FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper K

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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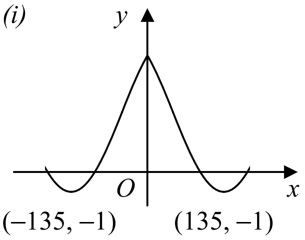
C3 Paper K – Marking Guide

1.	*(a)*	$\arctan(x-2) = -\frac{\pi}{3}$	M1	
		$x - 2 = \tan(-\frac{\pi}{3}) = -\sqrt{3}$	M1	
		$x = 2 - \sqrt{3}$	A1	
	(b)	$1 - 2\sin^2\theta - \sin\theta - 1 = 0$	M1	
		$2\sin^2\theta + \sin\theta = 0$		
		$\sin\theta(2\sin\theta + 1) = 0$	M1	
		$\sin\theta = 0$ or $-\frac{1}{2}$	A1	
		$\theta = 0$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$		
		$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$	A2	**(8)**
2.	*(a)*	$= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3}$	M1	
		$= \frac{4x - 2(x-3)}{(x+3)(x-3)}$	M1	
		$= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)}$	M1	
		$= \frac{2}{x-3}$	A1	
	(b)	$2^3 - 8 = 0 \therefore (x-2)$ is a factor of $(x^3 - 8)$	B1	
		$$\begin{array}{r\|l} & x^2 + 2x + 4 \\ \hline x-2 & x^3 + 0x^2 + 0x - 8 \\ & x^3 - 2x^2 \\ & 2x^2 + 0x \\ & 2x^2 - 4x \\ & 4x - 8 \\ & 4x - 8 \end{array}$$	M1 A1	
		$\therefore\ x^3 - 8 = (x-2)(x^2 + 2x + 4)$		
		$\therefore\ \frac{x^3-8}{3x^2-8x+4} = \frac{(x-2)(x^2+2x+4)}{(3x-2)(x-2)} = \frac{x^2+2x+4}{3x-2}$	M1 A1	**(9)**
3.	*(a)*	$= -\operatorname{cosec}^2 x^2 \times 2x = -2x \operatorname{cosec}^2 x^2$	M1 A1	
	(b)	$= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2 - x)$	M1 A2	
	(c)	$= \frac{\cos x \times (3 + 2\cos x) - \sin x \times (-2\sin x)}{(3 + 2\cos x)^2}$	M1 A1	
		$= \frac{3\cos x + 2\cos^2 x + 2\sin^2 x}{(3+2\cos x)^2} = \frac{3\cos x + 2}{(3 + 2\cos x)^2}$	M1 A1	**(9)**
4.	*(a)*	$(e^x - 3)(e^x - 5) = 0$	M1 A1	
		$e^x = 3, 5$		
		$x = \ln 3, \ln 5$	M1 A1	
	(b)	assume $\log_2 3$ is rational	B1	
		$\therefore\ \log_2 3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$	M1	
		$\Rightarrow\ 2^{\frac{p}{q}} = 3$	M1	
		$\Rightarrow\ 2^p = 3^q$	A1	
		2 and 3 are co-prime $\therefore$ only solution is $p = q = 0$	M1	
		but $q \neq 0$ $\therefore$ contradiction $\therefore \log_2 3$ is irrational	A1	**(10)**

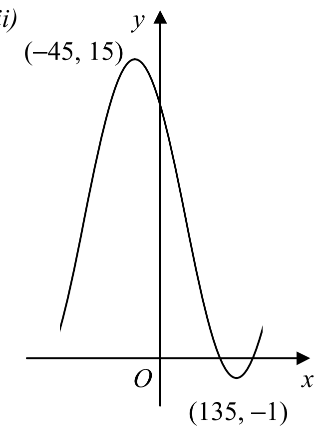
5.	*(a)*	$f(x) > 0$	B1	
	(b)	$y = 3e^{x-1}, \quad x - 1 = \ln \frac{y}{3}$	M1	
		$x = 1 + \ln \frac{y}{3}$		
		$f^{-1}(x) = 1 + \ln \frac{x}{3}, \; x \in \mathbb{R}, \; x > 0$	M1 A2	
	(c)	$f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$	M1 A1	
		$gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$	A1	
	(d)	$f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln \frac{5x-2}{3}$	M1 A1	
		$\therefore \; 1 + \ln \frac{5x-2}{3} = 4, \quad \frac{5x-2}{3} = e^3$	M1	
		$x = \frac{1}{5}(3e^3 + 2)$	A1	**(12)**
6.	*(a)*	$2x^2 + 3\ln(2 - x) = 0 \;\Rightarrow\; 3\ln(2 - x) = -2x^2$		
		$\ln(2 - x) = -\frac{2}{3}x^2$	M1	
		$2 - x = e^{-\frac{2}{3}x^2}$	M1	
		$x = 2 - e^{-\frac{2}{3}x^2} \quad [\, k = -\frac{2}{3} \,]$	A1	
	(b)	$x_1 = 1.90988, x_2 = 1.91212, x_3 = 1.91262, x_4 = 1.91273$	M1 A1	
		$\therefore \; \alpha = 1.913$ (3dp)	A1	
		$f(1.9125) = 0.0070, \; f(1.9135) = -0.020$	M1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(c)	$f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$	M1 A1	
		$\therefore \; 4x - \frac{3}{2-x} = 0, \quad 4x = \frac{3}{2-x}, \quad 4x(2 - x) = 3$	M1	
		$4x^2 - 8x + 3 = 0, \quad (2x - 3)(2x - 1) = 0$	M1	
		$x = \frac{1}{2}, \frac{3}{2}$	A1	**(13)**

7. *(a)* *(i)*



B3

(ii)



M1 A2

	(b)	$2\sqrt{2}\cos x - 2\sqrt{2}\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
		$R\cos\alpha = 2\sqrt{2}, \; R\sin\alpha = 2\sqrt{2}, \quad \therefore \; R = \sqrt{8+8} = 4$	M1 A1	
		$\tan\alpha = 1, \; \alpha = 45$	B1	
		$\therefore \; f(x) = A + 4\cos(x + 45)°$		
	(c)	3	B1	
	(d)	$3 + 4\cos(x + 45) = 0, \quad \cos(x + 45) = -\frac{3}{4}$	M1	
		$x + 45 = 180 - 41.4, \; 180 + 41.4 = 138.6, \; 221.4$	M1	
		$x = 93.6, \; 176.4$ (1dp)	A2	**(14)**

Total **(75)**

Performance Record – C3 Paper K

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	trigonometry	rational expressions	differentiation	exponentials and logarithms, proof	functions	numerical methods, differentiation	functions, trigonometry	
Marks	8	9	9	10	12	13	14	75
Student								