Mark Scheme 1	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed *Where appropriate, leave your answers to 3 s.f.*	
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1. It is not necessary to multiply out the denominator.

Obtain common factors in both denominators $\dfrac{\ldots}{(x-1)(x+2)(x+3)}$ — M1

Combine to single denominator — M1

Multiply out numerator to $(x^2+4x+3)-(6x+12)$ — M1

Simplify to $\dfrac{x^2-2x-9}{(x-1)(x+2)(x+3)}$ — A1 (4)

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2. a) i) If f(any positive integer) attempted — M1
Show that f(1) = –1, f(2) = 13 — M1
Obtain answer $1 < x < 2$, or n = 1 — A1 (3)

ii) f(x) is continuous — M1
If f(1) < 0 and f(2) > 0 — M1 for both
Then there exists x in the interval $1 < x < 2$ such that f(x) = 0 — M1 (3)
Accept also a generalized solution with n and (n+1) or a good sketch with clear argument!

b) Show formula $x_{n+1} = \sqrt[4]{(1+x_n)}$ — M1
Construct a table showing x_n and x_{n+1} — M1
Iterate formula and show values in table — M1
Obtain answer x = 1.2207… = 1.221 (3 d.p.) — A1 (4)

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3. a) g(x) = 1 + f(x + 1)
= 1 + $(x + 1 - 3)^2 + 4$ — M1
= $(x - 2)^2 + 5$ or equivalent — A1 (2)

b) i) f(x) to g(x) is a **translation** 1 up and *1 left.* — **A1**A1*A1* (3)

ii) g(x) to f(x) is a translation 1 down and 1 right. — A1 (1)

c) h(x) = 1 = $|x + 2| - 3$
4 = $|x + 2|$ — M1
Therefore x = 2 or –6 — A1A1(3)

d) h(–3) = $|-3 + 2| - 3$
= $|-1| - 3$
= 1 – 3 = –2 — A1
f(h(–3)) = f(–2) — M1
= $(-2 - 3)^2 + 4$
= 29 — A1 (3)

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4. a) Use formula $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ — M1
Show C = $2x$ — A1 (2)

b) Show that 2 cos 45° cos 15° = cos 60° + cos 30° — M1

Write down results; $\cos 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, $\cos 60° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$ M1

Substitute into equation $\frac{2}{\sqrt{2}}\cos 15° = \frac{1}{2} + \frac{\sqrt{3}}{2}$ M1

Simplify to $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$ A1 (4)

c) From a) $2\cos 3x\cos x = \cos 4x + \cos 2x$
Solve $\cos 4x + \cos 2x = 1$ M1
Let $X = 2x$
$\cos 2X + \cos X = 1$
Using $\cos^2 X + \sin^2 X = 1$ and $\cos^2 X - \sin^2 X = \cos 2X$ to give $\cos 2X = 2\cos^2 X - 1$
So $2\cos^2 X + \cos X - 2 = 0$ M1
Let $Y = \cos X$

Therefore $Y^2 + \frac{Y}{2} - 1 = 0$ M1

Solve to find $Y = -\frac{1}{4} \pm \frac{\sqrt{17}}{4}$ A1

There $\cos X = -\frac{1}{4} \pm \frac{\sqrt{17}}{4}$

$X = \cos^{-1}\left(-\frac{1}{4} \pm \frac{\sqrt{17}}{4}\right)$ A1

$-1 \le \cos X \le 1$, so we ignore negative root since its value is –1.28
X = 38.668…° or 321.331…°
Therefore $2x$ = 38.668…°, 321.33…°, 398.66…°, 681.33…°
Therefore x = 19.334…°, 160.66…°, 199.33…°, 340.66…°
=19.3°,160.7°, 199.3°, 340.7° (1 d.p.) A1 (6)

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5. a) Substitute $g(x)$ into $f(x)$ to obtain $fg(x) = (4x-2)^3$ or $= [8(8x^3 - 8x^2 + 4x - 1)]$ A1
Substitute $f(x)$ into $g(x)$ to obtain $gf(x) = 4x^3 - 2$ A1 (2)

b) $y = 4\sin x - 2$ A1
Max at y = 2, min at y = –6 A1
Single sine shape A1

Minimum point occurs when $x = \frac{3\pi}{2}$ and y = –6

So coordinates of min are $\left(\frac{3\pi}{2}, -6\right)$ A1 (4)

c) Using equation $y = \frac{x+1}{x-1}$
Swap variables x and y M1

Rearrange the equation to show $x = \frac{y+1}{y-1}$ and state that $h^{-1}(x) = \frac{x+1}{x-1}$ i.e. self-inverse A1A1

State the domain; y: $\in \mathbb{R}$, $y \neq 1$ A1
State range; $h^{-1}(x)$: $-\infty < h^{-1}(x) < 1$, $1 < h^{-1}(x) < +\infty$ A1 (5)

