FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Given that

$$x = \sec^2 y + \tan y,$$

show that

$$\frac{dy}{dx} = \frac{\cos^2 y}{2\tan y + 1}. \qquad \textbf{(4)}$$

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2. The functions f and g are defined by

$$f : x \to 3x - 4, \quad x \in \mathbb{R},$$

$$g : x \to \frac{2}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(a) Evaluate fg(1). **(2)**

(b) Solve the equation gf(x) = 6. **(4)**

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3. Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0 \qquad \textbf{(8)}$$

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4. *(a)* Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{d}{dx}(\tan x) = \sec^2 x. \qquad \textbf{(4)}$$

The tangent to the curve $y = 2x \tan x$ at the point where $x = \frac{\pi}{4}$ meets the y-axis at the point P.

(b) Find the y-coordinate of P in the form $k\pi^2$ where k is a rational constant. **(6)**

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5. *(a)* Express $3\cos x° + \sin x°$ in the form $R\cos(x - \alpha)°$ where $R > 0$ and $0 < \alpha < 90$. **(4)**

(b) Using your answer to part *(a)*, or otherwise, solve the equation

$$6\cos^2 x° + \sin 2x° = 0,$$

for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place where appropriate. **(6)**

6.

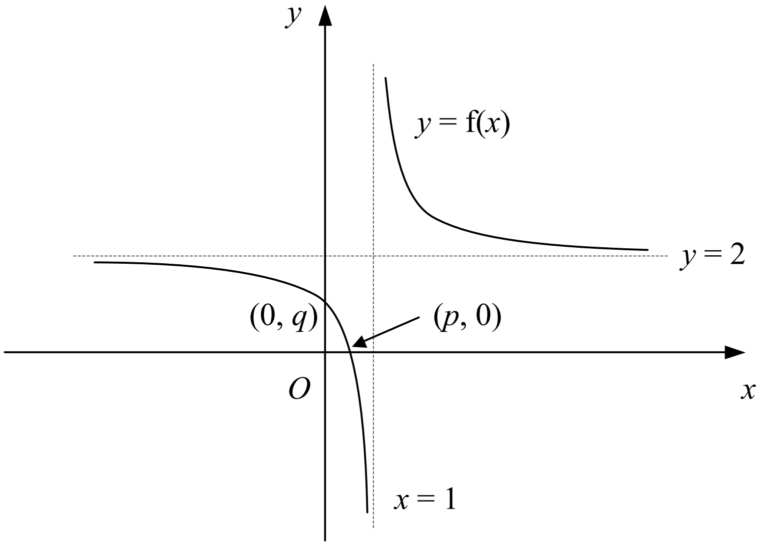

Figure 1

Figure 1 shows the curve with equation $y = \mathrm{f}(x)$. The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.

(a) Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of

(i) $y = |\mathrm{f}(x)|$,

(ii) $y = 2\mathrm{f}(x + 1)$. **(6)**

Given also that

$$\mathrm{f}(x) \equiv \frac{2x-1}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1,$$

(b) find the values of p and q, **(3)**

(c) find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

Turn over

7. *(a)* *(i)* Show that

$$\sin (x + 30)° + \sin (x - 30)° \equiv a \sin x°,$$

where a is a constant to be found.

(ii) Hence find the exact value of $\sin 75° + \sin 15°$, giving your answer in the form $b\sqrt{6}$. **(6)**

(b) Solve, for $0 \leq y \leq 360$, the equation

$$2 \cot^2 y° + 5 \operatorname{cosec} y° + \operatorname{cosec}^2 y° = 0.$$ **(6)**

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8. $$\mathrm{f}(x) = \frac{x^4 + x^3 - 5x^2 - 9}{x^2 + x - 6}.$$

(a) Using algebraic division, show that

$$\mathrm{f}(x) = x^2 + A + \frac{B}{x + C},$$

where A, B and C are integers to be found. **(5)**

(b) By sketching two suitable graphs on the same set of axes, show that the equation $\mathrm{f}(x) = 0$ has exactly one real root. **(3)**

(c) Use the iterative formula

$$x_{n+1} = 2 + \frac{1}{x_n^2 + 1},$$

with a suitable starting value to find the root of the equation $\mathrm{f}(x) = 0$ correct to 3 significant figures and justify the accuracy of your answer. **(5)**

END