Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Bronze Level B2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
70	63	57	53	48	42

1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3},$$

find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

January 2013

2.

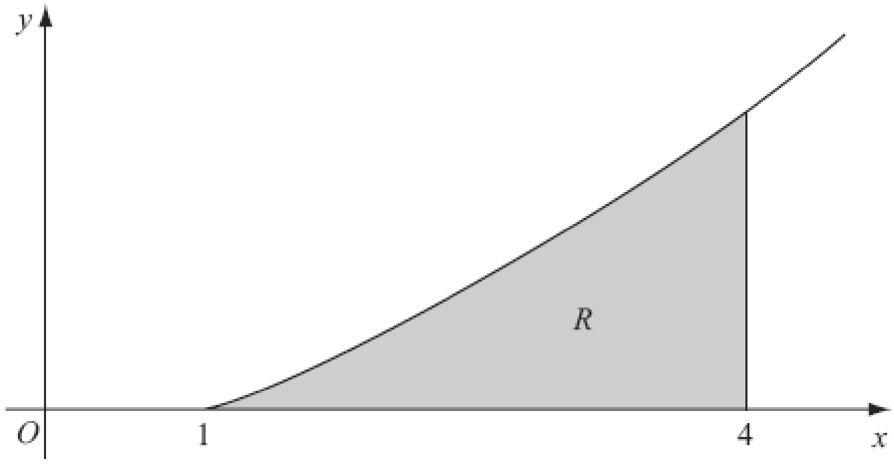

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) (i) Use integration by parts to find $\int x \ln x \; dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

January 2010

3. $$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*.

(4)

(*b*) (i) Hence find $\int f(x)\ dx$.

(3)

(ii) Find $\int_0^2 f(x)\ dx$ in the form ln *k*, where *k* is a constant.

(3)

June 2009

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4. $$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*.

(4)

(*b*) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\ dx$ is 2 + ln *k*, giving the value of the constant *k*.

(6)

June 2007

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5. (*a*) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of *x* up to and including the term in x^2. Simplify each term.

(5)

(*b*) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of *x*.

(4)

June 2008

6.

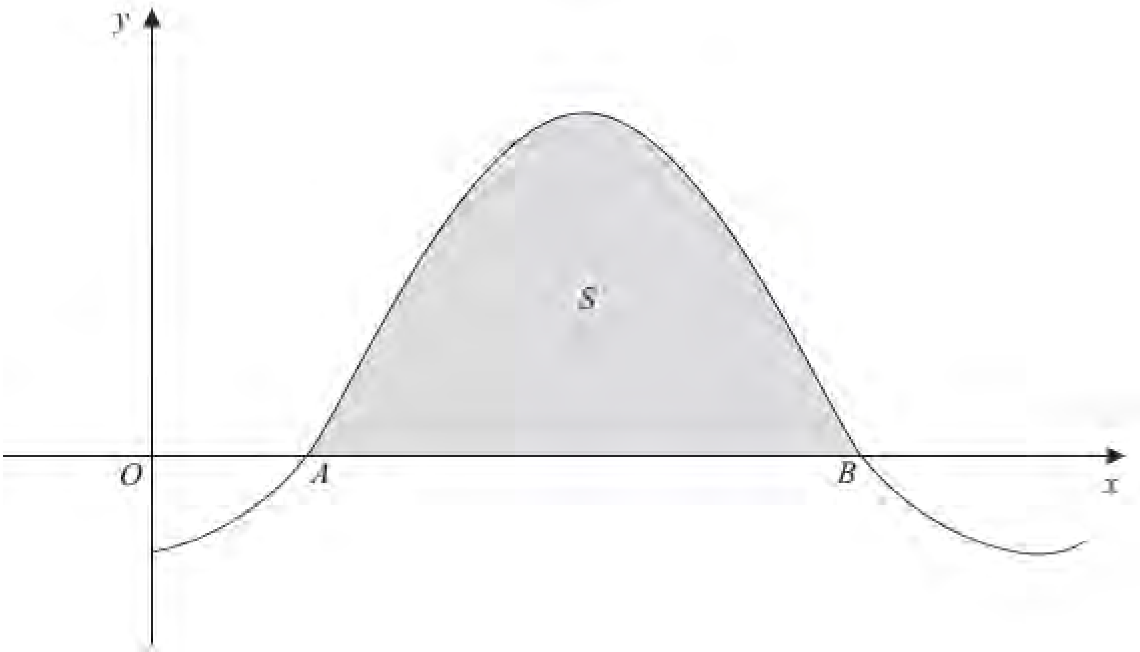

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.

