FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. The function f is defined by

$$\mathrm{f}(x) \equiv 2 + \ln(3x - 2), \quad x \in \mathbb{R}, \quad x > \tfrac{2}{3}.$$

(a) Find the exact value of ff(1). **(2)**

(b) Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

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2. Find, to 2 decimal places, the solutions of the equation

$$3\cot^2 x - 4\operatorname{cosec} x + \operatorname{cosec}^2 x = 0$$

in the interval $0 \leq x \leq 2\pi$. **(6)**

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3. *(a)* Given that $y = \ln x$, find expressions in terms of y for

(i) $\log_2 x$,

(ii) $\ln \frac{x^2}{\mathrm{e}}$. **(4)**

(b) Hence, or otherwise, solve the equation

$$\log_2 x = 4 - \ln \frac{x^2}{\mathrm{e}},$$

giving your answer to 2 decimal places. **(4)**

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4. *(a)* Use the identities for $(\sin A + \sin B)$ and $(\cos A + \cos B)$ to prove that

$$\frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} \equiv \tan(x + y).$$ **(4)**

(b) Hence, show that

$$\tan 52.5° = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2.$$ **(5)**

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5. $$\mathrm{f}(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2-5x-3}, \quad x \in \mathbb{R}, \quad x < -1.$$

(a) Show that

$$\mathrm{f}(x) = \frac{4x-1}{2x+1}.$$ **(5)**

(b) Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

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6. A curve has the equation $y = \mathrm{e}^{3x} \cos 2x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

(b) Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{3x}(5\cos 2x - 12\sin 2x)$. **(3)**

The curve has a stationary point in the interval $[0, 1]$.

(c) Find the x-coordinate of the stationary point to 3 significant figures. **(4)**

(d) Determine whether the stationary point is a maximum or minimum point and justify your answer. **(2)**

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7. *(a)* Sketch on the same diagram the graphs of $y = 4a^2 - x^2$ and $y = |2x - a|$, where a is a positive constant. Show, in terms of a, the coordinates of any points where each graph meets the coordinate axes. **(6)**

(b) Find the exact solutions of the equation

$$4 - x^2 = |2x - 1|.$$ **(6)**

Turn over

8.

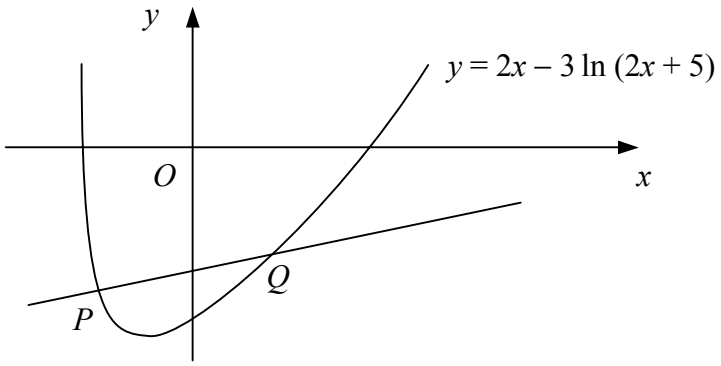

Figure 1

Figure 1 shows the curve with equation $y = 2x - 3\ln(2x + 5)$ and the normal to the curve at the point $P(-2, -4)$.

(a) Find an equation for the normal to the curve at P. **(4)**

The normal to the curve at P intersects the curve again at the point Q with x-coordinate q.

(b) Show that $1 < q < 2$. **(3)**

(c) Show that q is a solution of the equation

$$x = \tfrac{12}{7}\ln(2x + 5) - 2.$$ **(2)**

(d) Use the iterative formula

$$x_{n+1} = \tfrac{12}{7}\ln(2x_n + 5) - 2,$$

with $x_0 = 1.5$, to find the value of q to 3 significant figures and justify the accuracy of your answer. **(5)**

END