FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

# Core Mathematics C3

Paper I

Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

*Written by Shaun Armstrong*

**1.** Express

$$\frac{2x}{2x^2+3x-5} \div \frac{x^3}{x^2-x}$$

as a single fraction in its simplest form. **(5)**

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**2.**

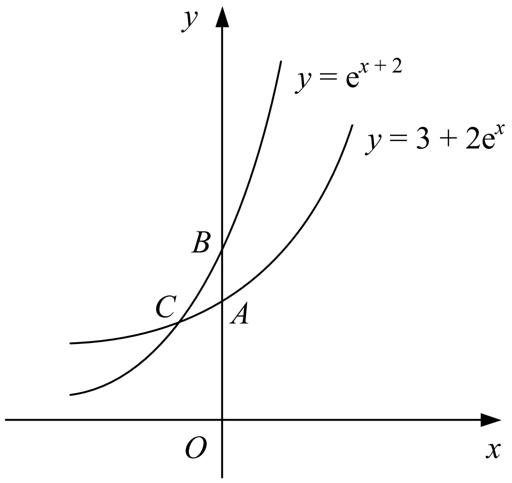


**Figure 1**

Figure 1 shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$ which cross the $y$-axis at the points $A$ and $B$ respectively.

*(a)* Find the exact length $AB$. **(3)**

The two curves intersect at the point $C$.

*(b)* Find an expression for the $x$-coordinate of $C$ and show that the $y$-coordinate of $C$ is $\frac{3e^2}{e^2-2}$. **(5)**

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**3.**

$$f(x) = \frac{x^2+3}{4x+1}, \quad x \in \mathbb{R}, \quad x \neq -\tfrac{1}{4}.$$

*(a)* Find and simplify an expression for $f'(x)$. **(3)**

*(b)* Find the set of values of $x$ for which $f(x)$ is increasing. **(5)**

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**4.** The curve $C$ has the equation $y = x^2 - 5x + 2\ln\frac{x}{3}$, $x > 0$.

*(a)* Show that the normal to $C$ at the point where $x = 3$ has the equation

$$3x + 5y + 21 = 0. \qquad \textbf{(5)}$$

*(b)* Find the $x$-coordinates of the stationary points of $C$. **(3)**

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**5.** The functions f and g are defined by

$$\mathrm{f}(x) \equiv 6x - 1, \quad x \in \mathbb{R},$$

$$\mathrm{g}(x) \equiv \log_2(3x + 1), \quad x \in \mathbb{R}, \quad x > -\tfrac{1}{3}.$$

*(a)* Evaluate gf(1). **(2)**

*(b)* Find an expression for $\mathrm{g}^{-1}(x)$. **(3)**

*(c)* Find, in terms of natural logarithms, the solution of the equation

$$\mathrm{fg}^{-1}(x) = 2. \qquad \textbf{(4)}$$

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**6.** *(a)* Use the identities for $\cos(A + B)$ and $\cos(A - B)$ to prove that

$$\cos P - \cos Q \equiv -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}. \qquad \textbf{(4)}$$

*(b)* Hence find all solutions in the interval $0 \le x < 180$ to the equation

$$\cos 5x^\circ + \sin 3x^\circ - \cos x^\circ = 0. \qquad \textbf{(7)}$$

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***Turn over***

7. The function f is defined by

$$f(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where $a$ is a positive constant.

*(a)* Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of

*(i)* $y = |f(x)|$,

*(ii)* $y = f(|x|)$. **(6)**

The function g is defined by

$$g(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

*(b)* Find $fg(a)$ in terms of $a$. **(2)**

*(c)* Solve the equation

$$gf(x) = 9a^3.$$ **(4)**

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8. $$f(x) = 2x + \sin x - 3\cos x.$$

*(a)* Show that the equation $f(x) = 0$ has a root in the interval $[0.7, 0.8]$. **(2)**

*(b)* Find an equation for the tangent to the curve $y = f(x)$ at the point where it crosses the $y$-axis. **(4)**

*(c)* Find the values of the constants $a$, $b$ and $c$, where $b > 0$ and $0 < c < \frac{\pi}{2}$, such that

$$f'(x) = a + b\cos(x - c).$$ **(4)**

*(d)* Hence find the $x$-coordinates of the stationary points of the curve $y = f(x)$ in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places. **(4)**

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**END**