Core Mathematics C3 For Edexcel
Advanced Level

Paper L
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Express

$$\frac{x}{x^2 - 9} - \frac{1}{x^2 - 4x + 3}$$

as a single fraction in its simplest form. **(6)**

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2. The function f is given by

$$\mathrm{f}\colon x \mapsto \mathrm{e}^{2x+3}, \qquad x \in \mathbb{R}.$$

(*a*) Find the exact value of ff(0). **(2)**

(*b*) Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

(*c*) Write down the domain of f^{-1}. **(1)**

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3. Given that

$$x = \ln\left(y^2 + 4\right),$$

show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{2} + \dfrac{2}{y}$. **(6)**

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4. $$\mathrm{f}(x) = \ln x - 3x + 5, \qquad x > 0$$

(*a*) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval [1, 2]. **(2)**

The root α is to be estimated using the iterative formula

$$x_{n+1} = \frac{1}{3}\left(\ln x_n + 5\right), \quad x_0 = 2.$$

(*b*) Calculate the values of x_1, x_2, x_3 and x_4 giving your answers to 4 significant figures. **(3)**

(*c*) Prove that α is 1.876, to 4 significant figures. **(2)**

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5. (*a*) Given that $y = \tan x + \sin 2x$, find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \dfrac{\pi}{4}$. **(4)**

(*b*) Find the equation of the tangent to the curve at the point where $x = \dfrac{\pi}{4}$. **(3)**

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6. (*a*) Prove that

$$\sin 2\theta \equiv \frac{2\tan\theta}{1+\tan^2\theta}.$$

(4)

(*b*) Hence solve the equation

$$\tan\theta(4-\tan\theta)=1, \quad 0<\theta<\frac{\pi}{2}.$$

(5)

7.

Figure 1

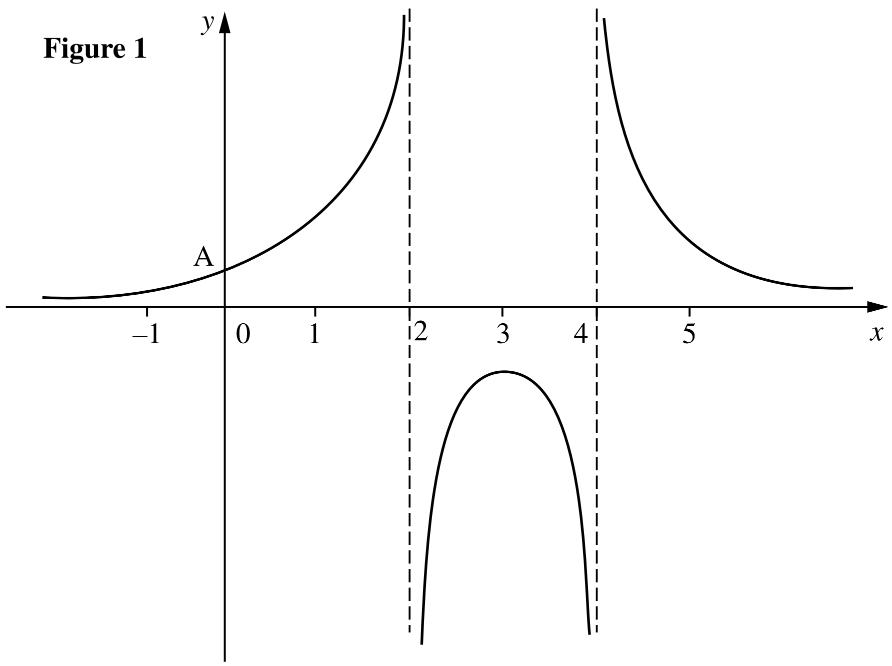

Figure 1 shows a sketch of the curve with the equation $y = \mathrm{f}(x), x \in \mathbb{R}$.

The curve has a maximum point at $(3, -1)$ and meets the y-axis at the point $A\,(0,\ 0.125)$.

The lines $x = 2, x = 4$ and the x axis are asymptotes to the curve as shown in Fig. 1.

On a separate diagram sketch the graphs of

(*a*) $y = |4\mathrm{f}(x)|$

(5)

(*b*) $y = \mathrm{f}(x+3)$

(4)

In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

(ii) how the curve approaches the asymptotes of the curve,

(iii) the coordinates of A.

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8. (*a*) On the same pair of axes sketch the graphs of

$$y = |x - a| \quad \text{and} \quad y = 2a - |x - a| \quad \text{where } a > 0. \text{ Label the graphs clearly.}$$ **(5)**

(*b*) Write down the coordinates of the points of intersection of the two graphs. **(2)**

(*c*) Find the area of the quadrilateral formed. **(3)**

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9. (*a*) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the values of R and α to 3 significant figures. **(4)**

(*b*) Find the maximum and minimum values of $\cos\theta + 2\sin\theta$ and the smallest possible value for θ for which the maximum occurs. **(2)**

The depth d metres, of water in a lake is modelled using the equation

$$d = 15 + \cos\left(\frac{\pi t}{12}\right) + 2\sin\left(\frac{\pi t}{12}\right), \quad 0 \le t < 24,$$

where t hours is the number of hours after 1200.

(*c*) Calculate the maximum depth of water predicted by this model and the value of t when this maximum occurs. **(4)**

(*d*) Calculate the depth of the water at 1200. **(1)**

(*e*) Calculate, to the nearest half hour, the time in the evening when the depth of the water is 15 metres. **(4)**

END **TOTAL 75 MARKS**