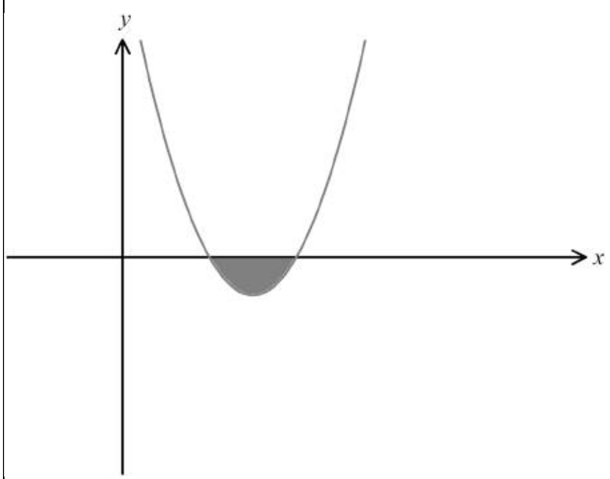
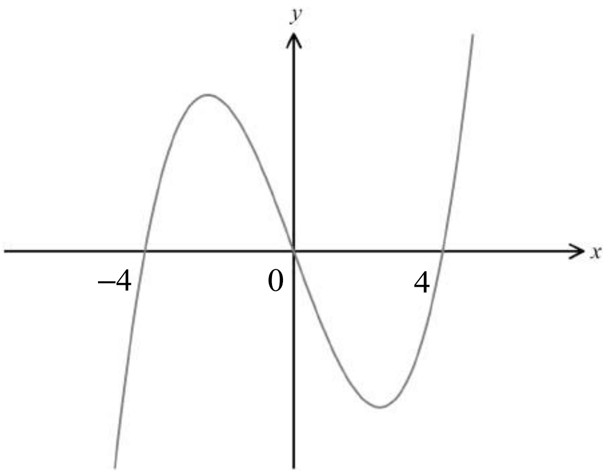
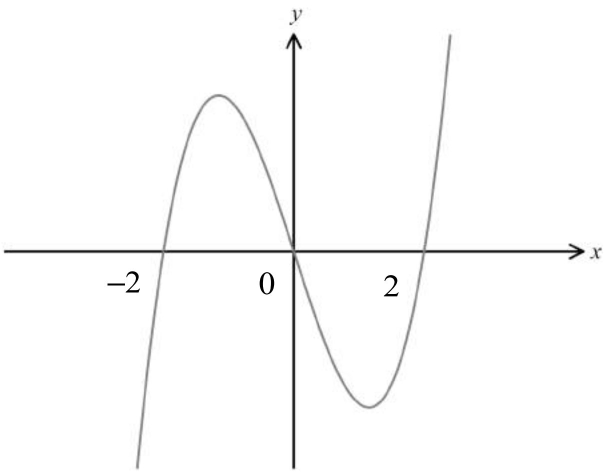
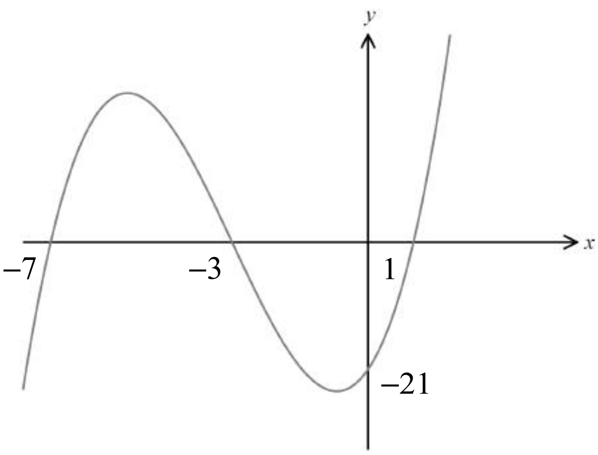
mark scheme

Practice Paper C : Core Mathematics 1

Question Number	General Scheme		Marks
1	$(y+1)^2 + y^2 = 5$ $(y^2 + 2y + 1 = 5)$ $2y^2 + 2y - 4 = 0$	**M1:** uses a correct substitution **M1:** attempts to form a 3TQ **A1:** correct equation oe	M1 M1 A1
	$(y-1)(y+2) = 0$ $y = 1,\ y = -2$	**M1:** attempts to solve 3TQ **A1:** correct values for y	M1 A1
	$x = 1 + 1,\ x = -2 + 1$ $x = 2\ ,\ x = -1$	**M1:** uses *their* values to find x **A1:** cao	M1 A1
NOTE	**Be sure to reward and not discredit candidates who choose to find x first and then y.**		
		Total	7