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6. a) Use the formula $R = \sqrt{a^2 + b^2}$ M1
Obtain the result $R = \sqrt{5}$ A1

Use the formula $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$ M1

Obtain the result $\alpha = \tan^{-1}(2) = 63.434\ldots° = 63.4°$ (3sf) A1 (4)

b) Substitute $R\cos(x - \alpha)$ for $\cos x + 2\sin x$ and equate to 1

$\therefore \cos(x - \alpha) = \frac{1}{\sqrt{5}}$ M1

take $\cos^{-1}$ and add α to obtain the results M1

$x - 63.435\ldots = 63.435\ldots°$ or $-63.435\ldots°$

$x = 0°, 126.86\ldots° = 127°$ (3 s.f). A1A1 (4)

c) Substitute $R\cos(x - \alpha)$ for $\cos x + 2\sin x$ into bottom of equation M1

State that the equation is a maximum when $\cos(x - \alpha) = -1$ M1

Obtain the result $x = 243.43\ldots° = 243°$ (3 s.f.) A1 (3)

d) Solve to the result, max $= 6 \div (6 - \sqrt{5}) = 1.5941\ldots = 1.59$ (3 s.f). A1 (1)

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7. a) $\frac{dx}{dy} = 2y - 1$ M1A1

Therefore $\frac{dy}{dx} = \frac{1}{2y-1}$ A1 ft

When $x = 6$, $6 = y^2 - y$, $y > 0$, so $y = 3$ by inspection or other method M1

Therefore $\frac{dy}{dx} = \frac{1}{2\times 3 - 1} = \frac{1}{5}$ A1 (5)

b) i) $\frac{dy}{dx}\sin 3x\frac{d}{dx}(\cos 6x) + \cos 6x\frac{d}{dx}(\sin 3x)$ M1

$= -6\sin 3x\sin 6x + 3\cos 3x\cos 6x$ A1

When $x = \frac{\pi}{3}$, $\frac{dy}{dx} = 0 + 3\times -1\times 1 = -3$ A1

Therefore $y = -3x + c$ M1

When $x = \frac{\pi}{3}$, $y = 0$, so $c = \pi$

Therefore $y = -3x + \pi$ A1 (5)

ii) When $x = \frac{\pi}{6}$, $\frac{dy}{dx} = 0$ A1

When $x = \frac{\pi}{6}$, $y = -1$ A1

Therefore tangent is $y = -1$ A1 (3)

iii) The equation of the normal is $x = \frac{\pi}{6}$ A1 (1)

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1. $\ln y = 6x - 2,\ e^{3x} = ey \Rightarrow y = \dfrac{e^{3x}}{e}$ M1

Substitute in: $\ln\left(\dfrac{e^{3x}}{e}\right) = 6x - 2$ M1

$3x - 1 = 6x - 2$ (simplify LHS) M1

Obtain result $x = 1/3$ A1

Back substitution; $y = \dfrac{e^{3\times\frac{1}{3}}}{e} = 1$ M1A1(6)

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2. a) $\dfrac{\frac{\tan\phi}{1}}{\frac{\tan\phi}{1} + \frac{1}{\tan\phi}} = \dfrac{\tan^2\phi}{\tan^2\phi + 1} = \dfrac{\tan^2\phi}{\sec^2\phi} = \tan^2\phi\cos^2\phi = \dfrac{\sin^2\phi}{\cos^2\phi}\cos^2\phi = \sin^2\phi$ M1M1M1A1

(4)

b) Since this equation is the square of the one in part a),
the answer is also the square of the answer in part a); $\mathbf{\sin^4\phi}$ A1***A1*(2)**

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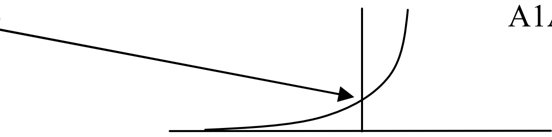
3. a) Curve sketch which cuts the y-axis at **y = 3** A1**A1**(2)



b) $y = 3e^x,\ \dfrac{dy}{dx} = 3e^x$ M1

$3e^{\ln 3} = 9$ = tangent gradient M1

normal gradient = $-1/9$ M1

$y = mx + c$, $y - 9 = -1/9(x - \ln 3)$, $9y - 81 = \ln 3 - x$, $9y = \ln 3 + 81 - x$ M1

$y = -\dfrac{1}{9}x + \dfrac{\ln 3 + 81}{9}$, c = 9.1220… = 9.12 (3 s.f.) A1 (5)

