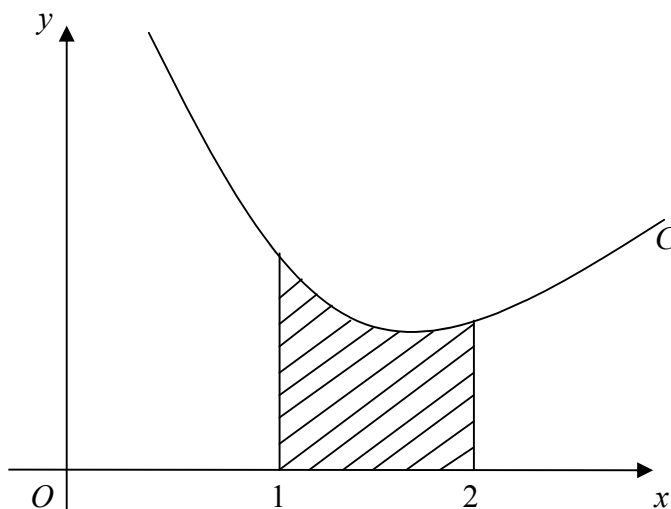




1. The curve  $C$  has equation  $5x^2 + 2xy - 3y^2 + 3 = 0$ . The point  $P$  on the curve  $C$  has coordinates  $(1, 2)$ .
- (a) Find the gradient of the curve at  $P$ . (5)
- (b) Find the equation of the normal to the curve  $C$  at  $P$ , in the form  $y = ax + b$ , where  $a$  and  $b$  are constants. (3)
- 

2. **Figure 1**



In Fig. 1, the curve  $C$  has equation  $y = f(x)$ , where

$$f(x) = x + \frac{2}{x^2}, \quad x > 0.$$

The shaded region is bounded by  $C$ , the  $x$ -axis and the lines with equations  $x = 1$  and  $x = 2$ . The shaded region is rotated through  $2\pi$  radians about the  $x$ -axis.

Using calculus, calculate the volume of the solid generated. Give your answer in the form  $\pi(a + \ln b)$ , where  $a$  and  $b$  are constants. (8)

---

3.

Figure 2

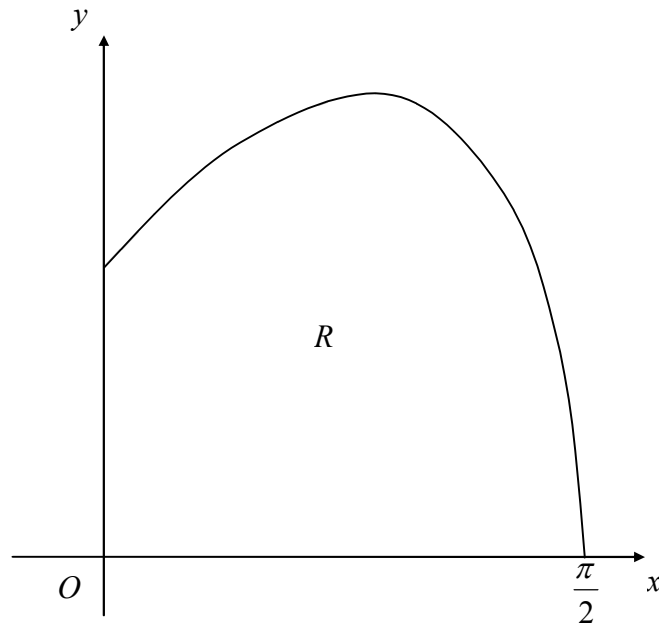


Figure 2 shows part of the curve with equation

$$y = e^x \cos x, \quad 0 \leq x \leq \frac{\pi}{2}.$$

The finite region  $R$  is bounded by the curve and the coordinate axes.

(a) Calculate, to 2 decimal places, the  $y$ -coordinates of the points on the curve where  $x = 0, \frac{\pi}{6}, \frac{\pi}{3}$  and  $\frac{\pi}{2}$ . (3)

(b) Using the trapezium rule and all the values calculated in part (a), find an approximation for the area of  $R$ . (4)

(c) State, with a reason, whether your approximation underestimates or overestimates the area of  $R$ . (2)

---

4. A curve is given parametrically by the equations

$$x = 5 \cos t, \quad y = -2 + 4 \sin t, \quad 0 \leq t < 2\pi.$$

(a) Find the coordinates of all the points at which  $C$  intersects the coordinate axes, giving your answers in surd form where appropriate. (4)

(b) Sketch the graph of  $C$ . (2)

$P$  is the point on  $C$  where  $t = \frac{1}{6}\pi$ .