2 (a)	$\frac{1}{25^{\frac{3}{2}}} = \frac{1}{125}$	**M1:** uses the negative power rule at **any** stage **A1:** cao	M1 A1 (2)
(b)	$= \left(\frac{4}{25x^2(1-x)^2}\right)^{\frac{3}{2}}$ $= \left(\frac{2}{5x(1-x)}\right)^3$ $= \frac{8}{125x^3(1-x)^3}$	**M1:** uses the negative power rule at **any** stage **M1:** attempts to square root the fraction (**two** terms correct) **A1:** cao isw	M1 M1 A1 (4)
		Total	6

3 (a)	$2x+4 \geq x-6$ $x \geq -10$	**M1:** correct method to solve linear inequality **A1:** cao	**M1** **A1** **(2)**
(b)	Critical values: $x^2-6x+8=0$ $(x-4)(x-2)=0$ $x=4,\ x=2$	**M1:** attempts to solve 3TQ **A1:** correct CVs	**M1** **A1**
		**M1:** graph drawn with 'inside' region chosen. Can be implied by shading	**M1**
	$2<x<4$	**A1:** cao	**A1** **(4)**
(c)	$2<x<4$	**B1ft:** chooses the values of x that satisfies *their* (a) and (b)	**B1** **(1)**
		Total	**7**

4 (a)		**B1:** correct shape (must be a **positive** cubic) **B1:** $(0,0)$ shown **B1:** $(-4,0)$ and $(4,0)$ shown	B1 B1 B1 **(3)**
(b)		**B1:** correct shape (must be a **positive** cubic) **B1:** $(0,0)$, $(-2,0)$ and $(2,0)$ shown	B1 B1 **(2)**
(c)		**B1:** correct shape (must be a **positive** cubic) **B1:** $(1,0)$, $(-3,0)$ and $(-7,0)$ shown **M1:** attempt to work out y intercept **A1:** $(0,-21)$.	B1 B1 M1 A1 **(4)**
		Total	**9**

5	$\dfrac{2}{(1+\sqrt{5})+\sqrt{6}}\times\dfrac{(1+\sqrt{5})-\sqrt{6}}{(1+\sqrt{5})-\sqrt{6}}$	**M1:** multiplies top and bottom by $(1+\sqrt{5})-6$	**M1**
	$=\dfrac{2(1+\sqrt{5})-2\sqrt{6}}{(1+\sqrt{5})^2-6}$	**M1:** attempts to combine the fractions	**M1**
	$=\dfrac{\dots}{1+2\sqrt{5}+5-6}$	**M1:** attempts to manipulate the denominator **A1:** correct denominator	**M1** **A1**
	$=\dfrac{2+2\sqrt{5}-2\sqrt{6}}{1+2\sqrt{5}+5-6}$	**A1:** correct numerator	**A1**
	$=\dfrac{2+2\sqrt{5}-2\sqrt{6}}{2\sqrt{5}}$ $=\dfrac{1+\sqrt{5}-\sqrt{6}}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}$	**M1:** rationalises the denominator	**M1**
	$=\dfrac{\sqrt{5}+5-\sqrt{30}}{5}$ $=1+\dfrac{1}{5}\sqrt{5}-\dfrac{1}{5}\sqrt{30}$	**A1:** cao. **The answer <u>must</u> be in the required form.**	**A1**
		Total	**7**
ALT	Candidates may write the given fraction as $\dfrac{2}{(1+\sqrt{6})+\sqrt{5}}$ or $\dfrac{2}{1+(\sqrt{6}+\sqrt{5})}$ These are also correct. See the end of this mark scheme for these methods.		

