Worked Solutions

Edexcel C3 Paper J

1. $\dfrac{\cancel{(2x-5)}(2x+5)}{x\cancel{(x+1)}} \times \dfrac{\cancel{(x+2)}\cancel{(x+1)}}{\cancel{(2x-5)}\cancel{(x+2)}} = \dfrac{2x+5}{x}$ **(6)**

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2. (*a*) $\dfrac{dx}{dy} = \sec^2 y = 1 + x^2$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ **(4)**

(*b*) $\dfrac{d^2y}{dx^2} = \dfrac{-2x}{(1+x^2)^2}$

$(1+x^2)\dfrac{(-2x)}{(1+x^2)^2} + 2x \cdot \dfrac{1}{1+x^2} = 0$ **(3)**

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3. (*a*) $n = 1, T_1 = \ln p$ **(1)**

(*b*) $T_2 = \ln(pq) = \ln p + \ln q$

$\therefore d = \ln pq - \ln p = \ln q$ **(2)**

(*c*) $S_n = \dfrac{n}{2}\left(\ln p + \ln pq^{n-1}\right)$

$= \dfrac{n}{2}(\ln p + \ln p + (n-1)\ln q) = \dfrac{n}{2}(2\ln p + (n-1)\ln q)$ **(3)**

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4. (*a*) $\left.\begin{array}{l} f(1) = 2 + \ln 3 - 5 = -1.901 \\ f(2) = 4 + \ln 6 - 5 = 0.79 \end{array}\right\}$ change in sign $\Rightarrow$ root in interval **(2)**

(*b*) $5 - \ln 3x = 2x \quad \therefore x = \dfrac{1}{2}(5 - \ln 3x)$ **(2)**

(*c*) $x_1 = 1.74796, \quad x_2 = 1.67147, \quad x_3 = 1.69384, \quad x_4 = 1.68719$ **(2)**

(*d*) $\alpha = 1.689$(3 d.p.) **(1)**

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5. (*a*) $\cos 2x \cos 60 + \sin 2x \sin 60 = 2\sin 2x \cos 30 + 2\cos 2x \sin 30$

$\cos 2x \cdot \dfrac{1}{2} + \sin 2x \cdot \dfrac{\sqrt{3}}{2} = 2\sin 2x \cdot \dfrac{\sqrt{3}}{2} + 2\cos 2x \cdot \dfrac{1}{2}$

$-\cos 2x = \sqrt{3}\sin 2x$

$\tan 2x = -\dfrac{1}{\sqrt{3}}$ **(5)**

(*b*) $2x = 150, 330$

$x = 75, 165$ **(3)**

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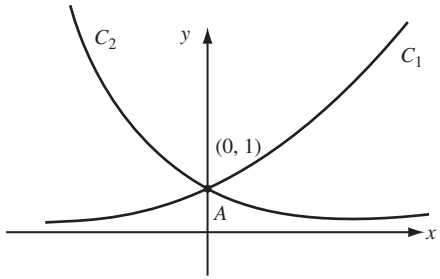
6. (*a*)



C_2 is steeper than C_1

(*b*) A is at (0, 1)

(*c*) $C_1 : \dfrac{dy}{dx} = \dfrac{1}{2}e^{\frac{1}{2}x}$, at $x = 0$, $\dfrac{dy}{dx} = \dfrac{1}{2}$

$C_2 : \dfrac{dy}{dx} = -2e^{-2x}$, at $x = 0$, $\dfrac{dy}{dx} = -2$

gradient of normal to $C_2 = +\dfrac{1}{2}$(product $= -1$)

$\therefore$ gradient of tangent to $C_1 =$ gradient of normal to C_2 **(6)**

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7. (*a*) $7(3x+1)^6 \times 3$ **(3)**

(*b*) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{2}\ln(4x+1)\right) = \dfrac{1}{2}\left(\dfrac{4}{4x+1}\right)$ **(3)**

(*c*) $-7\sin 7x$ **(3)**

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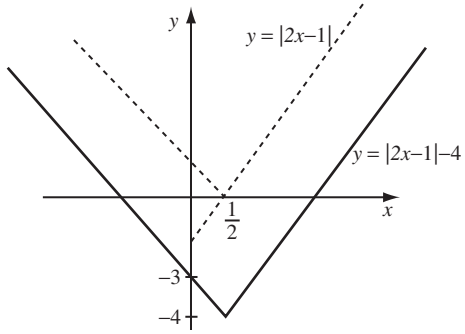
8. (*a*)



(2)

(*b*) $|2x-1|-4=3$

$2x-1-4=3, x=4$ or $-2x+1-4=3 \qquad x=-3$ **(3)**

(*c*) $g(x) = x^2 - 8x + 16 + 1 \qquad g(x) = (x-4)^2 + 1 \qquad g(x) \geq 1$ **(3)**

(*d*) $f(3) = 5 - 4 = 1 \qquad gf(3) = g(1) = 10$ **(2)**

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9. (*a*) (i) $9\cos\theta - 40\sin\theta$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$R^2 = 9^2 + 40^2 \Rightarrow R = 41$

$\tan\alpha = \dfrac{40}{9} \Rightarrow \alpha = 1.35^{\mathrm{c}}$ **(4)**

(ii) $\cos(\theta + 1.35) = \dfrac{6}{41}$

$\theta + 1.35 = 1.42$

$\theta = 0.07^{\mathrm{c}}$ **(3)**

(*b*) $13 + \dfrac{10}{\tan\theta} = 3\tan\theta$

$3\tan^2\theta - 13\tan\theta - 10 = 0$

$(3\tan\theta + 2)(\tan\theta - 5) = 0$

$\tan\theta = 5$

or $-\dfrac{2}{3}$ (no solution)

$\tan\theta = 5 \Rightarrow \theta = 1.37^{\mathrm{c}}$ **(5)**