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4. a) Crosses y-axis at y = 1 and touches x-axis at $x = -3$, $x = -2$, $x = 2$ and $x = 3$ A1A1(2)

b) Sketch $2f(x + 1)$ A1
Graph is stretched by 2 in the y-direction and translated 1 to left.
Graph touches x-axis at x =1 and 2. A1
Graph cuts y-axis at y =1 A1 (3)

c) The functions are the same A1 (1)

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5. a) Using product rule where $u = 2x^4$ $v = \cos^4 x$ M1

$u` = 8x^3$ $v` = -4\cos^3 x\sin x$ A1

$\dfrac{d}{dx}(f(x)) = 8x^3\cos^4 x + -4\cos^3 x\sin x \times 2x^4$ A1

$= 8x^3\cos^4 x - 8x^4\cos^3 x\sin x$ A1 (4)

b) Rearrange to obtain $e^{-3x} + x^3e^{-3x}$ M1

$\frac{d}{dx}(f(x)) = -3e^{-3x} + \frac{d}{dx}(x^3e^{-3x})$ A1

Using the product rule:

$\frac{d}{dx}(x^3e^{-3x}) = 3x^2e^{-3x} - 3x^3e^{-3x}$ A1

$\therefore \frac{d}{dx}(f(x)) = 3e^{-3x}(x^2 - x^3 - 1)$ A1 (4)

c) $\ln(x^x) = x\ln x$ M1

$\frac{d}{dx}(\ln(x^x)) = \ln x + x\frac{d}{dx}(\ln x)$ A1

$= \ln x + \frac{x}{x}$ A1

$= 1 + \ln x$ A1 (4)

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6. a) $fg(x) = 2 + \ln(2 + e^{2x})$ A2

$gf(x) = 2 + e^{2(2 + \ln x)}$ M1

$= 2 + e^{(4 + 2\ln x)}$ M1

$= 2 + e^4x^2$ A1 (5)

b) $f(x) = 2 + \ln x \Rightarrow y = 2 + \ln x$

$y - 2 = \ln x$ M1

$x = e^{(y-2)}$ M1

$f^{-1}(x) = e^{(x-2)}$ A1

Range: $f^{-1}(x) > 0$ A1 (4)

c) $g(x) = 2 + e^{2x} \Rightarrow y = 2 + e^{2x}$

$y - 2 = e^{2x}$

$2x = \ln(y - 2)$ M1

$x = \ln(y - 2)/2$ M1

$g^{-1}(x) = \frac{\ln(x-2)}{2}$ A1

Domain: $x > 2, x \in \mathbb{R}$ A1 (4)

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7. a) $f(x) = \sin 3x$ M1

$\therefore f(x + {}^{\pi}/_{6}) = \sin[3x + \pi/2]$ M1

$= [\sin 3x\cos(\pi/2) + \cos 3x\sin(\pi/2)]$ M1

$= \cos 3x$ A1

$f(x - {}^{\pi}/_{6}) = \sin(3x - \pi/2)$

$= (\sin 3x\cos(\pi/2) - \cos 3x\sin(\pi/2))$ M1

$= -\cos 3x$ A1

$\therefore f(x + {}^{\pi}/_{6}) = -f(x - {}^{\pi}/_{6})$ A1 (7)

b) By differentiation or considering $\frac{dy}{dx}$ M1

$= \cos^2 x - \sin^2 x$ A1

$\cos x > \sin x$ for $0 < x < \frac{\pi}{4}$ M1

$\therefore \cos^2 x - \sin^2 x \geq 0$ for $0 < x < \frac{\pi}{4}$ A1 (4)

OR g(x) = cos2x and cos2x ≥ 0 for $0 < x < \frac{\pi}{4}$ *OR clear sketch of* $g`(x)$ *in required region.*

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8. a) Let $f(x) = 10x^3 - \left(\frac{1}{1-x}\right)$ M1

f(0) = –1 so f(0) < 0 : f(0.9) = –2.71 so f(0.9) < 0 M1

Look at other x values between extremes i.e. Attempt to find x s.t. f(x) > 0, M1
for example; f(0.7) = 0.096666… = 0.0967 (3 s.f.), f(0.8) = 0.12
[Note: f(0.6) = –0.34]
so; $0.6 < x_1 < 0.7$, $0.8 < x_2 < 0.9$ are 2 suitable intervals
- other answers possible A1A1(5)

b)
	(0.69336…)	(0.68832…)	(0.68459…)	(0.68188…)	
$x_0 = 0.7$,	$x_1 = 0.6934$,	$x_2 = 0.6883$,	$x_3 = 0.6846$,	$x_4 = 0.6819$ (4dp)	A1A1A1A1

(4)

c) f(0.675) = $-1.4543\ldots \times 10^{-3} = -1.45 \times 10^{-3}$ (3 s.f.) A2 (2)

d) [note a) equation is linked to the iteration, ie same equation rearranged]
part b) specifies an answer below 0.68188… = 0.6819 (4 dp) M1
part c) specifies an answer above 0.675, this means that the answer is 0.68
(2 dp as required) M1A1(3)

