C4 June 2004 Mark Scheme

1. $3x^2 - 2y^2 + 2x - 3y + 5 = 0.$

$6x + 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$

$6(0)+4(1)\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -2$, gradient $\frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 1)$

$x - 2y - 1 = 0$

2. $f(x) = \frac{3x \quad 1}{(1 \quad 2x)^2}, \qquad x \quad < \tfrac{1}{2}.$

$(a)\ \frac{3x \quad 1}{(1 \quad 2x)^2} = \frac{A}{(1 \quad 2x)} + \frac{B}{(1 \quad 2x)^2} \Rightarrow 3x\text{-}1 = A(1\text{-}2x)+B$

Using two values of x $A = \frac{3}{2}$, $B = \frac{1}{2}$

(b) 1st method:

$f(x) = \frac{3}{2} \ 1 \quad 2x \ ^{1} + \frac{1}{2} \ 1 \quad 2x \ ^{2}$

$\frac{3}{2} \ 1 \quad 2x \ ^{1} = \frac{3}{2} \ 1 \quad 2x \quad 4x^2 \quad 8x^3 \quad \ldots = \frac{3}{2}\text{-}3x\text{-}6x^2\text{-}12x^3 \quad \ldots$

$\frac{1}{2} \ 1 \quad 2x \ ^{2} = \frac{1}{2} \ 1 \quad 4x \quad 12x^2 \quad 32x^3 \quad \ldots = \frac{1}{2} \quad 2x \quad 6x^2 \quad 16x^3 \quad \ldots$

$f(x) = \frac{3}{2}\text{-}3x\text{-}6x^2\text{-}12x^3 \quad \ldots + \frac{1}{2} \quad 2x \quad 6x^2 \quad 16x^3 \quad \ldots = -1 - x + 4x^3 + \ldots$

2nd method:

$f(x) = 3x \quad 1 \ 1 \quad 2x \ ^{2} = 3x \quad 1 \ 1 \quad 4x \quad 12x^2 \quad 32x^3 \quad \ldots$

$= 3x + 12x^2 + 36x^3 + \ldots - 1 - 4x - 12x^2 - 32x^3 + \ldots$

$= -1 - x + 4x^3 + \ldots$

3. $y = 3\sin\frac{x}{2}, 0 \quad x \quad 2$

$(a)\ A \ \ _0^2 \ 3\sin\frac{x}{2}dx \quad 6 \quad \cos\frac{x}{2} \ _0^2 \quad 6(\ \cos\ ) \quad 6(\ \cos 0) \quad 12s.u.$

$(b)\ \ 3\sin\frac{x}{2} \ ^2 dx \quad 9 \ \sin^2\frac{x}{2}dx \quad \frac{9}{2} \quad 1 \quad 2\cos x\, dx \quad \frac{9}{2} \ x \quad 2\sin x \quad c$

$V \ \ _0^2 \ 3\sin\frac{x}{2} \ ^2 dx \quad \frac{9}{2} \ x \quad 2\sin x \ _0^2 \quad \frac{9}{2}(0 \quad 2\sin 0) \quad (2 \quad 2\sin 2\ ) \quad 9 \ ^2 c.u.$

4. $x = \sin t,\ \ y = \sin\ \ t\ \ \frac{}{6}\ ,\qquad \frac{}{2} < t < \frac{}{2}.$

$(a)\ \ t = \frac{}{6} \Rightarrow x = \sin\frac{}{6} = \frac{1}{2},\ y = \sin\ \ \frac{}{6}\ \ \frac{}{6} = \frac{\sqrt{3}}{2}$

$$\frac{dy}{dx}\ \ \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\ \ \frac{\cos\ t\ \ \frac{}{6}}{\cos t},\ \ \left.\frac{dy}{dx}\right|_{t\ \frac{}{6}}\ \ \frac{\cos\ \frac{}{6}\ \ \frac{}{6}}{\cos\ \frac{}{6}}\ \ \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\ \ \frac{\sqrt{3}}{3}$$

$$y\ \ \frac{\sqrt{3}}{2}\ \ \frac{\sqrt{3}}{3}\ x\ \ \frac{1}{2}\qquad y\ \ \frac{\sqrt{3}}{3}x\ \ \frac{\sqrt{3}}{3}$$

$(b)\ \ \ y = \sin\ \ t\ \ \frac{}{6}\ \ \ \sin t\cos\frac{}{6}\ \ \cos t\sin\frac{}{6}\ \ \frac{\sqrt{3}}{2}\sin t\ \ \frac{1}{2}\sqrt{1\ \ \sin^2 t}$

$$y = \frac{3}{2}x + \frac{1}{2}\ \ (1 - x^2),\qquad -1 < x < 1.$$

5. $A(0, a, b)$

$$r\ \ \begin{matrix}6\\19\\1\end{matrix}\qquad\begin{matrix}1\\4\\2\end{matrix}$$

$$(a)\ \ \overrightarrow{OA}\ \ \begin{matrix}6\\19\\1\end{matrix}\qquad\begin{matrix}1\\4\\2\end{matrix}\qquad\begin{matrix}0\\a\\b\end{matrix}$$

