Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Gold Level (Hard) G1

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
66	59	48	43	37	29

1. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x \, \mathrm{d}x$.

(6)

January 2011

2.

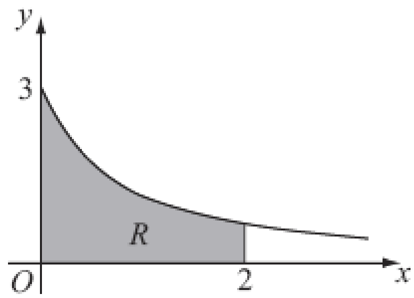

Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

January 2009

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3. Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}} \, \mathrm{d}x$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

June 2013 (R)

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4. A curve C has parametric equations

$$x = 2\sin t, \qquad y = 1 - \cos 2t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $t = \frac{\pi}{6}$.

(4)

(*b*) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), \qquad -k \leq x \leq k,$$

stating the value of the constant k.

(3)

(*c*) Write down the range of f(x).

(2)

June 2013

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5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(*a*) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(*b*) Find the cosine of the acute angle between AB and l_1.

(6)

June 2007

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6. (*a*) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = \int \frac{2}{u(2u-1)}\,\mathrm{d}u$$

(3)

(*b*) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

June 2013

7.

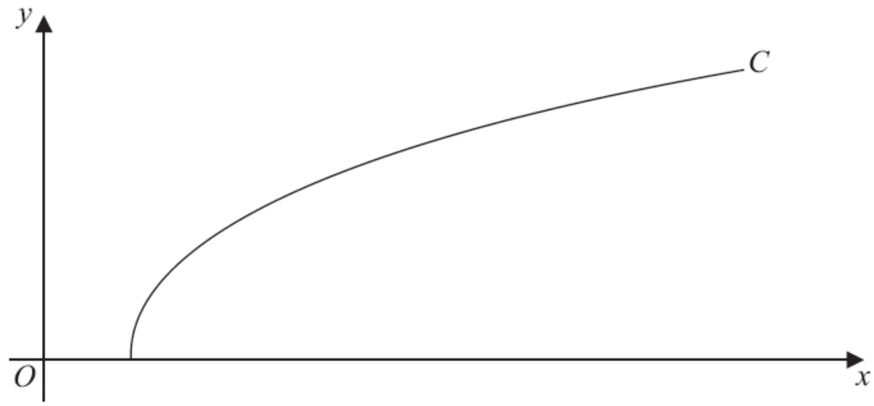

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$.

(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad a \leq x \leq b$$

stating values of a and b.

(3)

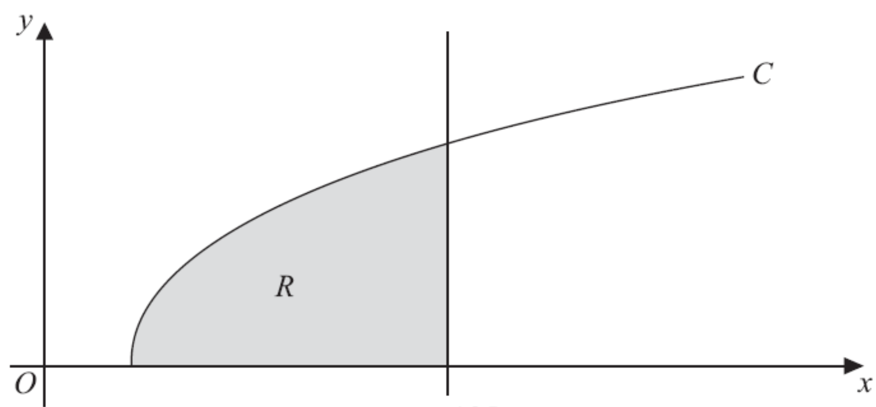

Figure 3

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.

(5)

June 2013 (R)

8.

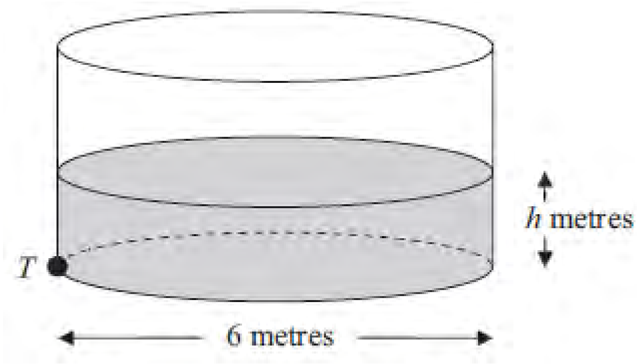

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that, t minutes after the tap has been opened,

