mark scheme

Practice Paper A : Core Mathematics 2

Question Number	General Scheme		Marks
1 (a)	$\frac{4}{5}$	**M1** – use of a triangle or $\sin^2\theta = 1 - \cos^2\theta$ **A1** – cao	**M1** **A1** **(2)**
(b)	$\frac{3}{5}$	**M1** – use of a triangle or $\tan\theta = \frac{\sin\theta}{\cos\theta}$ with *their* $\sin\theta$ **A1** – cao	**M1** **A1** **(2)**
Note	Award **M1** in **both** parts if candidates draw a right angled triangle with the hypotenuse of length 5, the opposite of length 4 and the adjacent of length 3 in part (a), **but do not award this** if they draw the triangle in part (b)		
		Total	**4**

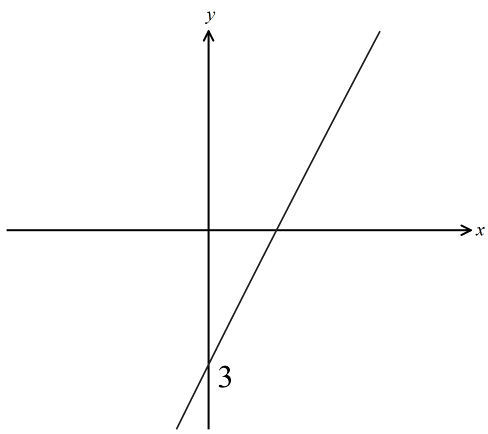
2 (a)	$$(2a+b)^8 = \underbrace{(2a)^8}_{\text{B1}} + \underbrace{{}^8C_1(2a)^7(b)^1 + {}^8C_2(2a)^6(b)^2 + {}^8C_3(2a)^5(b)^3}_{\text{M1 A2}} + \ldots$$ **B1** – first term correctly expressed (need not be evaluated). Accept equivalent forms, i.e. ${}^8C_0(2a)^8(b)^0$ **M1** – second, third and fourth terms of the expansion of the form ${}^8C_{k/8-k}(2a)^{8-k}(b)^k$ isw **A2** – correct **unsimplified** expansion in terms of binomial coefficients $$(2a+b)^8 = 256a^8 + 1024a^7b + 1792a^6b^2 + 1792a^5b^3 + \ldots$$ **A1** – correct **simplified** expansion **Note:** accept $\binom{n}{r}$ as alternative notation for ${}^nC_r$ throughout		**B1** **M1** **A2** **A1** **(5)**
(b)	$\left(a+\frac{b}{2}\right)^8 = \left[\frac{1}{2}(2a+b)\right]^8 = \frac{1}{256}(2a+b)^8$	**B1** – cso **The value of k must be clearly obtained and not just stated**	**B1** **(1)**
(c)	$\dfrac{\frac{1}{256}\left(256a^8 + 1024a^7b + 1792a^6b^2 + 1792a^5b^3 + \ldots\right)}{4ab}$		
	$\dfrac{1}{1024ab}\left(256a^8 + 1024a^7b + 1792a^6b^2 + 1792a^5b^3 + \ldots\right)$	**M1** – correct method using (b)	**M1**
	$= \dfrac{256a^8}{1024ab} + \dfrac{1024a^7b}{1024ab} + \dfrac{1792a^6b^2}{1024ab} + \dfrac{1792a^5b^3}{1024ab} + \ldots$	**A1ft** – correct expansion unsimplified	**A1**
	$\frac{1}{4}a^7b^{-1} + a^6 + \frac{7}{4}a^5b + \frac{7}{4}a^4b^2 + \ldots$	**A1** – cao	**A1** **(3)**
		Total	**9**

Question	Scheme	Guidance	Marks
3 (a)	x: 2, 3, 4, 5 y: 2.4142..., 1.3660..., 1, 0.8090...	**B2** – correct values for x and y shown in a table or implied (**B1** for one correct value)	**B2**
	$h = \frac{5-2}{3} = 1$	**B1** – correct value for the height of each strip	**B1**
	$\therefore$ Area $\approx \frac{1}{2}\left[2.4142 + 0.8090 + 2(1.3660 + 1)\right]$	**M1** – attempts to apply the trapezium rule using *their* values **A1ft** – correct expression ft of their values	**M1** **A1**
	$\approx \frac{1}{2}(7.9552)$ ≈ 3.98	**A1** – cao	**A1** **(6)**
(b)	It is an overestimate because the trapezia are above the curve	**B1** – overestimate **B1** – *idea that* the trapezia will be above the curve. Award this mark if candidates draw a diagram to illustrate this	**B1** **B1** **(2)**
		Total	**8**

