Practice Paper A

CORE THREE

crashMATHS

Name	

Duration	**1 HOUR & 30 MINUTES**
Total Marks Available	**75 MARKS**

Targets

For examiner's use only

Question Number	*Leave Blank*
1	
2	
3	
4	
5	
6	
7	
8	
/	
Total marks	

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1 The equation

$$x^4 + x^2 - 8x - 9 = 0$$

has two roots. One of its roots is α.

(a) Show that α lies in the interval $[2.1, 2.2]$. **(2)**

(b) Show also that

$$x = \sqrt[4]{9 + 8x - x^2}$$ **(1)**

(c) Starting with $x_0 = 2.1$, use the iterative formula

$$x_{n+1} = \sqrt[4]{9 + 8x_n - x_n^{\,2}}$$

to find x_1, x_2 and x_3 to three decimal places. **(3)**

(d) Hence, state the value of α to one decimal place. **(1)**

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Question 1 continued

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Question 1 continued

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Question 1 continued

TOTAL 7 MARKS

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2 (a) Show that

$$\frac{x^3+6x^2-8}{x+1} \equiv ax^2+bx+c+\frac{d}{x+1}$$

where a, b, c and d are constants to be found. **(4)**

(b) Hence, or otherwise, given that

$$f(x)=\frac{x^3+6x^2-8}{x+1}$$

Find $f'(x)$, giving each term in its simplest form. **(4)**

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Question 2 continued

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Question 2 continued

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Question 2 continued

TOTAL 8 MARKS

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3 The curve $y = f(x)$ is defined such that

$$y = \frac{5}{e^{2x}} - 4$$

(a) Solve $f(x) = 0$. **(3)**

(b) Sketch $y = f(x)$.

On your sketch, you should indicate clearly any coordinates at which the curve crosses or meets the coordinate axis.

You should state the equation of any asymptotes to the curve. **(3)**

(c) Find the range of values of x for which $f(x) = |f(x)|$. **(1)**

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Question 3 continued

TOTAL 7 MARKS

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4 A factory produces party hats in the shape of a cone.

The machines are programmed so that all of the party hats have a height of 16 cm and a variable radius x cm.

(a) Show that the total surface area A of the cone is given by

$$A = \pi x^2 + \pi x\sqrt{256 + x^2}$$ **(3)**

(b) Find $\frac{dA}{dx}$ when the radius of the cone is 12 cm. **(5)**

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Question 4 continued

TOTAL 8 MARKS

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5 It is given that

$$4\cos\theta - 7\sin\theta \equiv R\cos(\theta - \alpha)$$

where α is measured in radians.

(a) Find the value of

(i) α

(ii) R

(4)

(b) Solve, for $0 < \theta \leq 2\pi$,

$$8\cos 2\theta - 14\sin 2\theta = 15\sin(2\theta - \alpha)$$

(6)

(c) Given also that

$$p(\theta) = 10 - (7\sin\theta - 4\cos\theta)^2$$

Find

(i) the maximum value of $p(\theta)$.

(ii) the smallest value of θ for which $p(\theta)$ is maximum. **(4)**

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Question 5 continued

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Question 5 continued

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Question 5 continued

TOTAL 14 MARKS

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6 The mass M, in grams, of a radioactive isotope decreases exponentially with time. Initially, the mass of the isotope is 120.0g and, 4 years later, its mass is 72.9g. Find the mass of the substance at a time 10 years after the initial value. **(5)**

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Question 6 continued

TOTAL 5 MARKS

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7 (i) Differentiate with respect to x

(a) $e^{3x^2}\sin x$ **(4)**

(b) $\cos\left(\ln\left(2x^3\right)\right)$ **(4)**

(ii) Given that

$$y = \sec^3 x$$

Show that

$$\frac{d^2y}{dx^2} = 9y + 3y^{\frac{3}{5}} - 9y^{\frac{1}{3}}$$

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

TOTAL 14 MARKS

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8 (a) Solve, for $0 < x \leq 2\pi$,

$$\sec\left(x + \frac{\pi}{3}\right) = \operatorname{cosec}\left(x + \frac{\pi}{4}\right)$$

(8)

(b) Given that

$$\arcsin x = k, \ 0 < k < \frac{\pi}{2}$$

Find, in terms of x, $\tan k$, for $-\frac{\pi}{2} < k < 0$. **(4)**

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Question 8 continued

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Question 8 continued

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Question 8 continued

TOTAL 12 MARKS