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h).$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

June 2010

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\int x\sin 2x\,\mathrm{d}x = -\dfrac{x\cos 2x}{2} + \int \dfrac{\cos 2x}{2}\,\mathrm{d}x$	M1 A1 A1
	$= \quad \ldots \quad + \dfrac{\sin 2x}{4}$	M1
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$	M1 A1
		[6]

Question Number	Scheme		Marks
2 (a)	Area$(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1
		Correct integration. Ignore limits.	A1
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\text{ (units)}^2$	$\underline{3}$	A1
			(4)
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2}\,\mathrm{d}x$	Use of $V = \underline{\pi\int y^2}\,\mathrm{d}x$. Can be implied. Ignore limits and $\mathrm{d}x$.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$		
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$	M1
		$\frac{9}{4}\ln\lvert 1+4x\rvert$	A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	Note that $\ln 1$ can be implied as equal to 0.		
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.	Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	(5)
			[9]

Question Number	Scheme		Marks
3.	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\ ,\ u = 2+\sqrt{(2x+1)}$		
		Either	
	$\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = u-2$	$\frac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$	M1
		Either $\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \int \frac{1}{u}(u-2)\,\mathrm{d}u$	Correct substitution (Ignore integral sign and $\mathrm{d}u$).	A1
	$= \int\left(1-\frac{2}{u}\right)\mathrm{d}u$	An attempt to divide each term by u.	dM1
	$= u - 2\ln u$	$\pm Au \pm B\ln u$	ddM1
		$u - 2\ln u$	A1 ft
	$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 **cao cso**
			[8]

Question Number	Scheme		Marks
4.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$ At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
		Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			(4)
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			(3)
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$		B1 B1
			(2)
			[9]

Question Number	Scheme		Marks
5. (a)	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of $\mathbf{i}: \quad 1+\lambda=1+2\mu \quad (1)$ $\mathbf{j}: \quad \lambda=3+\mu \quad (2)$ $\mathbf{k}: \quad -1=6-\mu \quad (3)$	Writes down any two of these equations correctly.	M1
	(1) & (2) yields $\lambda=6,\ \mu=3$ (1) & (3) yields $\lambda=14,\ \mu=7$ (2) & (3) yields $\lambda=10,\ \mu=7$	Solves two of the above equations to find … either one of λ or μ correct both λ and μ correct	 A1 A1
	Either checking eqn (3), $-1\neq 3$ checking eqn (2), $14\neq 10$ checking eqn (1), $11\neq 15$	Complete method of putting their values of λ and μ into a third equation to show a contradiction.	B1 $\sqrt{}$
	or for example: checking eqn (3), LHS = -1, RHS = 3 $\Rightarrow$ Lines l_1 and l_2 do not intersect	this type of explanation is also allowed for B1 $\sqrt{}$.	
			[4]

Question Number	Scheme		Marks
5. (b)	$\lambda = 1 \Rightarrow \overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ & $\mu = 2 \Rightarrow \overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$	<u>**Only one of either**</u> $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$ or $A(2,1,-1)$ or $B(5,5,4)$. (can be implied)	B1
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{\begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ or $\underline{\overrightarrow{BA} = \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix}}$	Finding the difference between their $\overrightarrow{OB}$ and $\overrightarrow{OA}$. (can be implied)	<u>M1</u> $\sqrt{}$
	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ & θ is angle		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_1}{\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_1\rvert} = \pm\left(\dfrac{3 + 4 + 0}{\sqrt{50} . \sqrt{2}}\right)$	Applies dot product formula between $\mathbf{d}_1$ and their $\pm\overrightarrow{AB}$. Correct expression.	M1 $\sqrt{}$ A1
	$\cos\theta = \frac{7}{10}$	$\frac{7}{10}$ or <u>0.7</u> or $\frac{7}{\sqrt{100}}$ but not $\frac{7}{\sqrt{50}\sqrt{2}}$	A1 **cao** **[6]**
			10 marks

Question Number	Scheme	Marks
6. (a)	$\left\{x = u^2 \Rightarrow\right\} \dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$	B1
	$\left\{\int \dfrac{1}{x(2\sqrt{x}-1)}\,dx\right\} = \int \dfrac{1}{u^2(2u-1)}\,2u\,du$	M1
	$= \int \dfrac{2}{u(2u-1)}\,du$	A1 * **cso**
		(3)
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$	M1 A1
	$u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$	
	So $\int \dfrac{2}{u(2u-1)}\,du = \int \dfrac{-2}{u} + \dfrac{4}{(2u-1)}\,du$ — Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda \ln u$ or $\pm\mu\ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$ — At least one term correctly followed through	A1 ft
	$-2\ln u + 2\ln(2u-1)$.	A1 **cao**
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$	
	$= \left(-2\ln 3 + 2\ln(2(3)-1)\right) - \left(-2\ln 1 + 2\ln(2(1)-1)\right)$ — Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\dfrac{5}{3}\right)$ — $2\ln\left(\dfrac{5}{3}\right)$	A1 **cso cao**
		(7)
		[10]