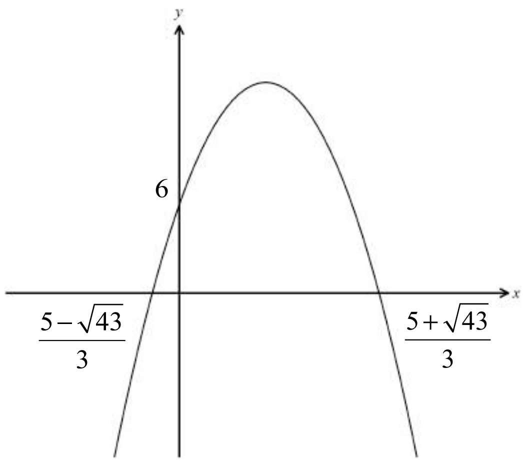
6 (a)	$\dfrac{x+x^{\frac{1}{3}}-3}{x^{\frac{1}{3}}}=x^{\frac{2}{3}}+1-3x^{-\frac{1}{3}}$	**M1:** attempts to write the given fraction in index form **A1:** correct expression	**M1** **A1**
	$\int\left(x^{\frac{2}{3}}+1-3x^{-\frac{1}{3}}\right)dx$ $\int\left(x^{\frac{2}{3}}+1-3x^{-\frac{1}{3}}\right)dx=\dfrac{x^{\frac{5}{3}}}{\frac{5}{3}}+x-\dfrac{3x^{\frac{2}{3}}}{\frac{2}{3}}+c$ $=\dfrac{3x^{\frac{5}{3}}}{5}-\dfrac{9x^{\frac{2}{3}}}{2}+x+c$	**M1:** correct attempt to integrate **one** term **A1:** one term integrated correctly (need not be simplified) **A1: all** terms integrated correctly + **constant**.	**M1** **A1** **A1** **(5)**
(b)	$5=\dfrac{3(8)^{\frac{5}{3}}}{5}-\dfrac{9(8)^{\frac{2}{3}}}{2}+8+c$	**M1:** substitutes (8,5) into *their* y from (a).	**M1**
	$5=\dfrac{96}{2}-18+8+c$ $c=15-\dfrac{96}{2}=-33$	**A1:** correct c with **no** algebraic slips	**A1**
	$\therefore y=\dfrac{3x^{\frac{5}{3}}}{5}-\dfrac{9x^{\frac{2}{3}}}{2}+x-33$	**A1ft:** y in terms of x, ft of *their* c	**A1** **(3)**
		Total	**8**

7	$\left(-\frac{3}{2},\frac{9}{2}\right)$	**B1:** correct midpoint of AB	**B1**
	$m_{AB}=\frac{3-6}{2--5}$ **or** $\frac{6-3}{-5-2}$ $m_{AB}=-\frac{3}{7}$	**M1:** attempts to work out the gradient of AB **A1:** correct gradient	**M1** **A1**
	$m_{\text{bisector}}=\frac{-1}{m_{AB}}\left(=\frac{7}{3}\right)$	**A1ft**: correct gradient for the bisector. Award ft for use an incorrect gradient of AB	**A1**
	$y-y_1=m(x-x_1)$ $y-\frac{9}{2}=\frac{7}{3}\left(x--\frac{3}{2}\right)$	**M1:** attempts to find the equation of the perpendicular bisector	**M1**
	$3\left(y-\frac{9}{2}\right)=7\left(x+\frac{3}{2}\right)$ **or** $7x-3y+24=0$ **or** $3y-7x-24=0$ **or** $y=\frac{7}{3}x+8$	**A1:** correct bisector in any of the forms. No ft	**A1**
		Total	**6**

