Core Mathematics C3 For Edexcel Advanced Level

Paper F
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (*a*) Express

$$\frac{2}{x+3} - \frac{1}{x^2+7x+12}$$

as a single fraction in its simplest form,

(*b*) Hence or otherwise solve the equation

$$\frac{2}{x+3} - \frac{1}{x^2+7x+12} = 0.$$

(6)

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2. Given $\mathrm{f}: x \mapsto \dfrac{2}{x-3}, \qquad x \in \mathbb{R}, \quad x \neq 3,$

(*a*) express f^{-1} in the same form.

(4)

(*b*) Evaluate f(4) and $\mathrm{ff}^{-1}(7)$.

(2)

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3. You are given $\mathrm{f}(x) = \ln(x+2), \qquad x \in \mathbb{R}, \quad x > -2.$

(*a*) On two separate diagrams sketch the graphs of

$$y = \mathrm{f}(x) \quad \text{and} \quad y = |\mathrm{f}(x)|.$$

(3)

(*b*) Explain how your graph shows that the equation

$$|\mathrm{f}(x)| - x = 0 \qquad \ldots(\mathrm{A})$$

has only one solution for x.

(1)

(*c*) Show that the solution to the equation $|\mathrm{f}(x)| - x = 0$ lies in the interval $[1, 2]$.

(2)

(*d*) Using the iteration

$$x_{n+1} = \ln(x_n + 2) \quad \text{and} \quad x_0 = 1,$$

find the values of x_1, x_2, x_3, x_4, x_5 and hence give the solution to equation (A) to 3 decimal places.

(3)

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4. Differentiate with respect to x,

(a) $x^2 \ln x$ **(4)**

(b) $\cos^2 3x$ **(3)**

(c) $\dfrac{\sin x}{x}$. **(3)**

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5. (a) Prove that

$$\cot 2\theta \equiv \frac{\cot^2 \theta - 1}{2 \cot \theta}$$ **(5)**

(b) Use the identity to find the values of $\theta, 0 < \theta < 2\pi$, which satisfy the equation

$$\cot^2 \theta - 2 \cot \theta - 1 = 0.$$ **(4)**

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6. (a) Show that the equation

$$e^x + 6e^{-x} = 5 \qquad \ldots(A)$$

can be written in the form

$$(e^x - 3)(e^x - 2) = 0$$ **(3)**

(b) Use this to find the values of x which satisfy equation (A). **(4)**

(c) Hence find the values of x which satisfy the equation

$$e^{2x+2} - 5e^{x+1} + 6 = 0.$$ **(4)**

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7. (*a*) Express

$$7\sin x + 24\cos x$$

in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < 90°$. The values of R and α are to be evaluated. Give α correct to 1 decimal place.

(4)

(*b*) Hence solve the equation

$$7\sin x + 24\cos x = 15, \quad \text{where } 0 < x < 360°.$$

(4)

(*c*) Prove that these values satisfy the equation

$$15\sec x - 7\tan x = 24.$$

(2)

(*d*) Find the maximum value of the function

$$7\sin x + 24\cos x$$

and give the smallest positive value for x for which this maximum value occurs.

(2)

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8. (*a*) Given $x = \sin y$, find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

The point $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{\pi}{4}\right)$ lies on the curve $y = \arcsin x$.

Using your answer to part (*a*) find,

(*b*) the gradient of the tangent to the curve at P,

(3)

(*c*) the equation of the tangent to the curve at P.

(2)

The tangent to the curve at P meets the x axis at the point Q.

(*d*) Show that the coordinates of the point Q are $\left(\dfrac{4-\pi}{4\sqrt{2}}, 0\right)$

(3)

(*e*) Find the exact value of the area of the triangle OPQ.

(2)

END **TOTAL 75 MARKS**