0 + λ = 6
a = 19 + 4 (-6) = -5
b = -1-2(-6) = 11

$$(b)\ \ \begin{matrix}6\\19\ \ \ 4\\1\ \ \ 2\end{matrix}\qquad\begin{matrix}1\\4\\2\end{matrix}\qquad 0$$

1(6 + λ) + 4(19 + 4λ) - 2(-1-2λ) = 0
λ = -4

$$\overrightarrow{OP}\ \ \begin{matrix}2\\3\\7\end{matrix}$$

(c) B (5, 15, 1),

$$\overrightarrow{OB} \quad \begin{matrix} 5 \\ 15 \\ 1 \end{matrix}$$

(6 + λ) = 5 ⇒ λ = -1

(19 + 4λ) = 15 ⇒ λ = -1

(-1-2λ) = 1 ⇒ λ = -1

⇒ B lies on l_i ⇒ A, P and B are collinear

$$|\overrightarrow{AP}| \quad \sqrt{(0 \quad 2)^2 \quad (5 \quad 3)^2 \quad (11 \quad 7)^2} \quad \sqrt{84}$$

$$|\overrightarrow{PB}| \quad \sqrt{(5 \quad 2)^2 \quad (15 \quad 3)^2 \quad (1 \quad 7)^2} \quad \sqrt{189}$$

$$\frac{AP}{PB} \quad \frac{\sqrt{84}}{\sqrt{189}} \quad \frac{2}{3} \qquad AP : PB \quad 2:3$$

6. $y = (x - 1) \ln x, \; x > 0.$

(a)

x	1	1.5	2	2.5	3
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3

$$I = {}_1^3 \; (x \quad 1) \ln x \quad dx$$

(b) (i) $I = {}_1^3 \; (x \quad 1) \ln x \quad dx \quad \frac{1}{2} \quad 1 \quad 0 \quad 2\ln 3 \quad 2\ln 2 \quad 1{,}792 \quad (4s.f.)$

(ii) ${}_1^3 \; (x \quad 1) \ln x \quad dx \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 2\ln 3 \quad 2 \; 0.5\ln 1.5 \quad 1.5\ln 2.5 \quad \ln 2 \quad 1{,}684 \quad (4s.f.)$

(d)

$$(x \quad 1) \ln x \quad dx \quad \frac{x^2}{2} \quad x \quad \ln x \qquad \frac{x^2}{2} \quad x \quad \frac{1}{x} dx \quad \frac{x(x \quad 2)}{2} \ln x \quad \frac{1}{2} \quad x \quad 2 \quad dx$$

$$\frac{x(x \quad 2)}{2} \ln x \quad \frac{1}{2} \; \frac{x^2}{2} \quad 2x \quad c \qquad \frac{x(x \quad 2)}{2} \ln x \quad \frac{1}{2} \; \frac{x(x \quad 4)}{2} \quad c$$

$${}_1^3 \; (x \quad 1) \ln x \quad dx \qquad \frac{x(x \quad 2)}{2} \ln x \quad \frac{1}{2} \; \frac{x(x \quad 4)}{2} \; {}_1^3$$

$$\frac{3(3 \quad 2)}{2} \ln 3 \quad \frac{1}{2} \; \frac{3(3 \quad 4)}{2} \qquad \frac{1(1 \quad 2)}{2} \ln 1 \quad \frac{1}{2} \; \frac{1(1 \quad 4)}{2} \qquad \frac{3}{2}\ln 3 \quad \frac{3}{2} \quad \frac{3}{2} \quad \frac{3}{2}\ln 3$$

7. (*a*) $\frac{dA}{dt} = 8$

$A = 6x^2 \Rightarrow \frac{dA}{dx} = 12x$

$\frac{dx}{dt} = \frac{dx}{dA} \quad \frac{dA}{dt} = \frac{1}{12x} \quad 8 = \frac{\frac{2}{3}}{x}$

(*b*) $V = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dV}{dt} = \frac{dV}{dt} \quad \frac{dx}{dt} = 3x^2 \quad \frac{\frac{2}{3}}{x} = 2x = 2V^{\frac{1}{3}}$

(*c*) $\frac{dV}{dt} = 2V^{\frac{1}{3}} \Rightarrow \quad V^{\frac{1}{3}} dV \quad 2 \;\; dt \qquad \frac{3}{2}V^{\frac{2}{3}} \quad 2t \quad c$

$V = 8$ when $t = 0 \Rightarrow \frac{3}{2}\,\sqrt[3]{8}^{\;2} \quad 2(0) \quad c \qquad c \quad 3$

$\frac{3}{2}V^{\frac{2}{3}} \quad 2t \quad 3$

$V = 16 \;\; 2 \Rightarrow \frac{3}{2}\sqrt[3]{16\sqrt{2}}^{\;2} \quad 2t \quad c \qquad 12 \quad 2t \quad 3 \qquad 4.5s$