

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C3

Paper 1

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.



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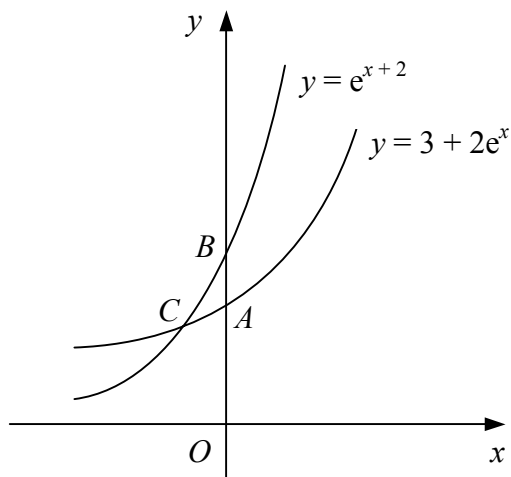
1. Express

$$\frac{2x}{2x^2 + 3x - 5} \div \frac{x^3}{x^2 - x}$$

as a single fraction in its simplest form.

(5)

2.



**Figure 1**

Figure 1 shows the curves  $y = 3 + 2e^x$  and  $y = e^{x+2}$  which cross the  $y$ -axis at the points  $A$  and  $B$  respectively.

(a) Find the exact length  $AB$ . (3)

The two curves intersect at the point  $C$ .

(b) Find an expression for the  $x$ -coordinate of  $C$  and show that the  $y$ -coordinate of  $C$  is  $\frac{3e^2}{e^2 - 2}$ . (5)

3.

$$f(x) = \frac{x^2 + 3}{4x + 1}, \quad x \in \mathbb{R}, \quad x \neq -\frac{1}{4}.$$

(a) Find and simplify an expression for  $f'(x)$ . (3)

(b) Find the set of values of  $x$  for which  $f(x)$  is increasing. (5)

4. The curve  $C$  has the equation  $y = x^2 - 5x + 2 \ln \frac{x}{3}$ ,  $x > 0$ .

(a) Show that the normal to  $C$  at the point where  $x = 3$  has the equation

$$3x + 5y + 21 = 0. \quad (5)$$

(b) Find the  $x$ -coordinates of the stationary points of  $C$ . (3)

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5. The functions  $f$  and  $g$  are defined by

$$f(x) \equiv 6x - 1, \quad x \in \mathbb{R},$$

$$g(x) \equiv \log_2(3x + 1), \quad x \in \mathbb{R}, \quad x > -\frac{1}{3}.$$

(a) Evaluate  $gf(1)$ . (2)

(b) Find an expression for  $g^{-1}(x)$ . (3)

(c) Find, in terms of natural logarithms, the solution of the equation

$$fg^{-1}(x) = 2. \quad (4)$$

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6. (a) Use the identities for  $\cos(A + B)$  and  $\cos(A - B)$  to prove that

$$\cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}. \quad (4)$$

(b) Hence find all solutions in the interval  $0 \leq x < 180$  to the equation

$$\cos 5x^\circ + \sin 3x^\circ - \cos x^\circ = 0. \quad (7)$$

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**Turn over**

7. The function  $f$  is defined by

$$f(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where  $a$  is a positive constant.

(a) Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of

(i)  $y = |f(x)|,$

(ii)  $y = f(|x|).$  (6)

The function  $g$  is defined by

$$g(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

(b) Find  $fg(a)$  in terms of  $a$ . (2)

(c) Solve the equation

$$gf(x) = 9a^3. \quad (4)$$

8.  $f(x) = 2x + \sin x - 3 \cos x.$

(a) Show that the equation  $f(x) = 0$  has a root in the interval  $[0.7, 0.8]$ . (2)

(b) Find an equation for the tangent to the curve  $y = f(x)$  at the point where it crosses the  $y$ -axis. (4)

(c) Find the values of the constants  $a, b$  and  $c$ , where  $b > 0$  and  $0 < c < \frac{\pi}{2}$ , such that

$$f'(x) = a + b \cos(x - c). \quad (4)$$

(d) Hence find the  $x$ -coordinates of the stationary points of the curve  $y = f(x)$  in the interval  $0 \leq x \leq 2\pi$ , giving your answers to 2 decimal places. (4)

**END**