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1. $e^{10x} - 2e^{5x} - 3 = 0$;
Let $y = e^{5x}$ $\therefore$ $y^2 - 2y - 3 = 0$ or $(e^{5x})^2 - 2e^{5x} - 3 = 0$ M1
$(y + 1)(y - 3) = 0$ M1
$y = -1$ is impossible as you cannot have a log of a negative number so $y = 3$ B1
$y = e^{5x} = 3$; $5x = \ln 3$; $x =$ **(ln 3)/5** (= 0.2197…) M1**A1**(5)

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2. a) $2\sin A\cos B = \sin(A + B) + \sin(A - B)$ M1
$2\sin 6x\cos 5x = \sin(11x) + \sin(x)$ M1
A = 11, B= 1 A1 (3)

b) $\dfrac{\cot 2\phi \operatorname{cosec} 2\phi}{\tan^2\phi \sec 2\phi + \sec 2\phi} = \dfrac{\cot 2\phi \operatorname{cosec} 2\phi}{\sec 2\phi(\tan^2\phi + 1)}$ M1

Use identity: $\tan^2 A + 1 \equiv \sec^2 A$

$= \dfrac{\cot 2\phi \operatorname{cosec} 2\phi}{\sec 2\phi \sec^2\phi}$

Use identity: $\cot 2A \equiv \dfrac{\cos 2A}{\sin 2A}$

$= \dfrac{\dfrac{\cos 2\phi}{\sin 2\phi} \times \dfrac{1}{\sin 2\phi}}{\dfrac{1}{\cos 2\phi} \times \dfrac{1}{\cos^2\phi}}$ M1

Correct manipulation of fractions; M1

$= \dfrac{\cos 2\phi}{\sin^2 2\phi} \div \dfrac{1}{\cos 2\phi \cos^2\phi}$

$= \dfrac{\cos 2\phi}{\sin^2 2\phi} \times \cos 2\phi \cos^2\phi$

$= \dfrac{\cos^2 2\phi}{\sin^2 2\phi}\cos^2\phi$

$= \cot^2 2\phi \cos^2\phi = (\cot 2\phi \cos\phi)^2$ $\therefore$ n = 2 or implied A2 (5)

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3. a) Sketch of curve M1
The curve is an *inverted exponential* which crosses the y-axis at **y = 1** M1**A1**
and the x-axis at x = ln (3/2) ≈ 0.40546… ≈ 0.405 (3 s.f.) A1 (4)

b) $\dfrac{dy}{dx} = -2e^x$, A1

when x = 1, $\dfrac{dy}{dx} = -2e$, $\therefore$ gradient of normal = $\dfrac{1}{2e}$ A1 ft

Substitute in values; $y = \dfrac{1}{2e}x + c$; $3 - 2e = \dfrac{1}{2e} + c$; $c = 3 - 2e - \dfrac{1}{2e}$ M1

$\therefore y = \dfrac{1}{2e}x + 3 - 2e - \dfrac{1}{2e}$ [≈0.18393…x – 2.6205… ≈ 0.184x – 2.62 (3 s.f.)] A1 (4)

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4. a) f(1) = –1 M1
f(2) = 59 M1
n = 1 or implied A1 (3)

b) $x_0 = 1$, $x_1 = 1.1225\ldots$, $x_2 = 1.1456\ldots$, $x_3 = 1.1499\ldots$, $x_4 = 1.1508\ldots$, M2

$x_5 = 1.1509\ldots$, $x_6 = 1.1510\ldots$, $x_7 = 1.1510\ldots$; so $x = 1.151$ (3 d.p.) A1 (3)

c) even function or $f(-x) = (-x)^6 - (-x)^2 - 1 = x^6 - x^2 - 1 = f(x)$ M1
$\therefore x = -1.151$ (3 d.p.) is also a solution A1 (2)

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5. a) $fg(x) = \dfrac{x^4 + 16}{x^4 - 16}$; domain: $x \in \mathbb{R}$, $\boldsymbol{x \neq \pm 2}$ M1A1**A1**

$gf(x) = \left(\dfrac{x+16}{x-16}\right)^4$; domain: $x \in \mathbb{R}$, $x \neq 16$ A2**A1**(6)

b) $f(x) = \dfrac{x+16}{x-16} = y$;

Swap variables; $x = \dfrac{y+16}{y-16}$; Attempt to rearrange; M1

$yx - 16x = y + 16$; $yx - y = 16x + 16$; $y(x - 1) = 16(x + 1)$; M1

$y = \dfrac{16(x+1)}{(x-1)} \Rightarrow f^{-1}(x) = \dfrac{16(x+1)}{(x-1)}$ *domain:* $x \in \mathbb{R}$, $x \neq 1$ *A1*A1(4)

