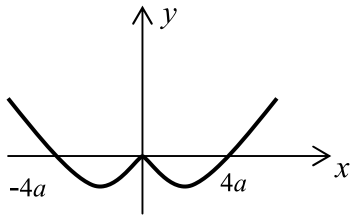
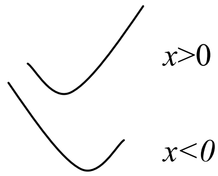
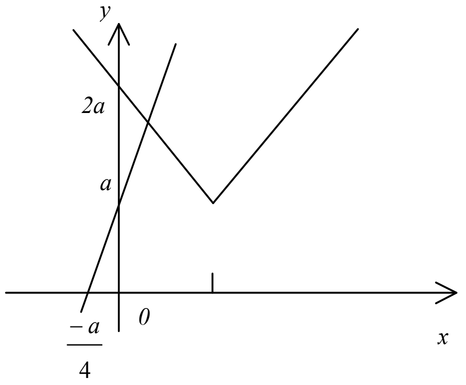
| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (*a*) | Using $x^2 - 1 \equiv (x-1)(x+1)$ somewhere in solution | M1 |
| | Using a common denominator e.g. $\dfrac{x-(x-1)}{(x-1)(x+1)}$ | M1 |
| | Clear, sound, complete proof of $f(x) = \dfrac{1}{(x-1)(x+1)}$ | A1 (3) |
| (*b*) | Range of f is $y$, where $y > 0$ | B2 (2) |
| | If $y \geq 0$ given allow B1. | |
| (c) | $gf(x) = g = g\left(\dfrac{1}{(x-1)(x+1)}\right) = 2(x-1)(x+1)$ | M1 A1 |
| | M1 requires correct order and $g(x) = \dfrac{2}{x}$ used | |
| | $2(x-1)(x+1) = 70$ | M1 |
| | M1 is independent of previous work | |
| | $x = 6$ (treat $-6$ extra as ISW) | A1 (4) |
| | | **(9 marks)** |
| **2.** | $\dfrac{y+3}{(y+1)(y+2)} - \dfrac{y+1}{(y+2)(y+3)} \equiv \dfrac{(y+3)^2-(y+1)^2}{(y+1)(y+2)(y+3)}$ | M1 |
| | $\equiv \dfrac{(y^2+6y+9)-(y^2+2y+1)}{(y+1)(y+2)(y+3)} \equiv \dfrac{4y+8}{(y+1)(y+2)(y+3)}$ | M1 A1 |
| | $\equiv \dfrac{4(y+2)}{(y+1)(y+2)(y+3)} \equiv \dfrac{4}{(y+1)(y+3)}$ or $\dfrac{4}{y^2+4y+3}$ | M1, A1 |
| | | **(5 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** (*a*) |     | B1<br>B1 ft |
| | $(4a, 0)$ & $(-4a, 0)$ and shape at $(0,0)$ | B1 (3) |
| (*b*) | $f(2a) = (2a)^2 - 4a(2a) = 4a^2 - 8a^2 = -4a^2$ | B1 |
| | $f(-2a)[\, = f(2a)$ ($\because$ even function) $] = -4a^2$ | B1 (2) |
| (*c*) | $a = 3$ and $f(x) = 45 \Rightarrow 45 = x^2 - 12x$ $\quad (x > 0)$ | M1 |
| | $0 = x^2 - 12x - 45$ | |
| | $0 = (x - 15)(x + 3)$ | M1 |
| | $x = 15$ (or $-3$) | A1 |
| | $\therefore$ Solutions are $x = \pm 15$ only | A1 (4) |
| | | **(9 marks)** |
| **4.** (*a*) | Attempting to reach at least the stage $x^2(x + 1) = 4x + 1$ | M1 |
| | Conclusion (no errors seen) $x = \sqrt{\dfrac{4x+1}{x+1}}$ (*) | A1 (2) |
| | [Reverse process: need to square and clear fractions for M1] | |
| (*b*) | $x_2 = \sqrt{\dfrac{4+1}{1+1}} =$ 1.58… | M1 |
| | $x_3 = 1.68$, $x_4 = 1.70$ | A1A1 (3) |
| | [Max. deduction of 1 for more than 2 d.p.] | |
| (*c*) | Suitable interval; e.g. [1.695, 1.705] (or "tighter") | M1 |