Question	Scheme		Notes	Marks
4 (a)	$x^2+(y-5)^2=5^2$		**B1** – LHS correct oe **B1** – RHS correct oe	**B1** **B1** **(2)**
(b)	$x^2+(3x-5)^2=25$ $x^2+9x^2-30x+25=25$		**M1** – correct method to find the coordinates of intersection	**M1**
	$x^2-3x=0$ $x=0$, $x=3$		**A1** – correct values of x	**A1**
	$y=3(3)=9$ $\therefore P(3,9)$		**M1** – substitutes $x=3$ to find the y coordinate of P **A1** – $P(3,9)$ cao	**M1** **A1** **(4)**
(c)	**Method 1** $\cos\theta=\dfrac{5^2+5^2-\left(3\sqrt{10}\right)^2}{2(5)(5)}$ $=-\dfrac{4}{5}$	**Alternative 1** $\theta=180-2\left(90-\tan^{-1}\dfrac{9}{3}\right)$ $\therefore\cos\theta=-\dfrac{4}{5}$	**Accept other methods** **M1** – use of cosine rule or geometry **A1** – cao	**M1** **A1** **(2)**
(d)	**Method 1** $\dfrac{1}{2}\times5\times5\times\sin\theta=7.5$	**Alternative 1** $\dfrac{9\times3}{2}-\dfrac{(9-5)\times3}{2}=7.5$	**Accept other methods** **M1** – correct method **A1** – 7.5	**M1** **A1** **(2)**
(e)	Area of sector = $\dfrac{\theta}{2}r^2=\dfrac{\cos^{-1}\left(-\dfrac{4}{5}\right)}{2}(5)^2=31.226...$		**M1** – correct method to work out the area of the sector (accept working in degrees) **A1** – correct area for the sector	**M1** **A1**
	Area of R = $31.226...-7.5=23.726...=23.7$		**M1** – area of sector – area of triangle **A1** – cao, no ft	**M1** **A1** **(4)**
			Total	**14**

5 (a)	0.988	**B1** – cao	**B1** **(1)**
(b)	$U_4 = 34000 \times 0.988^4 = £32000$ **AWRT**	**M1** – expression in the form $34000 \times 0.988^{3/4}$, ft their (a). Power can be 3 or 4. **A1** – cao (full answer = £32397.14)	**M1** **A1**
(c)	$34000 \times 0.988^N > 30000$ $0.988^N > \frac{15}{17}$	**M1** – forms an equation of the form $r^N = \frac{30000}{34000}$ ft their (a). r^{N-1} shows **M0** here	**M1**
	$N \log(0.988) > \log\left(\frac{15}{17}\right)$	**M1** – uses logarithms to find N	**M1**
	$\therefore N < \frac{\log\left(\frac{15}{17}\right)}{\log(0.988)} < 10.367...$ $\therefore N = 10$	**A1** – $N = 10$ Do not accept decimals.	**A1**
Note	Use of inequalities in not necessary in (c) – accept use of the equality sign. However, when the inequality sign is used, it **must** be reversed to score the final **A** mark.		
(d) (i)	$U_N = a + (N-1)d$ $= 150 + (10-1)(100) = 1050$	**M1** – use of $a + (N-1)d$, **A1ft** – correct answer ft their (c)	**M1** **A1** **(2)**
(ii)	Takings in year 1 = $34000 \times 150 = £5100000$	**B1** – cao	**B1**
	$34000 \times 0.988^{10} = £30133.42$ Takings in year N = £31640000	**M1** – calculates the price of the car in year N **M1** – calculates the takings in year N	**M1** **M1**
	$£31640000 > £5100000$, hence there were more takings in the year N	**A1** – correct answer **and** a justification	**A1** **(4)**
(e)	(Very) effective **because** more individuals are buying the car each year / the takings increased / owtte	**B1** – effective + an appropriate justification in context	**B1** **(1)**
		Total	**13**

6 (a)	$\log_3(3b)+\log_9(b)=2$	**M1** – attempt to obtain an equation with one variable	**M1**
	Since $\log_3(x)=2\log_9(x)$, $2\log_9(3b)+\log_9(b)=2$	**B1** – this principle seen anywhere (candidates may use the formula from the booklet to obtain this)	**B1**
	$\log_9(9b^3)=2$ $9b^3=81 \to b=\sqrt[3]{9}$	**M1** – use of multiplication rule for logs **M1** – removal of the logs **A1** – correct **exact** value for b	**M1** **M1** **A1**
	$\therefore a=3\sqrt[3]{9}$	**M1** – substitutes *their* b into an equation to find a **A1** – correct **exact** value for a	**M1** **A1**
		Total	**7**
ALT	If candidates use the formula to change the base, sight of $$\log_3(3b)=\frac{\log_9(3b)}{\log_9 3}=2\log_9(3b)$$ is *all* needed for **B1.** Candidates may then, alternatively, change the $\log_9 b$ term to $\frac{1}{2}\log_3 b$, which should also be accepted. Note also that candidates may make a substitution for b at the beginning of the question – although this is likely to prove tougher algebraically.		

