Worked Solutions

Edexcel C4 Paper B

1. (*a*) $xy - 2y + 5x - 10 = 12$

$x\frac{dy}{dx} + y.1 - 2\frac{dy}{dx} + 5 = 0$

$\frac{dy}{dx}(x-2) = -(y+5)$

$\frac{dy}{dx} = \frac{y+5}{2-x}$ **(2)**

(*b*) at $(4, 1)$, $\frac{dy}{dx} = \frac{1+5}{2-4} = -3$

$\therefore$ gradient of normal $= \frac{1}{3}$

equation of normal is $y - 1 = \frac{1}{3}(x-4)$

$3y = x - 1$ **(3)**

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2. (*a*) $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$ **(1)**

(*b*) given $\frac{dr}{dt} = \frac{1}{4}$.

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$r = 3$, $\frac{dA}{dt} = (2\pi \times 3) \times \left(\frac{1}{4}\right) = \frac{3\pi}{2}$ $\text{cm}^2\ \text{s}^{-1}$ **(4)**

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3. (*a*) $1 + \frac{1}{2}(8x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(8x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3.2}(8x)^3 + \ldots$

$= 1 + 4x - 8x^2 + 32x^3$ **(3)**

(*b*) $-\frac{1}{8} < x < \frac{1}{8}$ **(1)**

(*c*) $(1+ax)(1+4x-8x^2+32x^3) = 1 + 4x - 8x^2 + 32x^3 + ax + 4ax^2 - 8ax^3$

$4 + a = -8 + 4a$

$12 = 3a$, $a = 4$

coef. of $x^3 = 32 - 8a = 32 - 32 = 0$ **(6)**

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4. (*a*) $t = 10$, $m = 500\,e^{-0.2} = 409.4$ grams **(1)**

(*b*) $m = 300$, $300 = 500\,e^{-0.02t}$

$e^{-0.02t} = \frac{3}{5}$

$\ln e^{-0.02t} = \ln\left(\frac{3}{5}\right)$

$-0.02t = \ln\frac{3}{5}$

$t = 25.5$ years **(2)**

(*c*) $\frac{dm}{dt} = 500 \times (-0.02)\,e^{-0.02t}$

when $t = 1$, $\frac{dm}{dt} = 500 \times (-0.02) \times e^{-0.02} = -9.8$ g/year

mass is decreasing at 9.8 g/year **(3)**

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5. (*a*) (i) $\int (1 + e^x + e^{-x} + 1)dx = 2x + e^x - e^{-x} + c$ **(2)**

(ii) $\int (6x-1)^{-\frac{1}{2}}\ dx = 2 \times \frac{1}{6}(6x-1)^{\frac{1}{2}} + c = \frac{1}{3}\sqrt{6x-1} + c$ **(2)**

(*b*) $\int_0^{\frac{\pi}{6}} x\cos x\ dx = \int_0^{\frac{\pi}{6}} x\frac{d}{dx}(\sin x)\ dx$

$= \left[x\sin x\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x\ dx$

$= \frac{\pi}{6}\cdot\frac{1}{2} - 0 + \left[\cos x\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$ **(5)**

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6. (*a*) $\frac{dy}{d\theta} = 2.2\cos\theta(-\sin\theta)$

$\frac{dx}{d\theta} = \cos\theta$

$\frac{dy}{dx} = \frac{-4\sin\theta\cos\theta}{\cos\theta} = -4\sin\theta$

at $\theta = \frac{\pi}{6}$, gradient of tangent $= -4\cdot\frac{1}{2} = -2$

at $\theta = \frac{\pi}{6}, x = \frac{1}{2}, y = 2\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{2}$

equation of tangent is $y - \frac{3}{2} = -2\left(x - \frac{1}{2}\right)$

$2y + 4x = 5$ **(5)**

(*b*) $y = 2\cos^2\theta = 2(1 - \sin^2\theta)$

$y = 2(1 - x^2)$ **(2)**

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7. (*a*) $\frac{1}{(y-1)y} = \frac{1}{y-1} - \frac{1}{y}$ **(3)**

(*b*) $\int\left(\frac{1}{y-1} - \frac{1}{y}\right)dy = \int \cos x\ dx$

$\ln(y-1) - \ln y = \sin x + c$

$y = 5, \quad x = 0, \qquad \ln 4 - \ln 5 = 0 + c$

$\ln\frac{4}{5} = c$

$\therefore \quad \ln\frac{y-1}{y} = \sin x + \ln\frac{4}{5}.$

$\ln\frac{y-1}{y} - \ln\frac{4}{5} = \sin x$

$\ln\frac{5(y-1)}{4y} = \sin x$

$\frac{5y-5}{4y} = e^{\sin x}$

$5y - 5 = 4y\ e^{\sin x}$

$y\left(5 - 4e^{\sin x}\right) = 5$

$y = \frac{5}{5 - 4e^{\sin x}}$ **(7)**

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8. (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

equation of AB is $r = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ **(3)**

(b) $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$, $\quad \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ -7 \\ -3 \end{pmatrix} = 12 - 12 = 0$

so $12\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ is perpendicular to AC. **(2)**

(c) let $\angle BAC = \theta$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = \left|\overrightarrow{AB}\right| \times \left|\overrightarrow{AC}\right| \cos\theta$$

$$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \sqrt{2^2 + 3^2 + 1^2} \times \sqrt{1^2 + 4^2} \cos\theta$$

$$6 = \sqrt{14}\sqrt{17} \cos\theta$$

$$\cos\theta = \frac{6}{\sqrt{14}\sqrt{17}} \qquad \theta = 67°$$ **(4)**

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9. (*a*) At $A \quad x \ln x = 0$

$\therefore \quad \ln x = 0 \qquad (x \neq 0) \qquad x = 1$ **(1)**

(*b*) $\dfrac{dy}{dx} = x \cdot \dfrac{1}{x} + \ln x = 1 + \ln x$

at P $\dfrac{dy}{dx} = 0, \qquad \ln x = -1$

$x = e^{-1} = \dfrac{1}{e}$ and $y = \dfrac{1}{e} \ln e^{-1} = -\dfrac{1}{e}$

(c) area $= \displaystyle\int_{\frac{1}{e}}^{1} x \ln x \; dx = \int_{\frac{1}{e}}^{1} \ln x \frac{d}{dx}\left(\frac{x^2}{2}\right) dx$ (By parts)

$$= \left[\frac{x^2}{2} \ln x\right]_{\frac{1}{e}}^{1} - \int_{\frac{1}{e}}^{1} \frac{x^2}{2} \cdot \frac{1}{x} \; dx$$

$$= 0 - \frac{1}{2e^2} \ln e^{-1} - \left[\frac{x^2}{4}\right]_{\frac{1}{e}}^{1} = \frac{1}{2e^2} - \left[\frac{1}{4} - \frac{1}{4e^2}\right] = \frac{3}{4e^2} - \frac{1}{4}$$ **(6)**