| | $f(1.695) = -0.037\ldots$, $f(1.705) = +0.0435\ldots$ | M1 |
| | Change of sign, no errors seen, so root = 1.70 (correct to 2 d.p.) | A1 (3) |
| (*d*) | $x = -1$, "division by zero not possible", or equivalent | B1, B1 (2) |
| | or any number in interval $-1 < x < -\frac{1}{4}$, "square root of neg. no." | |
| | | **(10 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** (*a*) |  | |
| | V shape right way up | B1 |
| | vertex in first quadrant | B1 |
| | g | B1 |
| | $-1$ eeoo; $2a,\ a, -\frac{a}{4}$ | B2 (1, 0) (5) |
| (*b*) | $4x + a = (a - x) + a$ | M1 |
| | $5x = a, \qquad x = \frac{a}{5}$ | M1 |
| | $y = \frac{9a}{5}$ | A1 (3) |
| (*c*) | $\text{fg}(x) = \|4x + a - a\| + a = \|4x\| + a$ | M1 A1 (2) |
| (*d*) | $\|4x\| + a = 3a \quad \Rightarrow \quad \|4x\| = 2a$ | M1 |
| | $x = \frac{a}{2}, -\frac{a}{2}$ | A1, A1 (3) |
| | | **(13 marks)** |
| **6.** (*a*) | $\text{f}(3.1) = 10 + \ln 9.3 - \frac{1}{2}\text{e}^{3.1} = 1.131$ | |
| | $\text{f}(3.2) = 10 + \ln 9.6 - \frac{1}{2}\text{e}^{3.2} = -0.0045$ | M1 |
| | Sign change, so $3.1 < k < 3.2$ | A1 (2) |
| (*b*) | $\text{f}'(x) = \frac{1}{x} - \frac{1}{2}\text{e}^{x}$ | (3) |
| (*c*) | $\text{f}(1) = 10 + \ln 3 - \frac{1}{2}\text{e}$ | B1 |
| | $\text{f}'(x) = 1 - \frac{1}{2}\text{e}$ | B1 |
| (i) | $y - (10 + \ln 3 - \frac{1}{2}\text{e}) = (1 - \frac{1}{2}\text{e})(x - 1)$ | M1 |
| (ii) | $x = 0$: $y = 10 + \ln 3 - \frac{1}{2}\text{e} - 1 + \frac{1}{2}\text{e}$ | M1 |
| | $= 9 + \ln 3$ | A1 (5) |
| | | **(10 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (*a*) | $\sin x + \sqrt{3}\cos x = R\sin(x + \alpha)$ | |
| | $= R(\sin x\cos\alpha + \cos x\sin\alpha)$ | M1 |
| | $R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ | A1 |
| | Method for $R$ or $\alpha$, e.g. $R = \sqrt{(1+3)}$ or $\tan\alpha = \sqrt{3}$ | M1 |
| | Both $R = 2$ and $\alpha = 60$ | A1 (4) |
| (*b*) | $\sec x + \sqrt{3}\operatorname{cosec} x = 4 \Rightarrow \dfrac{1}{\cos x} + \dfrac{\sqrt{3}}{\sin x} = 4$ | B1 |
| | $\Rightarrow \sin x + \sqrt{3}\cos x = 4\sin x\cos x$ | M1 |
| | $= 2\sin 2x$ (*) | M1 (3) |
| (*c*) | Clearly producing $2\sin 2x = 2\sin(x + 60)$ | A1 (1) |
| | | **(8 marks)** |