(3)

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.

(6)

January 2013

7.

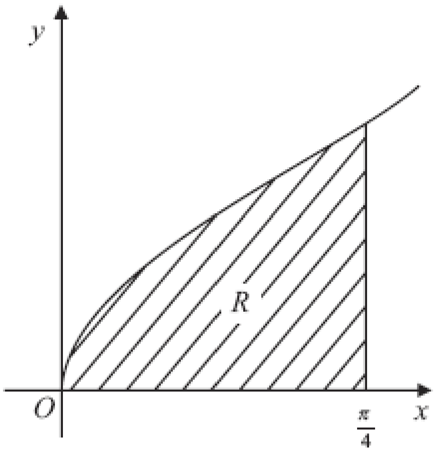

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(*a*) Given that $y = \sqrt{(\tan x)}$, copy and complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(*c*) Use integration to find an exact value for the volume of the solid generated.

(4)

June 2007

8. (*a*) Find $\int (4y+3)^{-\frac{1}{2}}\ \mathrm{d}y$.

(2)

(*b*) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{(4y+3)}}{x^2},$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

June 2011

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1.	$(2+3x)^{-3} = \underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$	$\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$=\left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\dots\right]$	see notes	M1 A1
	$=\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\dots}\right]$		
	$=\frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$	See notes below!	
	$=\frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\dots$		A1; A1
			[5]
			5

Q2	(a) 1.386, 2.291	awrt 1.386, 2.291	B1 B1	(2)
	(b) $A \approx \frac{1}{2}\times 0.5(\ \dots\ )$		B1	
	$= \dots\ \left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$		M1	
	$= 0.25\left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$	ft their (a)	A1ft	
	$=0.25\times 29.477\dots \approx 7.37$	cao	A1	(4)
	(c)(i) $\int x\ln x\,dx = \frac{x^2}{2}\ln x-\int\frac{x^2}{2}\times\frac{1}{x}\,dx$		M1 A1	
	$=\frac{x^2}{2}\ln x-\int\frac{x}{2}\,dx$			
	$=\frac{x^2}{2}\ln x-\frac{x^2}{4}\ (+C)$		M1 A1	
	(ii) $\left[\frac{x^2}{2}\ln x-\frac{x^2}{4}\right]_1^4=\left(8\ln 4-4\right)-\left(-\frac{1}{4}\right)$		M1	
	$=8\ln 4-\frac{15}{4}$			
	$=8\left(2\ln 2\right)-\frac{15}{4}$	$\ln 4 = 2\ln 2$ seen or implied	M1	
	$=\frac{1}{4}\left(64\ln 2-15\right)$	$a=64, b=-15$	A1	(7)
				[13]

Question	Scheme	Marks

Number		
3. (a)	$\mathrm{f}(x) = \dfrac{4-2x}{(2x+1)(x+1)(x+3)} = \dfrac{A}{2x+1} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$	
	$4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x \to -\frac{1}{2}, \quad 5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ any one correct constant	A1
	$x \to -1, \quad 6 = B(-1)(2) \Rightarrow B = -3$	
	$x \to -3, \quad 10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct	A1 (4)
(b) (i)	$\int \left(\dfrac{4}{2x+1} - \dfrac{3}{x+1} + \dfrac{1}{x+3} \right) \mathrm{d}x$	
	$= \dfrac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+*C*" ; ft constants	A1ft (3)
(ii)	$\left[2\ln(2x+1) - 3\ln(x+1) + \ln(x+3)\right]_0^2$	
	$= (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln 1 - 3\ln 1 + \ln 3)$	M1
	$= 3\ln 5 - 4\ln 3$	
	$= \ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$= \ln\left(\dfrac{125}{81}\right)$	A1 (3)
		(10 marks)

