

Question Number	Scheme	Marks
1. (a)	$10x, +(2y + 2x \frac{dy}{dx}), -6y \frac{dy}{dx} = 0$ At (1, 2) $10 + (4 + 2 \frac{dy}{dx}) - 12 \frac{dy}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{14}{10} = 1.4 \text{ or } \frac{7}{5} \text{ or } 1\frac{2}{5}$	M1, (B1), A1 M1 A1 (5)
(b)	The gradient of the normal is $-\frac{5}{7}$ Its equation is $y - 2 = -\frac{5}{7}(x - 1)$ (allow tangent) $y = -\frac{5}{7}x + 2\frac{5}{7} \text{ or } y = -\frac{5}{7}x + \frac{19}{7}$	M1 M1 A1cao (3)
		(8 marks)
2.	$[f(x)]^2 = x^2 + \frac{4}{x^4} + \frac{4}{x}$ $\int [f(x)]^2 dx = \left[\frac{x^3}{3} - \frac{4}{3x^3} + 4 \ln x \right]$ $\int_1^2 [f(x)]^2 dx = \left(\frac{8}{3} - \frac{4}{24} + 4 \ln 2 \right) - \left(\frac{1}{3} - \frac{4}{3} + 4 \ln 1 \right)$ $\left(= \frac{7}{2} + 4 \ln 2 \right)$ $V = \pi \int_1^2 [f(x)]^2 dx \Rightarrow V = \pi \left(\frac{7}{2} + \ln 16 \right) \text{ or } a = \frac{7}{2}, b = 16$	M1 M1 A1 B1 M1 M1 A1 A1
		(8 marks)

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3.	<p>(a) $x \quad 0 \quad \frac{\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2}$</p> <p>$y \quad 1 \quad 1.46 \quad 1.42 \quad 0$</p> <p>(b) $I \approx \frac{1}{2} \left(\frac{\pi}{6} \right) \dots$ $\approx \dots (1 + 2(1.46 + 1.42) + 0)$ ≈ 1.8</p> <p>(c) underestimates diagram or explanation</p> <p>NB. Exact answer is $\frac{1}{2} \left(e^{\frac{\pi}{2}} - 1 \right) \approx 1.905\dots$</p>	1, 0 1.146, 1.42 B1, B1 (3) B1 M1 A1 ft A1 (4) B1 B1 (2) (9 marks)
4.	<p>(a) $5 \cos t = 0 : t = \frac{\pi}{2}, y = -2 \quad (0, -2)$ $t = \frac{3\pi}{2}, y = -6 \quad (0, -6)$</p> <p>$4 \sin t = 2 : t = \frac{\pi}{6}, x = \frac{5\sqrt{3}}{2} \quad t = \frac{5\pi}{6}, x = -\frac{5\sqrt{3}}{2} \quad (\pm \frac{5\sqrt{3}}{2}, 0)$</p> <p>(b) </p> <p>$\frac{dx}{dt} = -5 \sin t, \quad \frac{dy}{dt} = 4 \cos t, \quad \frac{dy}{dx} = \frac{-4 \cos t}{5 \sin t}$</p> <p>(c) $y = \frac{5 \tan \frac{\pi}{6}}{4} \left(x - \frac{5\sqrt{3}}{2} \right), \quad \text{i.e. } 8\sqrt{3}y = 10x - 25\sqrt{3} \quad \star$</p>	M1 A1 M1 A1 (4) shape position B1 B1 (2) M1 A1 M1 A1 (4) (10 marks)

Question Number	Scheme	Marks
5. (a)	$f'(x) = 0$ for maximum (or stationary point or turning point) $f'(1.48) = e^{1.48} - 2 \times 1.48^2 = 0.0121\dots$ $f'(1.49) = \dots = -0.0031\dots$ change of sign \therefore root / maximum in range	B1 M1 A1 (3)
(b)	$y = e^x - \frac{2}{3}x^3 (+c)$ at $(0, 5)$ $5 = e^0 - 0 + c$ $c = 4$ $\left(y = e^x - \frac{2}{3}x^3 + 4 \right)$ $(c=4)$	M1 A1 M1 A1 (4)
(c)	Area $= \int_0^2 \left(e^x - \frac{2}{3}x^3 + 4 \right) dx$ $= \left[e^x - \frac{2}{12}x^4 + 4x \right]_0^2$ $= \left(e^2 - \frac{16}{6} + 8 \right) - (e^0 - 0 + 0)$ $= e^2 + 4\frac{1}{3}$ or $e^2 + \frac{13}{3}$	M1 A1 M1 A1 cao (4)
		(11 marks)

Question Number	Scheme	Marks
6. (a)	$na = -6, \quad \frac{n(n-1)}{2}a^2 = 27$ Attempts solution by eliminating variable e.g. $\frac{n(n-1)36}{n^2} = 54$ or $-\frac{6}{a}(-\frac{6}{a}-1)a^2 = 54$ $n = -2, \quad a = 3$	B1 B1 M1 A1 A1 (5)
(b)	$\frac{(-2)(-3)(-4)3^3}{6} = -108$ for M1 allow a instead of a^3	M1 A1 (2)
(c)	$ x < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$	B1 ft (1)
7. (a)	$1 \times 4 - 2 \times 1 - 2 \times 1 = 0$, i.e. $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = 0$, therefore perpendicular.	M1 A1 (2)
(b)	$3 + \lambda = 9 + 4\mu$ and either $4 - 2\lambda = 1 + \mu$ or $-5 + 2\lambda = -2 - \mu$ Eliminate to obtain $\mu = -1$ or $\lambda = 2$	M1 A1 M1 A1
	Point is $(5, 0, -1)$ Vector is $\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$	B1 (5)
(c)	$\lambda = -3 \Rightarrow$ point lies on first line l_1 show contradiction for $\mu \Rightarrow$ point not on l_2	M1 A1 B1 (3)
(d)	$\sqrt{(5^2 + 10^2 + 10^2)} = 15 \Rightarrow 1.5 \text{ km}$	M1 A1 (2)
		(12 marks)

Question Number	Scheme	Marks
8. (a)	$\int \frac{dx}{(1-2x)(1-4x)} = \int k dt$ $\int \frac{-1}{1-2x} + \frac{2}{1-4x} dx = \int k dt$ $\frac{1}{2} \ln(1-2x) - \frac{1}{2} \ln(1-4x) = kt (+\frac{1}{2}c)$ $\ln \frac{1-2x}{1-4x} = 2kt + c \quad (*)$	B1 M1 A1 M1 A1 A1 A1 cso (7)
(b)	Use $x=0$ when $t=0 \Rightarrow c=0$ $\therefore \frac{1-2x}{1-4x} = e^{2kt}$ $\therefore x(4e^{2kt}-2) = e^{2kt}-1, \quad \therefore x = \frac{e^{2kt}-1}{4e^{2kt}-2}$	B1 M1 M1 A1 (4)
(c)	As $t \rightarrow \infty, x \rightarrow \frac{1}{4}$	M1 A1 (2)
		(13 marks)