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6. a) As f(x) except for $3 < x < 6$ which is reflected about the x-axis, M1
crosses axis at y = 4, and touches at $x = 3$ and $x = 6$ A1 (2)

b) Quadrants 1&4 stay same, quadrants 2&3 reflection of quadrants 1&4 in y-axis, M1
crosses axis at y = 4, $x = 3$, $x = 6$, $x = -3$, $x = -6$ A1 (2)

c) Stretch $\times 2$ in the y-direction and $\times \frac{1}{3}$ in the x-direction A1

Crosses axis at y = 8, $\boldsymbol{x = 1}$, $x = 2$ A1**A1**A1 (4)

d) reflected in x-axis A1 (1)

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7. a) Using Product rule where $u = \sin^3 2x$ $v = \cos^4 3x$
$u' = 6\sin^2 2x\cos 2x$ $v' = \boldsymbol{-12\cos^3 3x\sin 3x}$ A1**A1**

$\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$ M1

$= 6\sin^2 2x\cos 2x\cos^4 3x - 12\sin^3 2x\cos^3 3x\sin 3x$ A1 (4)

b) Using Quotient rule where $u = e^{3x}$ $v = x^5$
$u' = 3e^{3x}$ $v' = 5x^4$ A1*A1*

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ M1

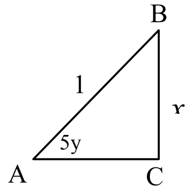
$\therefore \dfrac{dy}{dx} = \dfrac{3x^5e^{3x} - 5x^4e^{3x}}{x^{10}}$

$= \dfrac{e^{3x}(3x-5)}{x^6}$ A1 (4)

c) $x = \sin 5y$

$\dfrac{dx}{dy} = 5\cos 5y$ M1

$\dfrac{dy}{dx} = \dfrac{1}{5\cos 5y}$ M1



By Pythagoras AC = $\sqrt{1 - x^2}$ M1

$\cos 5y = \dfrac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$ M1

Therefore $\dfrac{dy}{dx} = \dfrac{1}{5\sqrt{1-x^2}}$ M1 (5)

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8. a) $R = \sqrt{6^2 + 8^2} = 10$ M1

tan α = 8/6 ⇒ α = 53.130… = 53.13° (2 d.p.) M1

∴ 6 cos x + 8 sin x = 10 cos (x – 53.13) A1 (3)

b) 6 cos 2y + 8 sin 2y = 1;

10 cos (2y – 53.130…) = 1; M1

cos (2y – 53.130…) = 1/10

2y – 53.130… = 84.26°, 275.74°, 444.26°, 635.74° M1

y = 68.695…, 164.43…, 248.67…, 344.43…

y = 68.70°, 164.43°, 248.68°, 344.43° (2 d.p.) A4 (6)

c) $\dfrac{10}{10 + 6\cos x + 8\sin x} = \dfrac{10}{10 + 10\cos(x - 53.130...)}$ M1

Minimum when cos (x – 53.130…) = 1 M1

∴ x – 53.130… = 0; x = 53.130… = 53.13° (2 d.p.) A1 (3)

d) Minimum value = $\dfrac{10}{10 + 10(1)} = \dfrac{1}{2}$ M1A1(2)

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1. a) Put over a common denominator — M1

$$\frac{3x^2 - x - 2 + 3x + 2}{3x^2 - x - 2}$$

$$= \frac{3x^2 + 2x}{3x^2 - x - 2}$$ — A1

$$= \frac{x(3x+2)}{(3x+2)(x-1)}$$ factorise denominator correctly — M1

$$= \frac{x}{x-1}$$ — A1 (4)

b) Try f(0) and f(1) — M1

$$f(0) = (0)^3 + \frac{23}{2}(0)^2 + 26(0)^2 - 16 = -16 < 0$$

$$f(1) = (1)^3 + \frac{23}{2}(1)^2 + 26(1)^2 - 16 = 22.5 > 0$$

There is a sign change so there is a solution between 0 and 1 — M1A1(3)

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2. a) –ve parts reflected in the x-axis. — M1
Max = 4
Touches x-axis at 1, 5, cuts y-axis at y = 2 — A1 (2)

b) Quadrants 1&4 stay the same, 2&3 are reflected in the y-axis — M1
Max = 4
Cuts x-axis at 1, 5, –1, –5, cuts y-axis at –2 — A1 (2)

c) Stretch ×2 in the y-direction, translate 1 to the left. — M1A1
Max = 8
Cuts x-axis at 0, 4, cuts y-axis at 0 — A1 (3)