Question Number	Scheme		Marks
4. (a)	A method of long division gives,		
	$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \frac{4}{(2x+1)(2x-1)}$	$A = 2$	B1
	$\frac{4}{(2x+1)(2x-1)} \equiv \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$		
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$ Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$	Forming any one of these two identities. Can be implied.	M1
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	**See note below** either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 **[4]**
4. (b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$ $A \to Ax$ $-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1) + \ln(2x-1)$ See note below.	**M1** * **B1** $\sqrt{}$ **A1** **cso & aef**
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	**depM1** *
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for their numerical expression.	**M1**
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$	**A1**
		Or $2 - \ln\left(\frac{5}{9}\right)$ and k stated as $\frac{9}{5}$.	**[6]**
			10 marks

Question Number	Scheme	Marks
5. (*a*)	$\dfrac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\dfrac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\dfrac{1}{2}}\left(1-\dfrac{3x}{4}\right)^{-\frac{1}{2}}$	B1
	$= \frac{1}{2}\left[\underline{1 + (-\frac{1}{2})(** x); + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(** x)^2 + \ldots}\right]$	M1; A1 ft
	with $** \neq 1$	
	$= \dfrac{1}{2}\left[\underline{1 + (-\frac{1}{2})(-\frac{3x}{4}) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2 + \ldots}\right]$	
	$= \frac{1}{2}\left[1 + \frac{3}{8}x; + \frac{27}{128}x^2 + \ldots\right]$	A1 A1 (5)
(*b*)	$(x+8)\left(\dfrac{1}{2} + \dfrac{3}{16}x + \dfrac{27}{256}x^2 + \ldots\right)$	M1
	$= \frac{1}{2}x + \frac{3}{16}x^2 + \ldots\ldots$ $+ 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \ldots\ldots$	M1
	$= 4 + 2x; + \dfrac{33}{32}x^2 + \ldots$	A1; A1 (4)
		(9 marks)

Question Number	Scheme		Marks
6. (a)	$\{y = 0 \Rightarrow\}\ 1 - 2\cos x = 0$	$1 - 2\cos x = 0$, seen or implied.	M1
	$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	At least one correct value of x. (See notes).	A1
		Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$	A1 cso
			[3]
(b)	$V = \pi\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \ \mathrm{d}x$	For $\pi\int(1 - 2\cos x)^2$. Ignore limits and $\mathrm{d}x$	B1
	$\left\{\int (1 - 2\cos x)^2 \ \mathrm{d}x\right\} = \int (1 - 4\cos x + 4\cos^2 x)\mathrm{d}x$		
	$= \int 1 - 4\cos x + 4\left(\frac{1 + \cos 2x}{2}\right) \mathrm{d}x$	$\cos 2x = 2\cos^2 x - 1$ See notes.	M1
	$= \int \left(3 - 4\cos x + 2\cos 2x\right) \mathrm{d}x$		
	$= 3x - 4\sin x + \frac{2\sin 2x}{2}$	Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$.	M1
		Correct integration.	A1
	$V = \{\pi\}\left(\left(3\left(\frac{5\pi}{3}\right) - 4\sin\left(\frac{5\pi}{3}\right) + \frac{2\sin\left(\frac{10\pi}{3}\right)}{2}\right) - \left(3\left(\frac{\pi}{3}\right) - 4\sin\left(\frac{\pi}{3}\right) + \frac{2\sin\left(\frac{2\pi}{3}\right)}{2}\right)\right)$	Applying limits the correct way round. Ignore π.	M1
	$= \pi\left(\left(5\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}\right) - \left(\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2}\right)\right)$		
	$= \pi\left((18.3060...) - (0.5435...)\right) = 17.7625\pi = 55.80$		
	$= \pi\left(4\pi + 3\sqrt{3}\right)$ or $4\pi^2 + 3\pi\sqrt{3}$	Two term exact answer.	A1
			[6] **9**