8	$y=\frac{x^2-12x+36}{x}$ $y=x-12+36x^{-1}$	**M1:** attempts converts y into index form **A1:** correct expression for y. Accept $\frac{36}{x}$ for $36x^{-1}$	**M1** **A1**
	$\frac{dy}{dx}=1-36x^{-2}$ $\frac{d^2y}{dx^2}=72x^{-3}$	**M1:** correct method to differentiate **one** term **A1:** correct first derivative **A1ft:** correct second derivative	**M1** **A1** **A1**
	$\left(\frac{x^4}{2}\right)\left(\frac{72}{x^3}\right)+x^3\left(1-\frac{36}{x^2}\right)+x-12+\frac{36}{x}+f(x)=0$	**M1:** attempts to substitute *their* derivatives into the given equation (one term correct) **A1ft:** correct substitution	**M1** **A1**
	$36x+x^3-36x+x-12+\frac{36}{x}+f(x)=0$ $x^3+x-12+\frac{36}{x}+f(x)=0$ $f(x)=12-x^3-x-\frac{36}{x}$	**M1:** for a **good** attempt to manipulate the algebra **A1:** $f(x)$ correct **cao**	**M1** **A1**
	Total		**9**
ALT	Some candidates may use the quotient rule to differentiate y. In such cases, the following scheme applies: $\frac{dy}{dx}=\frac{vu'-uv'}{v^2}=\frac{x(2)(x-6)-(x-6)^2(1)}{x^2}$ **M1 A1 M1** $\frac{dy}{dx}=\frac{(x-6)(2x-x+6)}{x^2}=1-36x^{-2}$ **A1** The rest is as shown in the standard scheme. If the quotient rule is used again for the second derivative, do not award any extra marks – the only mark is for a correct second derivative (ft of their first derivative).		

9	$u_n = a + (n-1)d$ $45 = a + (2-1)d$ $45 = a + d$	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $1750 = 10\left[2a + (20-1)d\right]$ $175 = 2a + 19d$	**M1:** an attempt to form an equation using one standard formula **A1:** one correct expression	**M1** **A1**
	$d = 45 - a$ $175 = 2a + 19(45 - a)$ $175 = -17a + 855$ $17a = 680 \rightarrow a = 40$		**dM1:** use of simultaneous equations to find a or d **A1:** a or d correct	**M1** **A1**
	$d = 45 - (40)$ $d = 5$		**ddM1**: correct method to find second variable **A1ft:** correct value for second variable	**M1** **A1**
			Total	**6**
NOTE	This scheme does not show when d is found first, which should also be credited.			

10 (a)	$b^2-4ac=(3-7k)^2-4(2k-k^2)(6k^4)$ $=9-42k+49k^2-48k^5+24k^6$	**M1:** use of the discriminant **A1:** correct expression **A1:** correct manipulation	M1 A1 A1
	$24k^6+49k^2+9$ is positive and since k is negative $-42k-48k^5$ is also positive. Hence $b^2-4ac>0$ and the curve has **two** intersections with the x axis.	**A2:** cso with a thorough explanation containing the underlined elements **Award A1 for a correct answer with a partial explanation.** **No attempt at an explanation loses both accuracy marks**	A2 **(5)**
(b)	$x=\dfrac{-(3-7k)\pm\sqrt{(3-7k)^2-4(2k-k^2)(6k^4)}}{2(2k-k^2)}$ $\therefore x=\dfrac{7k-3\pm\sqrt{24k^6-48k^5+49k^2-42k+9}}{4k-2k^2}$	**M1:** use of the quadratic formula **A1:** correct values except b^2-4ac, which (if wrong) should ft their working in (a) **A1:** cao. Accept equivalent forms, i.e. factorised terms etc	M1 A1 A1 **(3)**
	$f(k)=7k-3$ $g(k)=24k^6-48k^5+49k^2-42k+9$ $h(k)=4k-2k^2$	These are for reference only. Candidates need not state these	
(c)	$x=\dfrac{7(-1)-3\pm\sqrt{24+48+49+42+9}}{-6}$ $x=\dfrac{5\pm\sqrt{43}}{3}$	**M1:** substitutes values into *their* (b) to find out where the curve crosses the x axis	M1

