Worked Solutions

Edexcel C3 Paper H

1. $x^2 - y^2 = (x-y)(x+y)$

Assume both x and y are even then $x - y = 2m$ where m is an integer.
$x + y = 2n$ where n is an integer.

$\therefore\ x^2 - y^2 = 4mn \neq 18$

Assume both x and y are odd then $x - y = 2m'$

$x + y = 2n'$

$\therefore\ x^2 - y^2 = 4m'n' \neq 18$

Assume x odd, y even then $x - y = 2m_1 + 1$ odd

$x + y = 2m_2 + 1$ odd

$\therefore\ x^2 - y^2 = 4m_1m_2 + 2(m_1 + m_2) + 1$ which is odd $\therefore$ contradiction. **(6)**

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2. $\dfrac{\cancel{(2x-1)}\cancel{(x+3)}}{\cancel{(2x-1)}\cancel{(2x+1)}} \times \dfrac{x\cancel{(2x+1)}}{\cancel{(x+3)}(x+5)} = \dfrac{x}{x+5}$ **(6)**

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3. (*a*) $y = 2x - 1 \quad \therefore 2x = y + 1$

$f^{-1}: x \mapsto \dfrac{x+1}{2}, \quad x \epsilon \mathbb{R}$ **(2)**

(*b*) $gf^{-1}(x) = 3\left(\dfrac{x+1}{2}\right)^2 + 1$ **(2)**

(*c*) $3\left(\dfrac{x+1}{2}\right)^2 + 1 = \dfrac{7}{4}$

$3(x+1)^2 + 4 = 7$

$(x+1)^2 = 1$

$x = 0$ or -2 **(3)**

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4. (*a*) x must be obtuse.

$\therefore\ \tan x < 0, \sin x > 0$

$\cos^2 x = \dfrac{25}{169}$

$\tan^2 x = \sec^2 x - 1 = \dfrac{169}{25} - 1 = \dfrac{144}{25}$

$\tan x = -\dfrac{12}{5}$ **(4)**

(*b*) $\operatorname{cosec} 2x = \dfrac{1}{2\sin x \cos x}$

$= \dfrac{1}{2 \cdot \frac{12}{13} \cdot \frac{-5}{13}} = \dfrac{-169}{120}$ **(4)**

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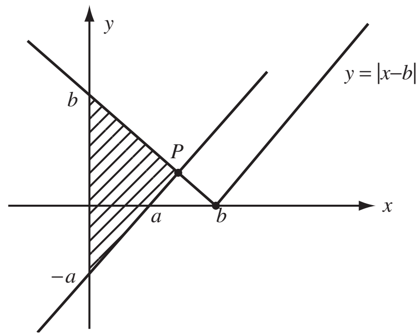
5. (*a*)



(*b*) at P $\quad x - a = -(x - b)$

$x - a = -x + b$

$x = \dfrac{a+b}{2}$

Area $= \dfrac{1}{2}(b+a)\left(\dfrac{a+b}{2}\right) = \dfrac{(a+b)^2}{4}$ **(4)**

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6. $y = \ln(3 - x)$

$y = \ln(x + 2) - 2$

$\ln(3 - x) = \ln(x + 2) - 2$

$\ln(x + 2) - \ln(3 - x) = 2$

$\frac{x + 2}{3 - x} = e^2$

$x + 2 = 3e^2 - xe^2$

$x(1 + e^2) = 3e^2 - 2$

$x = 2.40$ (2 d.p.)

$y = -0.52$ (2 d.p.) **(8)**

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7. (*a*) (i) $x \cdot \frac{3}{3x + 4} + \ln(3x + 4)$ **(3)**

(ii) $5\left(3x^2 + 7\right)^4 \cdot 6x$

$30x\left(3x^2 + 7\right)^4$ **(3)**

(*b*) $\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = -\text{cosec}^2 x$ **(4)**

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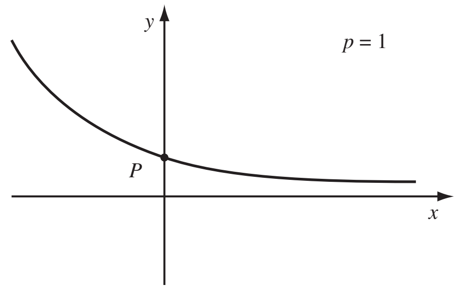
8. (*a*)



(2)

(*b*) $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}$ at $x = q$: $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}q}$

gradient of normal $= 2e^{\frac{1}{2}q}$

Equation of normal is $y = 2e^{\frac{1}{2}q}x$

pt. Q lies on curve $\therefore e^{-\frac{1}{2}q} = 2e^{\frac{1}{2}q}q$

$1 = 2e^q q$

$\therefore q$ satisifies $2e^x x - 1 = 0$ **(4)**

$2e^x x = 1$

$x = \frac{1}{2e^x}$ **(1)**

$x_1 = 0.33516, x_2 = 0.35761, x_3 = 0.34967, x_4 = 0.35246$

$q = 0.352$ (3 d.p.) **(3)**

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9. (*a*) (i) $R^2 = 9^2 + 40^2$

$R = 41$

$\tan\alpha = \dfrac{9}{40}$

$\alpha = 12.68$

(ii) $41\cos(\theta + 12.68) = 4$ **(4)**

$\theta + 12.68 = 84.40$

$\theta = 71.7(1\ \text{d.p.})$ **(3)**

(*b*) $6\sin^2\theta - \sin\theta - 2 = 0$

$(2\sin\theta + 1)(3\sin\theta - 2) = 0$

$\sin\theta = -\dfrac{1}{2}$, no solutions

$\sin\theta = \dfrac{2}{3}$, $\theta = 41.8°, 138.2°$ **(5)**
