x + \frac{1}{x}\right) dx = \left[x^2 + \ln x\right]_1^2 = 4 + \ln 2 - (1 + \ln 1) = 3 + \ln 2$ **(3)**

(*b*) (i) $\int x e^x dx = \int x \frac{d}{dx}(e^x) dx = xe^x - \int e^x dx = xe^x - e^x + c$ **(3)**

(ii) volume $= \pi \int_0^1 x e^x dx = \pi\left[e^x(x-1)\right]_0^1 = \pi\left[e \times 0 - 1(-1)\right] = \pi$ **(4)**

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7. (*a*) $f(x) = \frac{1}{x+3} + \frac{3}{x-1}$ (using 'cover up' rule) **(3)**

(*b*) $\frac{1}{3+x} = \frac{1}{3\left(1+\frac{x}{3}\right)}$

$$= \frac{1}{3}\left(1+\frac{x}{3}\right)^{-1}$$

$f(x) = \frac{1}{3}\left(1+\frac{x}{3}\right)^{-1} - 3(1-x)^{-1}$ [note change of sign]

$$= \frac{1}{3}\left[1 + (-1)\frac{x}{3} + \frac{(-1)(-2)}{2}\ \frac{x^2}{9} + \ldots\right]$$

$$-3\left[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \ldots\right]$$

$$= \frac{1}{3} - \frac{x}{9} + \frac{x^2}{27} - 3 - 3x - 3x^2$$

$$= -\frac{8}{3} - \frac{28}{9}x - \frac{80}{27}x^2$$ **(4)**

(*c*) valid for $|x| < 1$ (i.e. $-1 < x < 1$) **(1)**

(*d*) $f(x) = (x+3)^{-1} + 3(x-1)^{-1}$

$f'(x) = -(x+3)^{-2} - 3(x-1)^{-2}$

$$= -\frac{1}{(x+3)^2} - \frac{3}{(x-1)^2}$$

$f'(x) < 0$ as both $(x+3)^2$ and $(x-1)^2$ are always positive. **(3)**

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8. (*a*) $\dfrac{dy}{dx} = \dfrac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2}$

$= \dfrac{1-x^2}{(1+x^2)^2}$

$\dfrac{dy}{dx} = 0$ at $1 - x^2 = 0$

i.e. $x = 1, -1$

when $x = 1$, $y = \dfrac{1}{2}$

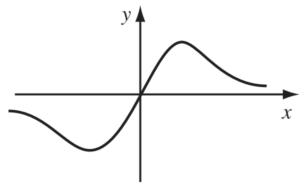
$x = -1$, $y = -\dfrac{1}{2}$ **(5)**

(*b*) $\dfrac{d^2y}{dx^2} = \dfrac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)\cdot 2x}{(1+x^2)^4}$

when $x = 1$, $\dfrac{d^2y}{dx^2} = \dfrac{-8-0}{2^4}$, which is < 0 $\quad\therefore$ max. value

$x = -1$, $\dfrac{d^2y}{dx^2} = \dfrac{8-0}{2^4}$, which is > 0 $\quad\therefore$ min. value **(3)**

(*c*)



(3)

(*d*) area $= \displaystyle\int_0^2 \frac{x}{1+x^2}\,dx = \left[\frac{1}{2}\ln(1+x^2)\right]_0^2 = \frac{1}{2}\ln 5$ **(3)**