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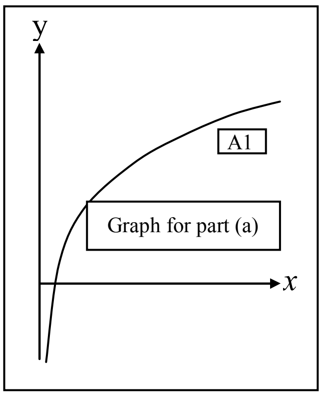
3. a) Sketch of graph (shape similar to ln x) — A1
Crosses x-axis when y = 0
$\therefore$ 0 = 3 + 2 ln x; ln x = –3/2
$x = e^{-3/2} = 0.22331\ldots$
$= 0.223$ (3 s.f.) — M1A1(3)



b) $\frac{dy}{dx} = \frac{2}{x}$ — A1
when x = 1, y' = 2/1 = 2 — A1 ft
$\therefore$ y = 2x + c
Substitute in (1,3) — M1
$\therefore$ 3 = 2 + c; c = 1
$\therefore$ y = 2x + 1 — A1 (4)

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4. a) begin: t = 0
T = 5(20 – e^0) — M1
= 5(20 – 1)
= 95 — A1
end: t = 1

T $= 5(20 - e^1)$

$= 100 - 5e$ A1 (3)

b) i) $\frac{dT}{dt} = -5e^t$ A1 (1)

ii) $\frac{dT}{dt} = -6 = -5e^t$ M1

Therefore $e^t = \frac{6}{5}$ A1

Therefore $t = \ln\left(\frac{6}{5}\right)$ A1 (3)

c) i) max when t = 1, $\frac{dT}{dt} = -5e^1$ = (–5e)°C/s ∴ maximum rate of cooling is 5e°C/s M1A1(2)

ii) min when t = 0, $\frac{dT}{dt} = -5e^0$ = –5°C/s ∴ minimum rate of cooling is 5°C/s M1A1(2)

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5. a) $fg(x) = f(x^2 - 2)$ M1

$= \frac{(x^2-2)^2 - 49}{x^2 - 2 + 7}$ A1

$= \frac{(x^2-9)(x^2+5)}{(x^2+5)}$ M1

$= (x + 3)(x - 3)$ A1 (4)

b) $gf(x) = g\left(\frac{x^2-49}{x+7}\right)$ M1

$= \left(\frac{x^2-49}{x+7}\right)^2 - 2$ A1

$= \frac{(x^2-49)^2 - 2(x+7)^2}{(x+7)^2}$ M1

$= \frac{x^4 - 100x^2 - 28x + 2303}{(x+7)^2}$ A1 (4)

$\therefore h(x) = x^4 - 100x^2 - 28x + 2303$

c) $g(x) > 23$ A1 (1)

d) Let $y = x^2 - 2$ M1

$\therefore y + 2 = x^2 \Rightarrow x = \sqrt{y+2} \quad \therefore g^{-1}(x) = (x+2)^{\frac{1}{2}}$ A1

Domain: $x > 23$ Range: $g^{-1}(x) > 5$ A1A1(4)

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6. a) $11 - \sqrt{11}\sqrt{10} + \sqrt{10}\sqrt{11} - 10 = 1$ A1 (1)

b) $R = \sqrt{a^2+b^2} = \sqrt{1^2+3^2} = \sqrt{10}$ A1

$\alpha = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(3) = 71.565\ldots = 71.6°$ (3 s.f.) M1A1

$\cos x + 3 \sin x = (\sqrt{10})\cos(x - 71.565\ldots)$ A1 (4)

c) $\cos x + 3\sin x = 1 \Rightarrow (\sqrt{10})\cos(x - 71.565\ldots) = 1$ M1

$\cos(x - 71.565\ldots) = \dfrac{1}{\sqrt{10}}$ M1

$x - 71.565\ldots = 0°, 71.565\ldots° \therefore x = 143.13\ldots° = 143°, 360°$ (3 s.f.) A1A1(4)

d) Minimum occurs when $\cos(x - 71.565\ldots) = 1$ M1
$\therefore x = 71.565\ldots = 71.6°$ (3 s.f.) A1 (2)

e) Minimum value = $\left(\dfrac{1}{\sqrt{10}+\sqrt{11}}\right)$ A1

$\left(\dfrac{1}{\sqrt{10}+\sqrt{11}}\right)\times\left(\dfrac{\sqrt{11}-\sqrt{10}}{\sqrt{11}-\sqrt{10}}\right) = \underline{\sqrt{11}-\sqrt{10}}$ M1<u>A1</u>(3)

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7. a) $\sin(A + B) = \sin A\cos B + \sin B\cos A$ M1
$\sin 3x = \sin(2x + x) = \sin 2x\cos x + \sin x\cos 2x$ M1

