

Worked Solutions

Edexcel C3 Paper B

1. (a) $\ln(3x + 7) = 1 \Rightarrow 3x + 7 = e \quad x = \frac{e - 7}{3}$ (3)

(b) $2e^{2y} + 5e^y - 3 = 0$
 $(2e^y - 1)(e^y + 3) = 0$
 $e^y = \frac{1}{2} \Rightarrow y = \ln \frac{1}{2}$ (or $-\ln 2$) (5)

2. (a) any valid pair of A & B e.g. $A = B = \pi/2$. (2)

(b) L.H.S. $= \frac{2}{\sin 2A} = \frac{2}{2 \sin A \cos A} = \operatorname{cosec} A \sec A$ (3)

3. (a) $\frac{2(x + 3) + 11}{(x - 5)(x + 3)} = \frac{2x + 17}{(x - 5)(x + 3)}$ (3)

(b) $\frac{2x + 17}{x^2 - 2x - 15} = 1$
 $2x + 17 = x^2 - 2x - 15$
 $x^2 - 4x - 32 = 0$
 $(x - 8)(x + 4) = 0$
 $x = 8$ or -4 (4)

4. (a) $y = k \ln x \quad \frac{y}{k} = \ln x$

$x = e^{\frac{y}{k}} \therefore f^{-1}(x) = e^{\frac{x}{k}}$ (3)

(b) $gf(x) = e^{k \ln x} = e^{\ln x^k} = x^k$ (3)

(c) $gf(2) = 16 \therefore 2^k = 16 \Rightarrow k = 4$. (2)

5. $\sin x = \frac{3}{5}$,

$\cos^2 x + \sin^2 x = 1$

$\therefore \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos x = -\frac{4}{5} \therefore x$ is obtuse.

$\therefore \tan x = -\frac{3}{4}$

$\cot 2x = \frac{1}{\tan 2x} = \frac{1 - \tan^2 x}{2 \tan x}$

$= \frac{1 - \frac{9}{16}}{2 \left(-\frac{3}{4}\right)}$

$= \frac{\frac{7}{16}}{\frac{-3}{2}} = \frac{-7}{24}$ (7)

6. (a) $\frac{dy}{dx} = x \cdot 2e^{2x} + e^{2x} \cdot 1$

$\frac{dy}{dx} = 0, \quad (2x + 1)e^{2x} = 0$

$\Rightarrow x = -\frac{1}{2}$ pt. $\left(-\frac{1}{2}, -\frac{1}{2e}\right)$

$\frac{d^2y}{dx^2} = 2(2x + 1)e^{2x} + e^{2x} \cdot 2$

$x = -\frac{1}{2}, \quad \frac{d^2y}{dx^2} > 0 \therefore \text{min.}$ (8)

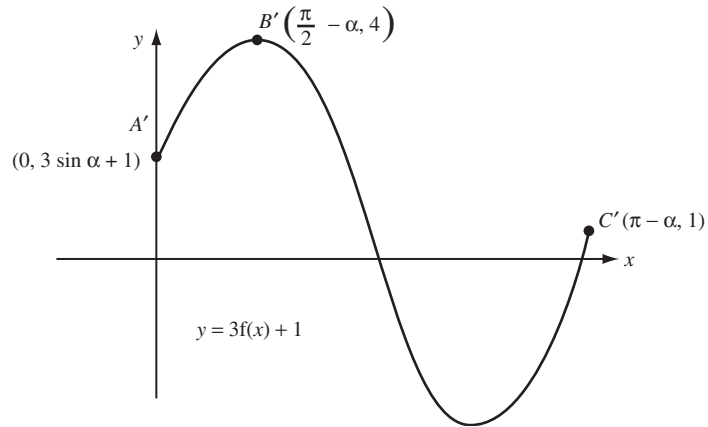
(b) $\frac{dy}{dx} = e^0 = 1$ at origin \therefore tangent is $y = x$. (2)

7. (a) $A(0, \sin \alpha)$

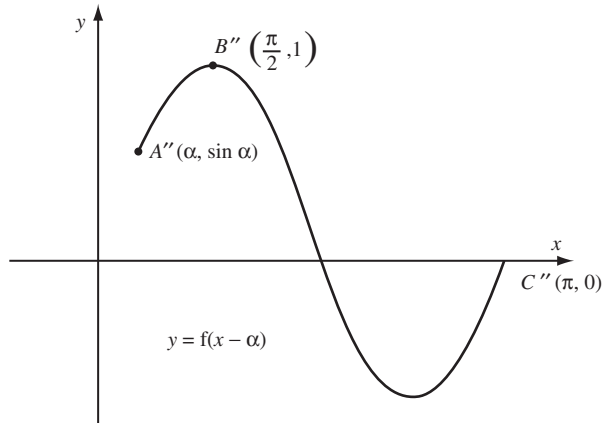
(b) $B\left(\frac{\pi}{2} - \alpha, 1\right)$

(c) $(\pi - \alpha, 0)$

(b) (i)

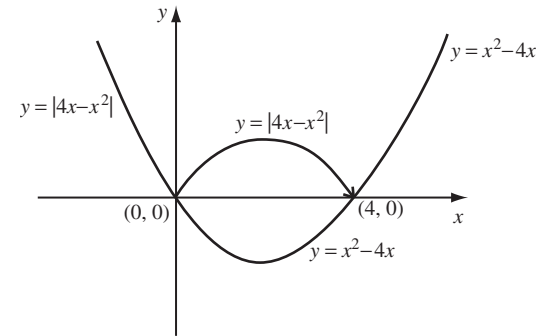


(ii)



(3)

8.



(4)

(a) $A = 2 \left| \int_0^4 x^2 - 4x \, dx \right|$

$= 2 \left| \left[\frac{x^3}{3} - \frac{4x^2}{2} \right]_0^4 \right|$

$= 2 \left| \frac{64}{3} - 32 \right|$

$= 21 \frac{1}{3} \text{ sq. units}$

(5)

(b) $y = x^2 - 4x$

$\frac{dy}{dx} = 2x - 4$

$x = 4$, gradient = 4.

$y = 4x - x^2$,

$\frac{dy}{dx} = 4 - 2x$

$x = 4$ gradient = -4

$\angle = 2 \arctan 4 = 151.9^\circ$

(8)