



1.  $f(x) = (1 + 3x)^{-1}, |x| < \frac{1}{3}.$

(a) Expand  $f(x)$  in ascending powers of  $x$  up to and including the term in  $x^3$ . (3)

(b) Hence show that, for small  $x$ ,

$$\frac{1+x}{1+3x} \approx 1 - 2x + 6x^2 - 18x^3. \quad (2)$$

(c) Taking a suitable value for  $x$ , which should be stated, use the series expansion in part (b) to find an approximate value for  $\frac{101}{103}$ , giving your answer to 5 decimal places. (3)

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2. (a) Express  $\frac{13 - 2x}{(2x - 3)(x + 1)}$  in partial fractions.

(4)

(b) Given that  $y = 4$  at  $x = 2$ , use your answer to part (a) to find the solution of the differential equation

$$\frac{dy}{dx} = \frac{y(13 - 2x)}{(2x - 3)(x + 1)}, \quad x > 1.5$$

Express your answer in the form  $y = f(x)$ .

(7)

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3.

Figure 2

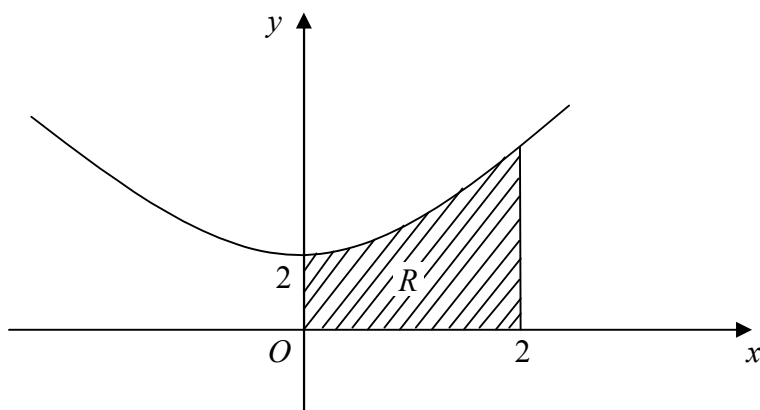


Figure 2 shows part of the curve with equation  $y = x^2 + 2$ .

The finite region  $R$  is bounded by the curve, the  $x$ -axis and the lines  $x = 0$  and  $x = 2$ .

- (a) Use the trapezium rule with 4 strips of equal width to estimate the area of  $R$ . (5)
- (b) State, with a reason, whether your answer in part (a) is an under-estimate or over-estimate of the area of  $R$ . (1)
- (c) Using integration, find the volume of the solid generated when  $R$  is rotated through  $360^\circ$  about the  $x$ -axis, giving your answer in terms of  $\pi$ . (6)
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4. A Pancho car has value  $\text{£}V$  at time  $t$  years. A model for  $V$  assumes that the rate of decrease of  $V$  at time  $t$  is proportional to  $V$ .

- (a) By forming and solving an appropriate differential equation, show that  $V = Ae^{-kt}$ , where  $A$  and  $k$  are positive constants. (3)

The value of a new Pancho car is  $\text{£}20\,000$ , and when it is 3 years old its value is  $\text{£}11\,000$ .

- (b) Find, to the nearest  $\text{£}100$ , an estimate for the value of the Pancho when it is 10 years old. (5)

A Pancho car is regarded as ‘scrap’ when its value falls below  $\text{£}500$ .

- (c) Find the approximate age of the Pancho when it becomes ‘scrap’. (3)
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5.