$\sin 2A = 2\cos A\sin A$ and $\cos 2A = \cos^2 A - \sin^2 A$ M1
Therefore $\sin 3x = 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$
$= 3\sin x\cos^2 x - \sin^3 x$ M1
$\cos^2 A = 1 - \sin^2 A$
Therefore $\sin 3x = 3\sin x(1 - \sin^2 x) - \sin^3 x$
$= 3\sin x - 4\sin^3 x$ A1 (5)

b) Let $y = \sin(ax)$, let $u = ax$, therefore $y = \sin(u)$ M1

$\dfrac{dy}{dx} = \dfrac{dy}{du}\times\dfrac{du}{dx}$ M1

$\dfrac{dy}{du} = \cos u$ M1

$\dfrac{du}{dx} = a$ M1

Therefore $\dfrac{dy}{dx} = a\cos u = a\cos(ax)$ A1 (5)

c) $\dfrac{d}{dx}(\sin 3x) = \dfrac{d}{dx}(3\sin x - 4\sin^3 x)$

$3\cos 3x = 3\cos x - 12\sin^2 x\cos x$ A2

$\cos 3x = \cos x - 4\sin^2 x\cos x$ A1 (3)

d) $\sin(75°) = \sin(30° + 45°)$ M1
$= \sin 30°\cos 45° + \sin 45°\cos 30°$

$= \dfrac{1}{2}\times\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\dfrac{\sqrt{3}}{2}$ M1

$= \dfrac{\sqrt{2}+\sqrt{6}}{4}$ A1 (3)

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(75)

1. a) $1-\dfrac{1}{1+\cot^2\phi}$

$=1-\dfrac{1}{\operatorname{cosec}^2\phi}$ M1

$=1-\sin^2\phi$ M1

$=\cos^2\phi$ A1 (3)

b) L.H.S. = $\cos\phi+\sin\phi\tan 2\phi=\cos\phi+\dfrac{\sin\phi\sin 2\phi}{\cos 2\phi}$ using $\tan 2A=\dfrac{\sin 2A}{\cos 2A}$ M1

$=\dfrac{\cos\phi\cos 2\phi+\sin\phi\sin 2\phi}{\cos 2\phi}$ M1

$=\dfrac{\cos\phi}{\cos 2\phi}$ = R.H.S. [using $\cos(A-B)=\cos A\cos B+\sin A\sin B$] A2 (4)

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2. a) $x_1=\sqrt{\dfrac{3}{2}+2}$ = 1.8708… = 1.871 (4 s.f.) A1

$x_2=\sqrt{\dfrac{3}{1.87...}+2}$ = 1.8983… = 1.898 (4 s.f.) A1

$x_3=\sqrt{\dfrac{3}{1.89...}+2}$ = 1.8921… = 1.892 (4 s.f.)

$x_4=\sqrt{\dfrac{3}{1.89...}+2}$ = 1.8935… = 1.894 (4 s.f.) A1 (3)

b) x_4 = 1.8935 (5 s.f)
x_5 = 1.8932 (5 s.f)
x_6 = 1.8933 (5 s.f)

f(1.8932) < 0 M1
f(1.8933) > 0 M1
Therefore as f(x) is continuous then there exists a solution f(n) = 0
with 1.8532 < n < 1.8933.
Therefore n = 1.893 (4 s.f.) A1 (3)

c) $0=x^3-2x^2-3$
$0=x(x^2-2)-3$
$3=x(x^2-2)$

$\dfrac{3}{x}=x^2-2$ M1

$\dfrac{3}{x}+2=x^2$ M1

$x=\sqrt{\dfrac{3}{x}+2}$ M1 (3)

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3. a) $|2x + 3| > 4$

With $x > -\frac{3}{2}$, $2x + 3 > 4 \rightarrow x > \frac{1}{2}$ M1

With $x < -\frac{3}{2}$, $2x + 3 < -4 \rightarrow x < -\frac{7}{2}$

Therefore $x > \frac{1}{2}$ or $x < -\frac{7}{2}$ A1A1(3)

b) i) Sketch of $z = (x - 1)(x - 3)$ M1
All points that lie below the x-axis are reflected to the +ve y-axis
Sketch of $y = |(x - 1)(x - 3)|$ A1 (2)

ii) For $1 < x < 3$, $y = -(x - 1)(x - 3)$ M1
$= -x^2 + 4x - 3$

$\frac{dy}{dx} = -2x + 4$ A1

When $\frac{dy}{dx} = 1$, $-2x + 4 = 1$, so $x = \frac{3}{2}$ $\therefore a = 3/2$ M1A1(4)

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4. a) Quadrants 1 and 4 remain the same. Quadrants 2 and 3 reflected in y-axis. A1
Cuts x-axis at 2 and –2, cuts y-axis at –1. **A1** (2)

b) Section between –1 and 2 is reflected in the x-axis. A1
Touches x-axis at –1 and 2, cuts y-axis at 1. **A1** (2)

c) Stretch ×3 in y-direction and $\times\frac{1}{2}$ in the x-direction M1

Cuts x-axis at $-\frac{1}{2}$ and 1, cuts y-axis at –3 A2 (3)

d) f(–1) = 0. Therefore 0 = k – 3e
Therefore k = 3e M1A1(2)

e) $\frac{dy}{dx} = -3e^{x+2}$ A1

Steepest when $x = -1$.

