FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper J

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

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1. *(a)* Given that $\cos x = \sqrt{3} - 1$, find the value of $\cos 2x$ in the form $a + b\sqrt{3}$, where a and b are integers. **(3)**

(b) Given that

$$2\cos(y + 30)° = \sqrt{3}\sin(y - 30)°,$$

find the value of $\tan y$ in the form $k\sqrt{3}$ where k is a rational constant. **(5)**

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2. The functions f and g are defined by

$$f(x) \equiv x^2 - 3x + 7, \quad x \in \mathbb{R},$$

$$g(x) \equiv 2x - 1, \quad x \in \mathbb{R}.$$

(a) Find the range of f. **(3)**

(b) Evaluate gf(−1). **(2)**

(c) Solve the equation

$$fg(x) = 17.$$ **(4)**

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3. $$f(x) = \frac{x^4 + x^3 - 13x^2 + 26x - 17}{x^2 - 3x + 3}, \quad x \in \mathbb{R}.$$

(a) Find the values of the constants A, B, C and D such that

$$f(x) = x^2 + Ax + B + \frac{Cx + D}{x^2 - 3x + 3}.$$ **(4)**

The point P on the curve $y = f(x)$ has x-coordinate 1.

(b) Show that the normal to the curve $y = f(x)$ at P has the equation

$$x + 5y + 9 = 0.$$ **(6)**

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4. *(a)* Given that

$$x = \sec \frac{y}{2}, \quad 0 \leq y < \pi,$$

show that

$$\frac{dy}{dx} = \frac{2}{x\sqrt{x^2 - 1}}.$$ **(5)**

(b) Find an equation for the tangent to the curve $y = \sqrt{3 + 2\cos x}$ at the point where $x = \frac{\pi}{3}$. **(6)**

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5. $$f(x) = 5 + e^{2x-3}, \quad x \in \mathbb{R}.$$

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

(c) Solve the equation $f(x) = 7$. **(2)**

(d) Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. **(4)**

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6. *(a)* Prove the identity

$$2 \cot 2x + \tan x \equiv \cot x, \quad x \neq \frac{n}{2}\pi, \quad n \in \mathbb{Z}.$$ **(5)**

(b) Solve, for $0 \leq x < \pi$, the equation

$$2 \cot 2x + \tan x = \operatorname{cosec}^2 x - 7,$$

giving your answers to 2 decimal places. **(6)**

Turn over

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7. The functions f and g are defined by

$$f : x \to |2x - 5|, \quad x \in \mathbb{R},$$

$$g : x \to \ln (x + 3), \quad x \in \mathbb{R}, \quad x > -3.$$

(a) State the range of f. **(1)**

(b) Evaluate fg(−2). **(2)**

(c) Solve the equation

$$fg(x) = 3,$$

giving your answers in exact form. **(5)**

(d) Show that the equation

$$f(x) = g(x)$$

has a root, α, in the interval [3, 4]. **(2)**

(e) Use the iteration formula

$$x_{n+1} = \tfrac{1}{2}[5 + \ln (x_n + 3)],$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures. **(3)**

(f) Show that your answer for x_4 is the value of α correct to 4 significant figures. **(2)**

END

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