Figure 1

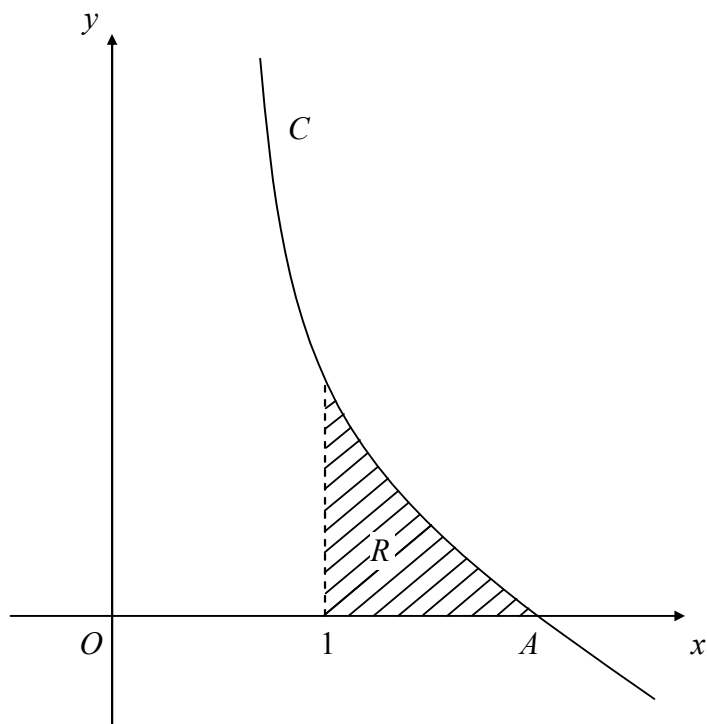


Figure 1 shows the curve  $C$  with equation  $y = f(x)$ , where

$$f(x) = \frac{8}{x} - x^2, \quad x > 0.$$

Given that  $C$  crosses the  $x$ -axis at the point  $A$ ,

(a) find the coordinates of  $A$ .

(2)

The finite region  $R$ , bounded by  $C$ , the  $x$ -axis and the line  $x = 1$ , is rotated through  $2\pi$  radians about the  $x$ -axis.

(b) Use integration to find, in terms of  $\pi$ , the volume of the solid generated.

(7)

6. Referred to a fixed origin  $O$ , the points  $A$  and  $B$  have position vectors  $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$  and  $(5\mathbf{i} - 3\mathbf{j})$  respectively.

(a) Find, in vector form, an equation of the line  $l_1$  which passes through  $A$  and  $B$ . (2)

The line  $l_2$  has equation  $\mathbf{r} = (4\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ , where  $\mu$  is a scalar parameter.

(b) Show that  $A$  lies on  $l_2$ . (1)

(c) Find, in degrees, the acute angle between the lines  $l_1$  and  $l_2$ . (4)

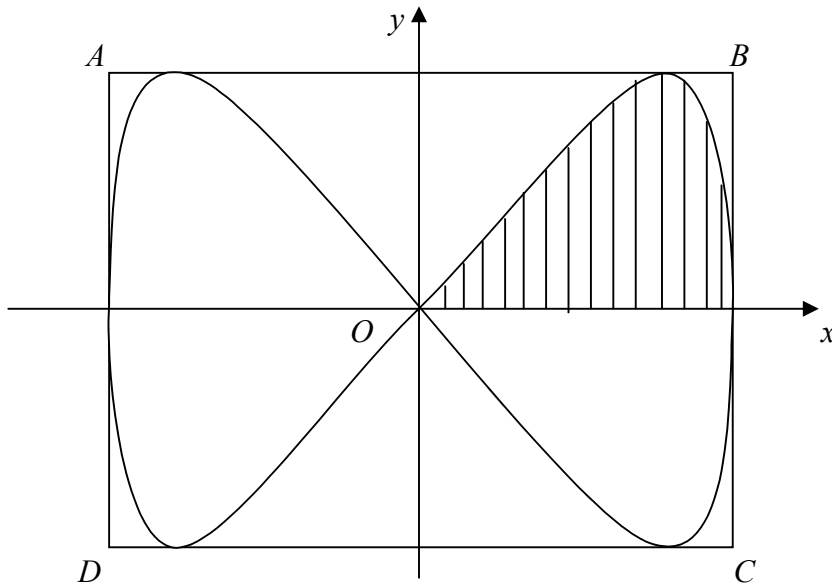
The point  $C$  with position vector  $(2\mathbf{i} - \mathbf{k})$  lies on  $l_2$ .

(d) Find the shortest distance from  $C$  to the line  $l_1$ . (4)

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7.

Figure 2



Part of the design of a stained glass window is shown in Fig. 2. The two loops enclose an area of blue glass. The remaining area within the rectangle  $ABCD$  is red glass.

The loops are described by the curve with parametric equations

$$x = 3 \cos t, \quad y = 9 \sin 2t, \quad 0 \leq t < 2\pi.$$

(a) Find the cartesian equation of the curve in the form  $y^2 = f(x)$ .

(4)

(b) Show that the shaded area in Fig. 2, enclosed by the curve and the  $x$ -axis, is given by

$$\int_0^{\frac{\pi}{2}} A \sin 2t \sin t \, dt, \text{ stating the value of the constant } A.$$

(3)

(c) Find the value of this integral.

(4)

The sides of the rectangle  $ABCD$ , in Fig. 2, are the tangents to the curve that are parallel to the coordinate axes. Given that 1 unit on each axis represents 1 cm,

(d) find the total area of the red glass.

(4)

END