7 (a)	$\frac{dy}{dx} = 3x^2 - 6x + 27$	**M1** – correct method to differentiate one term **A1** – correct derivative	**M1** **A1** **(2)**
(b)	If increasing, $\frac{dy}{dx} > 0$ $3x^2 - 6x + 27 = 3\left(x^2 - 2x + \frac{27}{3}\right)$ $= 3\left[(x-1)^2 - 1 + \frac{27}{3}\right] > 0$ for all x	**B1** – idea that, if increasing, $\frac{dy}{dx} > 0$ **M1** – completes the square to show that *their* $\frac{dy}{dx} > 0$ **A1** – correct proof	**B1** **M1** **A1** **(3)**
(c)	If there are no stationary points, $3x^2 - 6x + 27 = 0$ has no solutions.	**B1** – idea that, if there are no stationary points, there is no value of x such that $\frac{dy}{dx} = 0$	**B1**
	Method 1: $(x-1)^2 = -8$ $x - 1 \neq \sqrt{-8}$ Hence, the curve has no stationary points **Method 2:** $6^2 - 4(3)(27) = -288 < 0$ Since $b^2 - 4ac < 0$, the curve has no stationary points	**M1** – correct method to show the derivative has no solutions **A1** – correctly shows **and justifies** that the derivative has no solutions	**M1** **A1** **(3)**
(d)		**B1** – correct shape (see note) **B1** – passes through $y = 3$	**B1** **B1** **(2)**
Note	Ideally, the sketches should be *slightly* curvaceous. Although, due to scales, you should be prepared to accept completely straight lines here. **Do not**, however, accept sketches that imply the presence of stationary points or points of inflection.		
		Total	**7**

8 (a)	$\sin\theta - \frac{1}{\sin\theta} \equiv \frac{\sin^2\theta - 1}{\sin\theta}$	**M1** – use of a common denominator	**M1**
	$\equiv \frac{-\cos^2\theta}{\sin\theta} \equiv -\cos\theta \times \frac{\cos\theta}{\sin\theta}$	**M1** – use of $1-\sin^2\theta = \cos^2\theta$	**M1**
	$\equiv -\cos\theta \times \frac{1}{\tan\theta} = -\frac{\cos\theta}{\tan\theta}$	**A1** – correct proof showing all intermediate steps	**A1** **(3)**
(b)	$\tan^2 2x = \frac{2}{\cos 2x}\left(-\frac{\cos 2x}{\tan 2x}\right) + \tan^2 2x - 4$	**B1** – correct LHS **M1** – use of (a) to simplify the RHS	**B1** **M1**
	$4 = -\frac{2}{\tan 2x}$ $\tan 2x = -\frac{1}{2}$	**A1** – correct simplification	**A1**
	$2x = \tan^{-1}\left(-\frac{1}{2}\right) = -0.463...$	**dM1** – $2x = \tan^{-1}(n)$	**M1**
	$2x = \pi - 0.463...,\ 2\pi - 0.463...,$ **and** $2\pi + (\pi - 0.463...),\ 2\pi + (2\pi - 0.463...)$	**ddM1** – attempt to find values of $2x$ between 0 and 4π	**M1**
	$2x = 2.678...,5.820...,8.961...,12.103...$ $x = 1.3,\ 2.9,\ 4.5,\ 6.1$ **AWRT**	**ddM1** – divides at least one of their values of $2x$ by 2 **A1** – correct values of x	**M1** **A1** **(7)**
		Total	**10**

Notes on alternative methods:

This mark scheme may feature some alternative solutions, but, of course, at this level, there is likely to be questions that have many others. Where alternative methods are used, you should award full marks **if the method is correct** (do **not** award full marks for methods that coincidentally lead to the right answer). If the method is *not* correct, then you should aim to mark it by being as faithful to the original scheme as you can and ensure that you award the same amount of marks for the same amount of *progress* in a question as you would award using the general scheme.