Question Number	Scheme		Marks
7. (a)	x: 0, $\frac{\pi}{16}$, $\frac{\pi}{8}$, $\frac{3\pi}{16}$, $\frac{\pi}{4}$ y: 0, 0.445995927…, 0.643594252…, 0.817421946…, 1		
		0.446 or awrt 0.44600	B1
		awrt 0.64359	B1
		awrt 0.81742	B1
			[3]
(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{16}$; $\times \underline{\left\{0 + 2\left(0.44600 + 0.64359 + 0.81742\right) + 1\right\}}$	Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$	B1
		<u>For structure of trapezium rule</u> $\underline{\{.............\}}$;	<u>M1</u>$\sqrt{}$
		Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.	<u>A1</u>$\sqrt{}$
	$= \frac{\pi}{32} \times 4.81402... = 0.472615308... = \underline{0.4726}$ (4dp)	for seeing <u>0.4726</u>	A1 **cao**
			[4]
(c)	Volume $= (\pi)\underline{\int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x}\right)^2 \, dx} = (\pi)\underline{\int_0^{\frac{\pi}{4}} \tan x \, dx}$	$\underline{\int \left(\sqrt{\tan x}\right)^2 \, dx}$ or $\underline{\int \tan x \, dx}$ Can be implied. Ignore limits and (π)	**M1**
	$= (\pi)\left[\underline{\ln \sec x}\right]_0^{\frac{\pi}{4}}$ or $= (\pi)\left[\underline{-\ln \cos x}\right]_0^{\frac{\pi}{4}}$	$\tan x \to \underline{\ln \sec x}$ or $\tan x \to \underline{-\ln \cos x}$	**A1**
	$= (\pi)\left[\left(\ln \sec \frac{\pi}{4}\right) - \left(\ln \sec 0\right)\right]$		**dM1**
	or		
	$= (\pi)\left[\left(-\ln \cos \frac{\pi}{4}\right) - \left(\ln \cos 0\right)\right]$		
	$= \pi\left[\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right) - \ln\left(\frac{1}{1}\right)\right] = \pi\left[\ln\sqrt{2} - \ln 1\right]$		
	or		
	$= \pi\left[-\ln\left(\frac{1}{\sqrt{2}}\right) - \ln(1)\right]$		
	$= \underline{\pi \ln \sqrt{2}}$ or $\underline{\pi \ln \frac{2}{\sqrt{2}}}$ or $\underline{\frac{1}{2}\pi \ln 2}$ or $\underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)}$ or $\underline{\frac{\pi}{2} \ln\left(\frac{1}{2}\right)}$	$\underline{\pi \ln \sqrt{2}}$ or $\underline{\pi \ln \frac{2}{\sqrt{2}}}$ or $\underline{\frac{1}{2}\pi \ln 2}$ or $\underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)}$ or $\underline{\frac{\pi}{2} \ln\left(\frac{1}{2}\right)}$ must be exact.	**A1 aef**
			[4]
			11 marks

Question Number	Scheme	Marks
8.	(a) $\int (4y+3)^{-\frac{1}{2}}\,dx = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,dy = \int \frac{1}{x^2}\,dx$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,dy = \int x^{-2}\,dx$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ or equivalent	A1 **(6)**
		[8]

Statistics for C4 Practice Paper Bronze Level B2

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	5	5	88	4.42	4.90	4.71	4.49	4.22	3.72	3.33	2.46
2	13		77	10.06		11.88	9.64	8.27	6.96	4.98	3.39
3	10		77	7.66		9.01	7.97	7.00	5.97	4.76	3.04
4	10		73	7.31		8.92	7.56	6.44	5.17	3.96	2.28
5	9		76	6.80		8.12	7.16	6.26	5.13	3.70	1.89
6	9	9	67	6.02	8.47	7.04	5.84	4.45	3.41	2.76	1.38
7	11		75	8.23		9.84	8.60	7.43	6.09	4.88	3.11
8	8		52	4.19	7.32	5.79	4.04	2.50	1.48	0.91	0.52
	75		**73**	**54.69**		**65.31**	**55.30**	**46.57**	**37.93**	**29.28**	**18.07**