Worked Solutions

Edexcel C4 Paper F

1. $\dfrac{dy}{dt} = \cos t, \dfrac{dx}{dt} = 2 + \sin t$

$\dfrac{dy}{dr} = \dfrac{\cos t}{2 + \sin t}$

stationary points where $\dfrac{dy}{dx} = 0.$

i.e. $\cos t = 0 \Rightarrow t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}.$

when $t = \dfrac{\pi}{2}, \quad x = 2 \cdot \dfrac{\pi}{2} - \cos\dfrac{\pi}{2} = \pi; \quad y = 2$

$t = \dfrac{3\pi}{2}, \quad x = 2 \cdot \dfrac{3\pi}{2} - \cos\dfrac{3\pi}{2} = 3\pi; \quad y = 1 + \sin\dfrac{3\pi}{2} = 0$

The stationary points are $(\pi, 2)$ and $(3\pi, 0)$ **(5)**

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2. (*a*) $f(x) = \dfrac{(2+x)(3-x) - (2-x)(3+x)}{(3+x)(3-x)} = \dfrac{2x}{9-x^2}$ **(2)**

(*b*) $f(x) = \dfrac{2x}{9\left(1 - \dfrac{x^2}{9}\right)} = \dfrac{2}{9}x\left(1 - \dfrac{x^2}{9}\right)^{-1}$

$= \dfrac{2}{9}x\left[1 + (-1)\left(-\dfrac{x^2}{9}\right) + \dfrac{(-1)(-2)}{2}\left(-\dfrac{x^2}{9}\right)^2 + \dots\right]$

$= \dfrac{2}{9}x\left[1 + \dfrac{1}{9}x^2 + \dfrac{1}{81}x^4 + \dots\right]$

$= \dfrac{2}{9}x + \dfrac{2}{81}x^3 + \dfrac{2}{729}x^5 + \dots$ **(4)**

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3. (*a*) $\dfrac{dm}{dt} = Ak\,e^{kt}$

when $t = 0, \quad m = 4: \qquad 4 = A\,e^0 \Rightarrow A = 4$

$t = 0, \quad \dfrac{dm}{dt} = 8: \qquad 8 = 4k \cdot e^0 \Rightarrow k = 2.$ **(5)**

(*b*) $m = 4\,e^{2t}$

$20 = 4\,e^{2t}$

$\ln 5 = 2t$

$t = \dfrac{1}{2}\ln 5 \qquad (= 0.8047\dots)$ **(3)**

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4. $2x + x\dfrac{dy}{dx} + y + 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(x + 2y) = -(2x + y)$

$\dfrac{dy}{dx} = -\dfrac{(2x+y)}{x+2y}$

at $(1, 2)$ gradient $= -\dfrac{4}{5}$

equation of tangent is $y - 2 = -\dfrac{4}{5}(x - 1)$

$5y + 4x = 14$ **(6)**

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5. (*a*) area $= \displaystyle\int_0^4 (2x+1)^{\frac{1}{2}} = \left[\frac{2}{3} \times \frac{1}{2}(2x+1)^{\frac{3}{2}}\right]_0^4 = \frac{1}{3} \cdot 9^{\frac{3}{2}} - \frac{1}{3} = 8\frac{2}{3}$ units^2 **(4)**

(*b*) volume $= \pi\displaystyle\int_0^4 (2x+1)dx = \pi\left[x^2 + x\right]_0^4 = 20\pi$ units^3 **(4)**

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6. (*a*) At point of intersection $2 + \lambda = -3 + 2\mu$

$$3 + 4\lambda = 4 + \mu$$

$$-1 + 2\lambda = -2 + \mu$$

Solving first two equations, $\lambda = 1$ and $\mu = 3$.

The point of intersection is at $(3, 7, 1)$. **(4)**

(*b*) let angle between $\begin{pmatrix}1\\4\\2\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ be θ.

$$\begin{pmatrix}1\\4\\2\end{pmatrix} \cdot \begin{pmatrix}2\\1\\1\end{pmatrix} = \sqrt{21} \times \sqrt{6}\cos\theta$$

$$2 + 4 + 2 = \sqrt{21}\sqrt{6}\cos\theta$$

$\theta = 44.54\ldots = 45°$, to the nearest degree **(4)**

(*c*) line meets the yz-plane at $x = 0$.

$\therefore \quad 2 + \lambda = 0, \quad \lambda = -2$

line meets plane at $\begin{pmatrix}0\\-5\\-5\end{pmatrix}$ **(2)**

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7. (*a*) $$\int_a^{a+h} (x^2 - a^2)\mathrm{d}x = \left[\frac{1}{3}x^3 - a^2x\right]_a^{a+h}$$

$$= \frac{1}{3}(a+h)^3 - a^2(a+h) - \left(\frac{1}{3}a^3 - a^3\right)$$

$$= \frac{1}{3}\left(a^3 + 3a^2h + 3ah^2 + h^3\right) - a^3 - a^2h - \frac{1}{3}a^3 + a^3$$

$$= \frac{1}{3}h^2(a+h)$$ **(4)**

(*b*) $\int \tan^2 x \ \mathrm{d}x = \int (\sec^2 x - 1)\mathrm{d}x = \tan x - x + c$ **(2)**

(*c*) $$\int_0^{\frac{\pi}{3}} x\sec^2 x = \int_1^{\frac{\pi}{3}} x\frac{\mathrm{d}}{\mathrm{d}x}(\tan x)\mathrm{d}x$$

$$= \left[x\tan x\right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x \ \mathrm{d}x$$

$$= \left[x\tan x + \ln\cos x\right]_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{3}\sqrt{3} + \ln\frac{1}{2} - (0 + \ln 1) = \frac{\pi}{3}\sqrt{3} - \ln 2$$ **(5)**

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8. (*a*)

x	0	1	2	3
$\frac{4}{x+3}$	$\frac{4}{3}$	1	$\frac{4}{5}$	$\frac{2}{3}$

integral $= \frac{1}{2}\left[\frac{4}{3} + \frac{2}{3} + 2\left(1 + \frac{4}{5}\right)\right] = \frac{14}{5}$ **(4)**

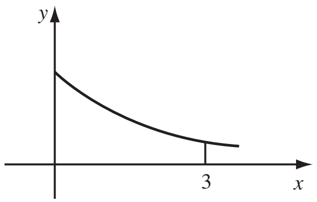
(*b*) $\int_0^3 \frac{4}{x+3}\,dx = \left[4\ln(x+3)\right]_0^3$

$= 4\ln 6 - 4\ln 3$

$= 4\ln 2$

$= \ln 16$ **(3)**

(*c*)



sum of trapezia > actual area under curve.

$\frac{14}{5} > \ln 16$ **(3)**

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9. (*a*) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx = \left[\ln \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \ln 1 - \ln \frac{1}{2}$

$= \ln 2$ **(4)**

(*b*) $\int \frac{1}{y}\ dy = \int \left(\frac{1-x}{1+x}\right) dx$

By division

$$\begin{array}{r} -1 \\ x+1\overline{)\,-x+1} \\ \underline{-x-1} \\ 2 \end{array}$$

$\int \frac{1}{y} dy = \int \left(-1 + \frac{2}{1+x}\right) dx$

$\ln y = -x + 2\ln(1+x) + c$

$y = 4, \quad x = 0:$

$\ln 4 = 2\ln 1 + c$

$c = \ln 4$

$\ln y = -x + 2\ln(1+x) + \ln 4$

$\ln y = \ln 4(1+x)^2 - x$ **(7)**