Question Number	Scheme		Marks
7.	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{dx}{dt} = 81\sec^2 t \sec t \tan t\,, \quad \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t \tan t}\left\{= \frac{1}{27\sec t \tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 **cao cso**
			(4)
(b)	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			(3)
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 dx$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)dx$	For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$		dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			(5) **[12]**

Question Number	Scheme		Marks	
8.	(a) $\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1	
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$		B1	
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1	
	Leading to $75\dfrac{dh}{dt} = 4 - 5h$ *	cso	A1	**(5)**
	(b) $\displaystyle\int \frac{75}{4-5h}\,dh = \int 1\,dt$	separating variables	M1	
	$-15\ln(4-5h) = t \quad (+C)$		M1 A1	
	$-15\ln(4-5h) = t + C$			
	When $t = 0,\ h = 0.2$			
	$-15\ln 3 = C$		M1	
	$t = 15\ln 3 - 15\ln(4-5h)$			
	When $h = 0.5$			
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1	**(6)**
				[11]

Question 1

This proved a good starting question and full marks were common. A few candidates differentiated the expression, using the product rule. However, the great majority realised that integration by parts was necessary and such errors as were seen usually arose from integrating $\sin 2x$ and $\cos 2x$ incorrectly. Both errors of sign and multiplying (rather than dividing) by 2 were not uncommon. Most knew how to use the limits and complete the question. In some cases the numerically correct answer, $\frac{\pi}{4}$, was obtained after incorrect working. In these cases, the final accuracy mark was not awarded.

Question 2

Q2 was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), a significant minority of candidates tried to integrate $3(1+4x)^{\frac{1}{2}}$. Many candidates, however, correctly realised that they needed to integrate $3(1+4x)^{-\frac{1}{2}}$. The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $k(1+4x)^{\frac{1}{2}}$. Few candidates applied incorrect limits to their integrated expression. A noticeable number of candidates, however, incorrectly assumed a subtraction of zero when substituting for $x = 0$ and so lost the final two marks for this part. A minority of candidates attempted to integrate the expression in part (a) by using a substitution. Of these candidates, most were successful.

In part (b), the vast majority of candidates attempted to apply the formula $\pi\int y^2\,\mathrm{d}x$, but a few of them were not successful in simplifying y^2. The majority of candidates were able to integrate $\frac{9}{1+4x}$ to give $\frac{9}{4}\ln|1+4x|$. The most common error at this stage was for candidates to omit dividing by 4. Again, more candidates were successful in this part in substituting the limits correctly to arrive at the exact answer of $\frac{9}{4}\pi\ln 9$. Few candidates gave a decimal answer with no exact term seen and lost the final mark.

Question 3

This question was generally well answered with about 57% of candidates gaining at least 7 of the 8 marks available and about 46% of candidates gaining all 8 marks. About 10% of candidates, however, did not score on this question.

Most candidates followed the advice given in the question and used the substitution $u = 2 + \sqrt{(2x + 1)}$. Most candidates differentiated this correctly to give either $\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = u - 2$ although a few differentiated incorrectly to give $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}$. The majority were then able to apply the substitution and reach an integral of the form $k\int\frac{(u-2)}{u}\mathrm{d}u$. Whilst most candidates reaching this stage then correctly divided through by u and integrated term by term to reach an expression of the form $k(\ln u - u)$, a few resorted to integration by parts or partial fractions and were generally less successful. Most candidates applied changed limits of 5 and 3 correctly to their integrated function in u and a few converted back to a function in x and used limits of 4 and 0 correctly. Disappointingly a number of candidates did not express their answer in the form $A + 2\ln B$ and gave answers such as $2 - 2\ln\frac{5}{3}$ or $2 + \ln\frac{9}{25}$.

Question 4

This question discriminated well between candidates of all abilities, with about 59% of candidates gaining at least 6 of the 9 marks available and about 17% of candidates gaining all 9 marks. Part (a) was found to be accessible to most, part (b) less well answered and part (c) often either not attempted or incomplete.

