Core Mathematics C3   For Edexcel
Advanced Level

Paper D
Time: 1 hour 30 minutes

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1. (*a*) Simplify $\dfrac{x^2 + 7x + 12}{x^2 + 3x}$.

(3)

(*b*) Find the value of x for which

$$\log_3\left(x^2 + 7x + 12\right) - \log_3\left(x^2 + 3x\right) = 2.$$

(4)

2. The functions f and g are defined by

$$\text{f}: x \mapsto \frac{1}{x - 1}, \qquad x \in \mathbb{R}, \quad x > 1$$

$$\text{g}: x \mapsto 3^x + k, \qquad x \in \mathbb{R} \text{ and } k \text{ is a constant.}$$

(*a*) Find the range of f.

(2)

(*b*) Given that $\text{gf}\left(\dfrac{3}{2}\right) = 14$, find the value of k.

(4)

3. (*a*) Prove that

$$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1.$$

(4)

(*b*) Using part (*a*) solve the equation

$$8\cos^2 x - 8\cos^4 x = 1, \quad \text{for} \quad 0 < x < 180°.$$

(3)

4. (*a*) Differentiate with respect to x

(i) $x^3 \mathrm{e}^{3x}$

(3)

(ii) $\dfrac{2x}{\cos x}$

(3)

(*b*) Given that $x = \cos y^2$,

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

(4)

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5. (*a*) Sketch on the same set of axes the graphs of

$$y = \ln(x - 3) \qquad x \in \mathbb{R}, \qquad x > 3.$$

$$\text{and } y = \frac{2}{x}, \qquad x \neq 0 \tag{3}$$

Given that $f(x) = \ln(x - 3) - \frac{2}{x}, \quad x > 3,$

(*b*) Explain how your graphs show that the equation $f(x) = 0$ has only one solution. **(1)**

(*c*) Show that the solution of $f(x) = 0$ lies between $x = 4$ and $x = 5$. **(2)**

(*d*) The iterative formula

$$x_{n+1} = e^{\frac{2}{x_n}} + 3$$

is used to solve the equation $f(x) = 0$.

Taking $x_0 = 5$, find the values of x_1, x_2, x_3 and x_4 and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 2 decimal places. **(3)**

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6. (*a*) Given that $\cot x = \frac{4}{3}$ and that x is a reflex angle, find the exact value of $\operatorname{cosec} x$. **(4)**

(*b*) Find the exact value of $\cos 2x$. **(4)**

(*c*) Find the exact value of $\tan 2x$. **(3)**

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7. (*a*) Sketch on the same pair of axes the graphs of

$$y = \left| x^2 - a^2 \right| \quad \text{and} \quad y = x + a; \quad a > 1$$

Mark clearly on your graphs the coordinates of the points of intersection with the x and y axes. **(4)**

(*b*) The curve and straight line intersect at three points.

Show that one of these points has coordinates $(-a, 0)$. **(1)**

(*c*) Show that the x coordinate of one of the other points satisfies the equation

$$x^2 - x - a^2 - a = 0 \tag{2}$$

(*d*) Write down an equation which is satisfied by the third point of intersection. **(2)**

(*e*) Given that $a = 3$, find the coordinates of the points of intersection of the curve and the straight line. **(2)**

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8. Show that $x = e^{-t}(\cos t + \sin t)$ satisfies the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0.$$

(6)

(*a*) Find the value of x, when $t = 0$, and show that $\frac{dx}{dt} = 0$, when $t = 0$.

(2)

(*b*) Find the next positive value for t for which $\frac{dx}{dt} = 0$.

(2)

(*c*) Hence find a maximum and minimum value for x.

(2)

(*d*) Justify your answers to (*c*) by evaluating $\frac{d^2x}{dt^2}$.

(2)

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END   **TOTAL 75 MARKS**

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