Therefore $\frac{dy}{dx} = -3e^1 = -3e$ M1A1(3)

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5. a) $fg(x) = (1 - x^2)^2 - 1$ M1

$= 1 - 2x^2 + x^4 - 1$

$= x^4 - 2x^2$ A1

$gf(x) = 1 - (x^2 - 1)^2$ M1

$= 1 - 1 - x^4 + 2x$

$= 2x^2 - x^4$ A1

$f(g(x)) = g(f(x))$

$x^4 - 2x^2 = 2x^2 - x^4$ M1

$2x^4 = 4x^2$

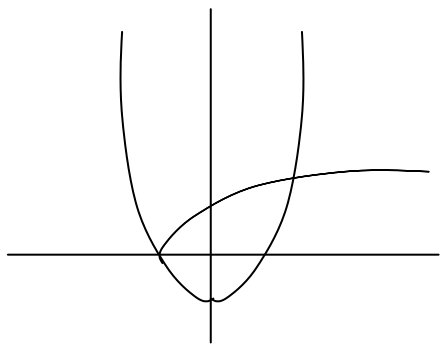
$x^4 = 2x^2$

$x^2 = 2$

$x = +\sqrt{2}$ or $-\sqrt{2}$ A1A1

or $x = 0$ A1 (8)

b) From sketch the required domain is $x \geq 0$ M1A1(2)



c) $f(x) = x^2 - 1$ Let $y = x^2 - 1$ M1

$x = y^2 - 1$ <switch variables> M1

$y^2 = x + 1$

$y = \sqrt{x+1}$ A1

$f^{-1}(x) = \sqrt{x+1}$ A1 (4)

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6. a) $f(x) = \ln x \;\therefore\; f'(x) = \frac{1}{x}$ A1

$g(x) = \ln 2x \;\therefore\; g'(x) = \frac{1}{x}$ A1 (2)

b) Gradient of $f'(x) = \frac{1}{3}$ M1

$\therefore$ when $x = 3$, $y = \ln 3$ M1

Tangent to curve is $y - \ln 3 = \frac{x}{3} - 1$

$\therefore\; y = \frac{x}{3} + \ln 3 - 1$ A1 (3)

c) Gradient of normal of $g(x) = -3$ M1

Co-ords to the normal = $(3, \ln 6)$ M1

$\therefore\; y - \ln 6 = -3(x - 3)$ M1

$y = \ln 6 - 3x + 9$ A1 (4)

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7. a) Using $\sin(A + B) = \sin A \cos B + \cos A \sin B \Rightarrow A = 2x, B = 4x$ M1

$\sin 2x \cos 4x + \cos 2x \sin 4x \equiv \sin 6x$ A1 (2)

b) $2 \sin 2x \cos 4x + \cos 2x \sin 4x \equiv \sin 2x \cos 4x + \sin 2x \cos 4x + \cos 2x \sin 4x$ M1

$\equiv \frac{1}{2}(\sin(6x) + \sin(-2x)) + \sin 6x = \frac{1}{2}(3\sin 6x - \sin 2x)$ A1 (2)

c) $y = e^{-x}\cos x$

$\frac{dy}{dx} = -e^{-x}\cos x - e^{-x}\sin x$ A1A1M1

Let $\frac{dy}{dx} = 0$ M1

So $-e^{-x}\cos x - e^{-x}\sin x = 0$

Therefore $e^{-x}(\cos x + \sin x) = 0$

e^{-x} is never zero, so $\cos x + \sin x = 0$

$\cos x = -\sin x$, or $\tan x = -1$ A1

Therefore $x = \frac{3\pi}{4}$ (2.3561… = 2.36 (3 s.f.)) A1

or $x = \frac{3\pi}{4} + \pi = \frac{7\pi}{4}$ (5.4977…= 5.50 (3 s.f.)) A1

$\frac{d^2y}{dx^2} = e^{-x}(\cos x + \sin x) - e^{-x}(-\sin x + \cos x) = 2e^{-x}\sin x$

When $x = \frac{3\pi}{4}$, $\frac{d^2y}{dx^2} > 0$. Therefore minimum point M1A1

When $x = \frac{7\pi}{4}$, $\frac{d^2y}{dx^2} < 0$. Therefore maximum point M1A1(11)

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(75)