		**B1:** correct y intersection labelled **B1:** correct shape x coordinates must be correct and shown on the sketch for full marks.	**B1** **B1** **(3)**
		Total	**11**
ALT	A tricky alternative method would be the use of completing the square for part (a): $y=\left(2k-k^2\right)\left(x^2+\left(\frac{3-7k}{2k-k^2}\right)x+\frac{6k^4}{2k-k^2}\right)$ **M1 A1** $y=\left(2k-k^2\right)\left[\left(x+\frac{3-7k}{4k-2k^2}\right)^2-\left(\frac{3-7k}{4k-2k^2}\right)^2+\frac{6k^4}{2k-k^2}\right]$ **M1** $\left(x+\frac{3-7k}{4k-2k^2}\right)^2-\left(\frac{3-7k}{4k-2k^2}\right)^2+\frac{6k^4}{2k-k^2}=0$ $\left(x+\frac{3-7k}{2k-k^2}\right)^2=\left(\frac{3-7k}{4k-2k^2}\right)^2-\frac{6k^4}{2k-k^2}$ $\left(x+\frac{3-7k}{2k-k^2}\right)^2=\left(\frac{3-7k}{4k-2k^2}\right)^2-\frac{6k^4}{1-\left(k-1\right)^2}$ $\left(\frac{3-7k}{4k-2k^2}\right)^2$ is always positive. *Idea that* $\frac{6k^4}{1-\left(k-1\right)^2}$ will always be negative for $k>0$, but $-\frac{6k^4}{1-\left(k-1\right)^2}$ is hence always positive. Hence, since the RHS is always positive, the equation has two solutions and the curve has two intersections with the x axis. **A2 for an explanation that fully conveys the consensus of the above (A1 for a partial explanation with the correct answer). Answer + no explanation loses both**		

	accuracy marks. **NOTE:** candidates must **show** that $2k-k^2$ is negative for all negative k by completing the square, or otherwise. Renounce the final to accuracy marks for candidates who simply state that it is negative.	

5 **ALT 1**	$\frac{2}{(1+\sqrt{6})+\sqrt{5}} \times \frac{(1+\sqrt{6})-\sqrt{5}}{(1+\sqrt{6})-\sqrt{5}}$	**M1:** multiplies top and bottom by $(1+\sqrt{6})-5$	**M1**
	$=\frac{2(1+\sqrt{6})-2\sqrt{5}}{(1+\sqrt{6})^2-5}$	**M1:** attempts to combine the fractions	**M1**
	$=\frac{\dots}{2+2\sqrt{6}}$	**M1:** attempts to manipulate the denominator **A1:** correct denominator	**M1** **A1**
	$=\frac{2+2\sqrt{6}-2\sqrt{5}}{2+2\sqrt{6}}$	**A1:** correct numerator	**A1**
	$=\frac{1+\sqrt{6}-\sqrt{5}}{1+\sqrt{6}}$ $=\frac{1+\sqrt{6}-\sqrt{5}}{1+\sqrt{6}} \times \frac{1-\sqrt{6}}{1-\sqrt{6}}$	**M1:** rationalises the denominator	**M1**
	$=\frac{1-\sqrt{6}+\sqrt{6}-6-\sqrt{5}+\sqrt{30}}{-5}$ $=\frac{-5-\sqrt{5}+\sqrt{30}}{-5}$ $=1+\frac{1}{5}\sqrt{5}-\frac{1}{5}\sqrt{30}$	**A1:** cao. **The answer <u>must</u> be in the required form.**	**A1**
		Total	7

5 **ALT 2**	$\frac{2}{1+(\sqrt{5}+\sqrt{6})} \times \frac{1-(\sqrt{5}+\sqrt{6})}{1-(\sqrt{5}+\sqrt{6})}$	**M1:** multiplies top and bottom by $(1+\sqrt{6})-5$	**M1**

	$= \dfrac{2(1+\sqrt{6})-2\sqrt{5}}{(1+\sqrt{6})^2-5}$	**M1:** attempts to combine the fractions	**M1**
	$= \dfrac{\ldots}{2+2\sqrt{6}}$	**M1:** attempts to manipulate the denominator **A1:** correct denominator	**M1** **A1**
	$= \dfrac{2+2\sqrt{6}-2\sqrt{5}}{2+2\sqrt{6}}$	**A1:** correct numerator	**A1**
	$= \dfrac{1+\sqrt{6}-\sqrt{5}}{1+\sqrt{6}}$ $= \dfrac{1+\sqrt{6}-\sqrt{5}}{1+\sqrt{6}} \times \dfrac{1-\sqrt{6}}{1-\sqrt{6}}$	**M1:** rationalises the denominator	**M1**
	$= \dfrac{1-\sqrt{6}+\sqrt{6}-6-\sqrt{5}+\sqrt{30}}{-5}$ $= \dfrac{-5-\sqrt{5}+\sqrt{30}}{-5}$ $= 1+\dfrac{1}{5}\sqrt{5}-\dfrac{1}{5}\sqrt{30}$	**A1:** cao. **The answer <u>must</u> be in the required form.**	**A1**
		Total	**7**