

Worked Solutions

Edexcel C3 Paper G

1. (a) $a + 2d = \ln 18$

$$a + 4d = \ln 162$$

$$2d = \ln 162 - \ln 18$$

$$d = \frac{1}{2} \ln \frac{162}{18} = \ln \sqrt{\frac{81}{9}} = \ln 3 \quad (4)$$

(b) $a + 2 \ln 3 = \ln 18$

$$a = \ln 18 - \ln 9 \quad a = \ln 2 \quad (2)$$

2.
$$\frac{(2x+5)}{(2x-3)(2x+3)} \times \frac{(2x-3)(x+1)}{2(x+1)}$$

$$\frac{(2x+5)}{2(2x+3)} \quad (6)$$

3. (a) $-1 \leq \cos x \leq 1$ (2)

(b) fg: $x \mapsto \cos\left(x + \frac{\pi}{2}\right); \quad 0 \leq x \leq \frac{\pi}{2}$ (3)

(c) $0 \geq fg(x) \geq -1$ (2)

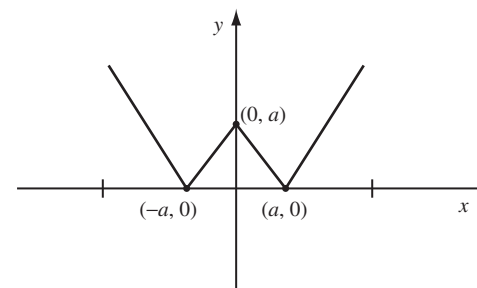
4. (a) $\cos x = -\frac{24}{25}$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2 \times \frac{7}{25} \times \frac{-24}{25} = \frac{-336}{625} \quad (5)$$

(b) $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} = \frac{1 - \tan \theta}{1 + \tan \theta}$ (3)

5. (a)



(4)

(b) $f(2a) = 2(2a) - 2a = 2a$

$$f(-2a) = 2a \quad (2)$$

(c) $a - x = \frac{1}{2}a \Rightarrow x = \frac{1}{2}a \quad x = -\frac{1}{2}a$

$$2x - 2a = \frac{1}{2}a \quad x = 1\frac{1}{4}a$$

$$x = -1\frac{1}{4}a \quad (3)$$

6. $\frac{dy}{dx} = e^{2x}(-3 \sin 3x + 3 \cos 3x) + 2e^{2x}(\cos 3x + \sin 3x)$

$$= e^{2x}(-\sin 3x + 5 \cos 3x)$$

$$\frac{d^2y}{dx^2} = 2e^{2x}(-\sin 3x + 5 \cos 3x) + e^{2x}(-3 \cos 3x - 15 \cos 3x)$$

$$= e^{2x}(-17 \sin 3x + 7 \cos 3x)$$

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13 = e^{2x}(-17 \sin 3x + 7 \cos 3x + 4 \sin 3x - 20 \cos 3x)$$

$$+ 13 \cos 3x + 13 \sin 3x = 0 \quad (11)$$

7. (a) $f'(x) = 4 - \frac{1}{x}$

$$f'(x) = 0 \Rightarrow x = \frac{1}{4}$$

$$f(x) = 1 - \ln \frac{1}{4} = 1 + \ln 4 \quad \left(\frac{1}{4}, 1 + \ln 4 \right)$$

$$f''(x) = +\frac{1}{x^2}$$

$$f''\left(\frac{1}{4}\right) > 0 \Rightarrow \text{min.} \quad (5)$$

(b) $B(1, 4) \quad f'(1) = 3$

tangent : $y - 4 = 3(x - 1)$

$$y = 3x + 1$$

normal $y - 4 = -\frac{1}{3}x + \frac{1}{3} \quad (6)$

(c) $P : \left(-\frac{1}{3}, 0\right) \quad Q : (13, 0)$

$$\Delta PBQ = \frac{1}{2} \left(13\frac{1}{3}\right) \times 4 = 26\frac{2}{3} \quad (3)$$

8. (a) $R^2 = 5^2 + 12^2$

$$\Rightarrow R = 13$$

$$\tan \alpha = \frac{12}{5}$$

$$\Rightarrow \alpha = 67.38^\circ \quad (4)$$

(b) minimum:

$$f(x) + 4 = -13 + 4 = -9 \quad (2)$$

(c) $13 \sin(x + 67.38) = -13$

$$x + 67.38 = 270$$

$$x = 202.6 \quad (3)$$

(d) $13 \sin(x + 67.38) = 6$

$$\sin(x + 67.38) = \frac{6}{13}$$

$$x + 67.38 = 27.48, 152.51, 387.48, \dots$$

$$x = 85.1, 320.1 \quad (5)$$