In part (a), the majority of candidates were able to apply the process of parametric differentiation followed by substitution of $t = \frac{\pi}{6}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Occasional sign errors were seen in the differentiation of both x and y and the 2 was not always treated correctly when differentiating $y = 1 - \cos 2t$ resulting in $\frac{\mathrm{d}y}{\mathrm{d}t} = \lambda \sin 2t\ (\lambda \neq 2)$. Other common mistakes included obtaining $t \pm 2\sin 2t$ or $2\sin t$ for $\frac{\mathrm{d}y}{\mathrm{d}t}$. A small number of candidates believed rewriting y as $2\sin^2 t$ made the differentiation of y easier. Most candidates showed the substitution of $t = \frac{\pi}{6}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ but many wrote down the numerical answer. Some candidates achieved $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ erroneously by either dividing $\frac{\mathrm{d}x}{\mathrm{d}t}$ by $\frac{\mathrm{d}y}{\mathrm{d}t}$ or from writing $\frac{\mathrm{d}x}{\mathrm{d}t} = -2\cos t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = -2\sin 2t$. Few candidates formed the Cartesian equation (required in part (b)) and used this to correctly find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

In part (b), sign errors, bracketing errors, manipulation errors, an inability to remember the double angle formulae for $\cos 2t$, and expanding $\left(\frac{x}{2}\right)^2$ to give $\frac{x^2}{4}$ were all common mistakes. The candidates who used $\cos 2t \equiv 1 - 2\sin^2 t$ were more successful in achieving $y = \frac{1}{2}x^2$ when compared to those who used either $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv \cos^2 t - \sin^2 t$. A small minority of candidates made t the subject in one of their parametric equations and found a correct alternative Cartesian form of $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$. Those candidates who attempted to find k usually found the correct answer of $k = 2$, although some incorrectly stated that $k = \frac{1}{2}$ or $k = \lambda\pi$.

Part (c) was not attempted by some and those that did often that gave partial or incorrect solutions such as $f(x) \geqslant 0$, $f(x) \leqslant 0$, $0 < y < 2$, $0 \leqslant x \leqslant 2$, $f(x) \leqslant 2$, $-2 \leqslant y \leqslant 2$, etc. A number of candidates also used incorrect notation for range.

Question 5

In part (a), a majority of candidates were able to prove that the two lines did not cross, although some candidates produced errors in solving relatively straightforward "simultaneous" equations. A small number of candidates tried to show a contradiction by substituting their values for λ and μ into one of the two equations they had already used, sometimes with apparent success, as they had already found an incorrect value of one of the parameters! There were a few candidates, however, who believed that the two lines were parallel and attempted to prove this.

Part (b) was less well answered. Many candidates found $\overrightarrow{OA}$ and $\overrightarrow{OB}$, although there was a surprising number of numeric errors seen in finding these vectors. A significant proportion of candidates did not subtract these vectors in order to find $\overrightarrow{AB}$. Most candidates then knew that a dot product was required but there was great confusion on which two vectors to use. A significant minority correctly applied the dot product formula between $\overrightarrow{AB}$ and the direction vector of l_1. Common errors here included applying the dot product formula between either $\overrightarrow{OA}$ and $\overrightarrow{OB}$; or the direction vector of l_1 and the direction vector of l_2; or the direction vector of l_1 and twice the direction vector of l_2; or $\overrightarrow{AB}$ and the direction vector of l_2. A few candidates did not state the cosine of the acute angle as question required but instead found the acute angle.

Question 6

This question discriminated well between candidates of all abilities, with about 44% of candidates gaining at least 5 of the 10 marks available and about 28% of candidates gaining all 10 marks. Part (a) was found to be accessible, but most of the marks in part (b) depended on candidates identifying the correct strategy for integrating $\dfrac{2}{u(2u-1)}$ with respect to u.

Part (a) required candidates to 'show that', so it was expected that solutions would show clear steps to the printed answer given. Most candidates found either $\dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$, although a few replaced dx with $\dfrac{du}{2u}$ instead of with $2u\,du$. In some cases dx was replaced erroneously by du or omitted throughout. The function of x was usually converted to a function of u correctly. There were some incorrect algebraic moves seen in attempting to reach the final answer especially in cases containing an error with dx. The final answer was sometimes written with the integral sign or du missing – thus not fully showing the answer required.

In part (b), only a minority realised that partial fractions were needed, often after attempting integration in several different ways. Once expressed in partial fractions, correct integration usually followed leading to a correct answer of $2\ln\left(\frac{5}{3}\right)$. Some candidates did make algebraic slips when forming their partial fractions or integrated $\frac{-4}{(2u-1)}$ incorrectly to give $-4\ln(2u-1)$.