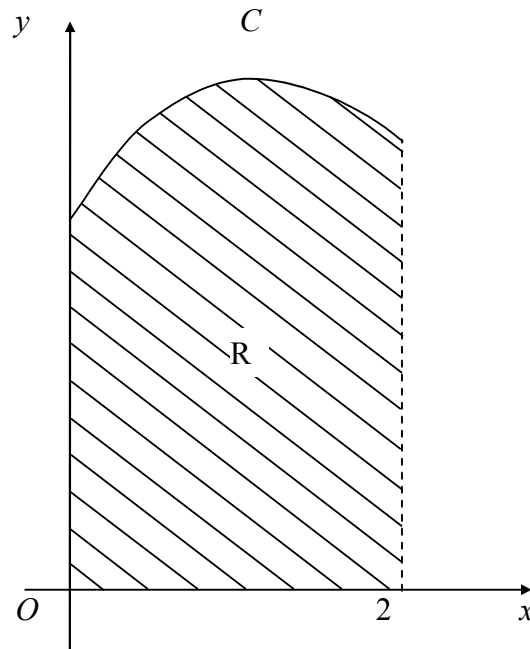
(c) Show that the normal to  $C$  at  $P$  has equation

$$8\sqrt{3}y = 10x - 25\sqrt{3}. \quad (4)$$

---

5.

Figure 1



The curve  $C$  has equation  $y = f(x)$ ,  $x \in \mathbb{R}$ . Figure 1 shows the part of  $C$  for which  $0 \leq x \leq 2$ .

Given that

$$\frac{dy}{dx} = e^x - 2x^2,$$

and that  $C$  has a single maximum, at  $x = k$ ,

(a) show that  $1.48 < k < 1.49$ .

(3)

Given also that the point  $(0, 5)$  lies on  $C$ ,

(b) find  $f(x)$ .

(4)

The finite region  $R$  is bounded by  $C$ , the coordinate axes and the line  $x = 2$ .

(c) Use integration to find the exact area of  $R$ .

(4)

6. When  $(1 + ax)^n$  is expanded as a series in ascending powers of  $x$ , the coefficients of  $x$  and  $x^2$  are  $-6$  and  $27$  respectively.
- (a) Find the value of  $a$  and the value of  $n$ . (5)
- (b) Find the coefficient of  $x^3$ . (2)
- (c) State the set of values of  $x$  for which the expansion is valid. (1)
- 

7. Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines,  $l_1$  and  $l_2$ , along which they travel are

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

and  $\mathbf{r} = 9\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mu(4\mathbf{i} + \mathbf{j} - \mathbf{k}),$

where  $\lambda$  and  $\mu$  are scalars.

- (a) Show that the submarines are moving in perpendicular directions. (2)
- (b) Given that  $l_1$  and  $l_2$  intersect at the point  $A$ , find the position vector of  $A$ . (5)

The point  $b$  has position vector  $10\mathbf{j} - 11\mathbf{k}$ .

- (c) Show that only one of the submarines passes through the point  $B$ . (3)
- (d) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance  $AB$ . (2)
-

8. In a chemical reaction two substances combine to form a third substance. At time  $t$ ,  $t \geq 0$ , the concentration of this third substance is  $x$  and the reaction is modelled by the differential equation

$$\frac{dx}{dt} = k(1 - 2x)(1 - 4x), \text{ where } k \text{ is a positive constant.}$$

- (a) Solve this differential equation and hence show that

$$\ln \left| \frac{1 - 2x}{1 - 4x} \right| = 2kt + c, \text{ where } c \text{ is an arbitrary constant.} \quad (7)$$

- (b) Given that  $x = 0$  when  $t = 0$ , find an expression for  $x$  in terms of  $k$  and  $t$ . (4)

- (c) Find the limiting value of the concentration  $x$  as  $t$  becomes very large. (2)

---

**END**