FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

# Core Mathematics C3

## Paper K

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

*Written by Shaun Armstrong*

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# C3 Paper K – Marking Guide

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | arctan $(x-2) = -\frac{\pi}{3}$ | M1 | |
| | | $x - 2 = \tan(-\frac{\pi}{3}) = -\sqrt{3}$ | M1 | |
| | | $x = 2 - \sqrt{3}$ | A1 | |
| | *(b)* | $1 - 2\sin^2\theta - \sin\theta - 1 = 0$<br>$2\sin^2\theta + \sin\theta = 0$ | M1 | |
| | | $\sin\theta(2\sin\theta + 1) = 0$ | M1 | |
| | | $\sin\theta = 0$ or $-\frac{1}{2}$ | A1 | |
| | | $\theta = 0$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$ | | |
| | | $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$ | A2 | **(8)** |

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|---|---|---|---|---|
| **2.** | *(a)* | $= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3}$ | M1 | |
| | | $= \frac{4x - 2(x-3)}{(x+3)(x-3)}$ | M1 | |
| | | $= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)}$ | M1 | |
| | | $= \frac{2}{x-3}$ | A1 | |
| | *(b)* | $2^3 - 8 = 0 \;\therefore\; (x-2)$ is a factor of $(x^3 - 8)$ | B1 | |
| | | $$\begin{array}{r\|l} & x^2 + 2x + 4 \\ \hline x-2 & x^3 + 0x^2 + 0x - 8 \\ & x^3 - 2x^2 \\ \hline & 2x^2 + 0x \\ & 2x^2 - 4x \\ \hline & 4x - 8 \\ & 4x - 8 \end{array}$$ | M1 A1 | |
| | | $\therefore\; x^3 - 8 = (x-2)(x^2 + 2x + 4)$ | | |
| | | $\therefore\; \frac{x^3-8}{3x^2-8x+4} = \frac{(x-2)(x^2+2x+4)}{(3x-2)(x-2)} = \frac{x^2+2x+4}{3x-2}$ | M1 A1 | **(9)** |

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|---|---|---|---|---|
| **3.** | *(a)* | $= -\text{cosec}^2 x^2 \times 2x = -2x\,\text{cosec}^2 x^2$ | M1 A1 | |
| | *(b)* | $= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2 - x)$ | M1 A2 | |
| | *(c)* | $= \frac{\cos x \times (3 + 2\cos x) - \sin x \times (-2\sin x)}{(3 + 2\cos x)^2}$ | M1 A1 | |
| | | $= \frac{3\cos x + 2\cos^2 x + 2\sin^2 x}{(3+2\cos x)^2} = \frac{3\cos x + 2}{(3 + 2\cos x)^2}$ | M1 A1 | **(9)** |

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|---|---|---|---|---|
| **4.** | *(a)* | $(e^x - 3)(e^x - 5) = 0$<br>$e^x = 3, 5$ | M1 A1 | |
| | | $x = \ln 3, \ln 5$ | M1 A1 | |
| | *(b)* | assume $\log_2 3$ is rational | B1 | |
| | | $\therefore\; \log_2 3 = \frac{p}{q}$ where $p$ and $q$ are integers and $q \neq 0$ | M1 | |
| | | $\Rightarrow\; 2^{\frac{p}{q}} = 3$ | M1 | |
| | | $\Rightarrow\; 2^p = 3^q$ | A1 | |
| | | 2 and 3 are co-prime $\therefore$ only solution is $p = q = 0$ | M1 | |
| | | but $q \neq 0 \;\therefore$ contradiction $\therefore\; \log_2 3$ is irrational | A1 | **(10)** |

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**5.** *(a)* $f(x) > 0$ — B1

*(b)* $y = 3e^{x-1}$, $\quad x - 1 = \ln\frac{y}{3}$ — M1

$x = 1 + \ln\frac{y}{3}$

$f^{-1}(x) = 1 + \ln\frac{x}{3}$, $x \in \mathbb{R}$, $x > 0$ — M1 A2

*(c)* $f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$ — M1 A1

$gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$ — A1

*(d)* $f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln\frac{5x-2}{3}$ — M1 A1

$\therefore\ 1 + \ln\frac{5x-2}{3} = 4$, $\quad \frac{5x-2}{3} = e^3$ — M1

$x = \frac{1}{5}(3e^3 + 2)$ — A1 **(12)**

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**6.** *(a)* $2x^2 + 3\ln(2 - x) = 0 \;\Rightarrow\; 3\ln(2 - x) = -2x^2$

$\ln(2 - x) = -\frac{2}{3}x^2$ — M1

$2 - x = e^{-\frac{2}{3}x^2}$ — M1

$x = 2 - e^{-\frac{2}{3}x^2}$ $\quad [\,k = -\frac{2}{3}\,]$ — A1

*(b)* $x_1 = 1.90988$, $x_2 = 1.91212$, $x_3 = 1.91262$, $x_4 = 1.91273$ — M1 A1

$\therefore\ \alpha = 1.913$ (3dp) — A1

$f(1.9125) = 0.0070$, $f(1.9135) = -0.020$ — M1

sign change, $f(x)$ continuous $\therefore$ root — A1

*(c)* $f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$ — M1 A1

$\therefore\ 4x - \frac{3}{2-x} = 0$, $\quad 4x = \frac{3}{2-x}$, $\quad 4x(2 - x) = 3$ — M1

$4x^2 - 8x + 3 = 0$, $\quad (2x - 3)(2x - 1) = 0$ — M1

$x = \frac{1}{2}, \frac{3}{2}$ — A1 **(13)**

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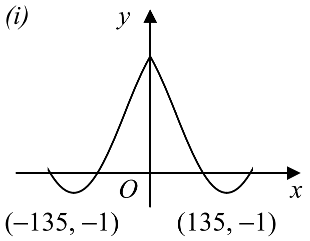
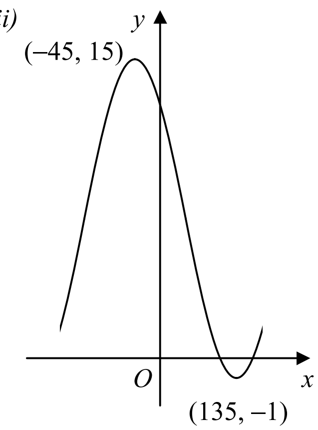
**7.** *(a)* *(i)*




B3

*(ii)*




M1 A2

*(b)* $2\sqrt{2}\cos x - 2\sqrt{2}\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 2\sqrt{2}$, $R\sin\alpha = 2\sqrt{2}$, $\quad \therefore\ R = \sqrt{8+8} = 4$ — M1 A1

$\tan\alpha = 1$, $\alpha = 45$ — B1

$\therefore\ f(x) = A + 4\cos(x + 45)°$

*(c)* 3 — B1

*(d)* $3 + 4\cos(x + 45) = 0$, $\quad \cos(x + 45) = -\frac{3}{4}$ — M1

$x + 45 = 180 - 41.4$, $180 + 41.4 = 138.6$, $221.4$ — M1

$x = 93.6$, $176.4$ (1dp) — A2 **(14)**

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Total **(75)**

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# Performance Record – C3 Paper K

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | Total |
|---|---|---|---|---|---|---|---|---|
| Topic(s) | trigonometry | rational expressions | differentiation | exponentials and logarithms, proof | functions | numerical methods, differentiation | functions, trigonometry | |
| Marks | 8 | 9 | 9 | 10 | 12 | 13 | 14 | 75 |
| Student | | | | | | | | |
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