Some candidates rewrote $\frac{2}{u(2u-1)}$ incorrectly as $\frac{2}{u}+\frac{2}{(2u-1)}$ or $2\left(\frac{1}{2u^2}-\frac{1}{u}\right)$. Other candidates wrote $\frac{2}{u}\times\frac{1}{(2u-1)}$ and integrated this to give $(2\ln u)\left(\frac{1}{2}\ln(2u-1)\right)$ or tried unsuccessfully to use a method of integration by parts. There were consequently many other incorrect versions of the integrated function which often, but not always, included a 'ln' term as suggested by the answer given in the question. Most candidates applied changed limits of 3 and 1 to their integrated function in u. Some candidates, however, converted back to a function in x and used limits of 9 and 1. Erroneous limits included 0 and 3 for u, and 1 and 81 for x.

Question 7

This question was challenging and discriminated well between candidates of all abilities, with about 40% of candidates gaining at least 9 marks of the 12 marks available and about 8% gaining all 12 marks. A significant number of candidates scored full marks in part (c).

In part (a), the majority of candidates were able to apply the process of parametric differentiation followed by substitution of $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$. Some candidates could not differentiate $27\sec^3 t$ correctly whilst others did not achieve $\frac{1}{18}$ after substituting $t = \frac{\pi}{6}$ into a correct $\frac{dy}{dx}$.

In part (b), many candidates were able to use a correct trigonometric identity in order to eliminate t and achieve an equation in x and y. Whilst most candidates used $1 + \tan^2 t \equiv \sec^2 t$, some candidates were successful with using $\tan t \equiv \frac{\sin t}{\cos t}$ and $\sin^2 t + \cos^2 t \equiv 1$. Inevitably there was some fudging to arrive at the correct answer and some candidates wrote down and applied an incorrect identity. Some candidates did not attempt to find the values of a and b and those that did often gave incorrect values such as $a = 0$, $b = 27$ or b as infinity.

In part (c), the majority of candidates were able to apply volume formula $\pi\int y^2 \; dx$ on $y = (x^{\frac{2}{3}}-9)^{\frac{1}{2}}$. A number of candidates, however, used incorrect formulae such as $2\pi\int y^2 \; dx$ or $\int y^2 \; dx$ or even $\int y \; dx$.

Most candidates where confident with integrating $x^{\frac{2}{3}}-9$ although some applied incorrect x-limits of 0 and 27 or 0 and 125 to their integrated function. A significant minority did not use the hint in part (b) and attempted to find the volume by parametric integration. Whilst a number of them were able to write down $729\pi\int \tan^3 t\sec^3 t \ \ \mathrm{d}t$, it was rare to see this integrated correctly.

Question 8

Many found part (a) difficult and it was quite common to see candidates leave a blank space here and proceed to solve part (b), often correctly. A satisfactory proof requires summarising the information given in the question in an equation, such as $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$, but many could not do this or began with the incorrect $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$. Some also found difficulty in obtaining a correct expression for the volume of water in the tank and there was some confusion as to which was the variable in expressions for the volume. Sometimes expressions of the form $V = \pi r^2 h$ were differentiated with respect to r, which in this question is a constant. If they started appropriately, nearly all candidates could use the chain rule correctly to complete the proof.

Part (b) was often well done and many fully correct solutions were seen. As noted in the introduction above, some poor algebra was seen in rearranging the equation but, if that was done correctly, candidates were nearly always able to demonstrate a complete method of solution although, as expected, slips were made in the sign and the constants when integrating. Very few candidates completed the question using definite integration. Most used a constant of integration (arbitrary constant) and showed that they knew how to evaluate it and use it to complete the question.

Statistics for C4 Practice Paper G1

					Mean score for students achieving grade:						
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6		76	4.55	5.92	5.34	4.47	3.21	2.22	1.41	0.64
2	9		70	6.27		7.98	6.00	4.33	3.11	2.49	0.91
3	8		69	5.54	7.67	6.55	4.78	3.02	2.21	1.16	0.34
4	9	9	63	5.63	8.28	7.06	5.89	4.72	3.59	2.44	1.17
5	10		60	5.95		8.06	6.01	4.55	3.23	2.10	1.10
6	10	10	54	5.43	9.51	7.34	5.44	3.86	2.63	1.58	0.65
7	12		55	6.63	10.35	7.67	5.03	3.19	2.83	1.78	0.36
8	11		40	4.36	10.25	6.65	3.33	1.38	0.48	0.18	0.06
	75		**59**	**44.36**		**56.65**	**40.95**	**28.26**	**20.30**	**13.14**	**5.23**