

6.

Figure 1

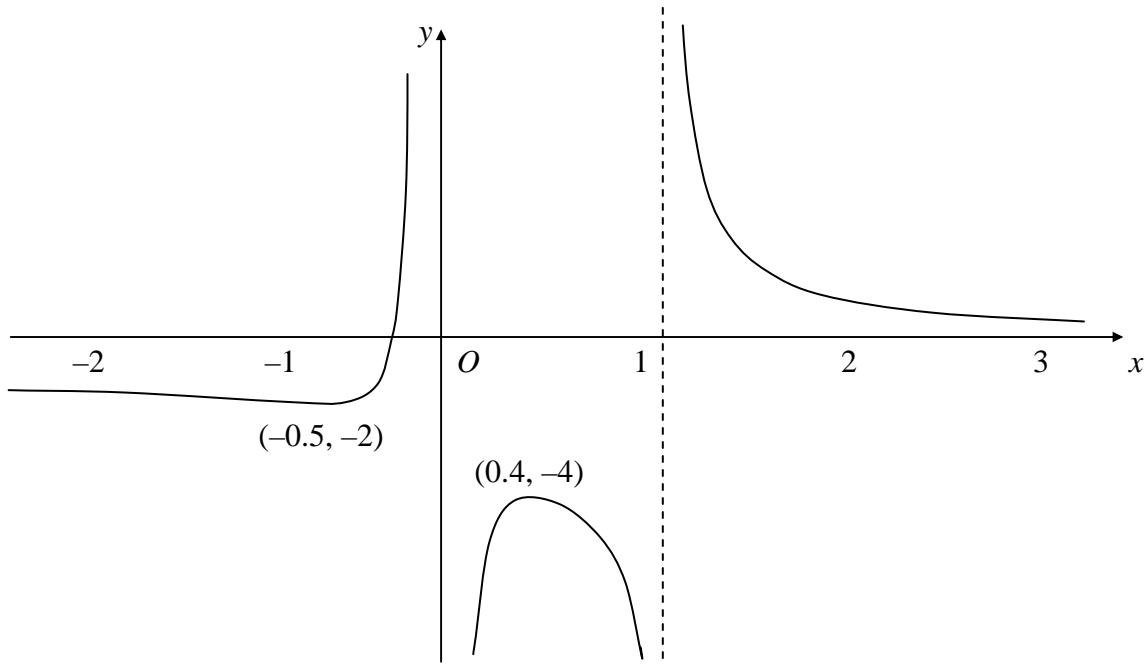


Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a minimum point at $(-0.5, -2)$ and a maximum point at $(0.4, -4)$. The lines $x = 1$, the x -axis and the y -axis are asymptotes of the curve, as shown in Fig. 1.

On a separate diagram sketch the graphs of

(a) $y = |f(x)|$, (4)

(b) $y = f(x - 3)$, (4)

(c) $y = f(|x|)$. (4)

In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

(ii) how the curve approaches the asymptotes of the curve.

6. continued

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7. (a) Sketch the curve with equation $y = \ln x$. (2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point $(e, 1)$ passes through the origin. (3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = e$ if $0 < m < \frac{1}{e}$. (2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = e^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5 , giving your answer to x_5 to 3 decimal places. (3)

The root of $\ln x - \frac{1}{3}x = 0$ is α .

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α , correct to 3 decimal places. (3)

8. In a particular circuit the current, I amperes, is given by

$$I = 4 \sin \theta - 3 \cos \theta, \quad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R \sin(\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha < 360^\circ$,

- (a) find the value of R , and the value of α to 1 decimal place. (4)
- (b) Hence solve the equation $4 \sin \theta - 3 \cos \theta = 3$ to find the values of θ between 0 and 360° . (5)
- (c) Write down the greatest value for I . (1)
- (d) Find the value of θ between 0 and 360° at which the greatest value of I occurs. (2)

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