

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

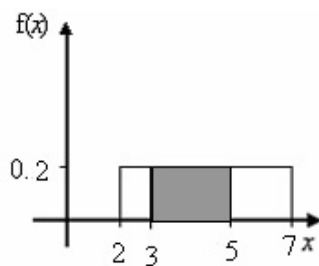
The continuous random variable $X \sim U[2, 7]$.

Find

- a $P(3 < X < 5)$,
- b $P(X > 4)$.

Solution:

$$\mathbf{a} \quad \frac{1}{b-a} = \frac{1}{7-2} = 0.2$$



$$P(3 < X < 5) = (5 - 3) \times 0.2 \\ = 0.4$$

$$\mathbf{b} \quad P(X > 4) = (7 - 4) \times 0.2 \\ = 0.6$$

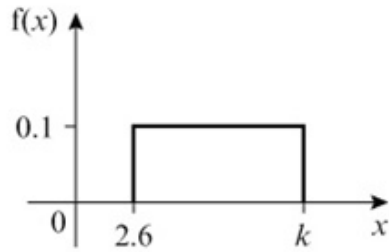
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Exercise A, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

- the value of k ,
- $P(4 < X < 7.9)$.

Solution:

a Area = 1
 $(k - 2.6) \times 0.1 = 1$
 $(k - 2.6) = 10$
 $k = 12.6$

b $P(4 < X < 7.9) = (7.9 - 4) \times 0.1$
 $= 0.39$

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Exercise A, Question 3

Question:

The continuous random variable X has p.d.f.

$$f(x) = \begin{cases} k, & -2 \leq x \leq 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find

- a the value of k ,
- b $P(-1.3 < X < 4.2)$.

Solution:

$$\begin{aligned} \mathbf{a} \quad & \text{Area} = 1 \\ & k \times (6 - (-2)) = 1 \\ & 8k = 1 \\ & k = \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad P(-1.3 < X < 4.2) &= \frac{1}{8} \times (4.2 - (-1.3)) \\ &= 0.6875 \end{aligned}$$

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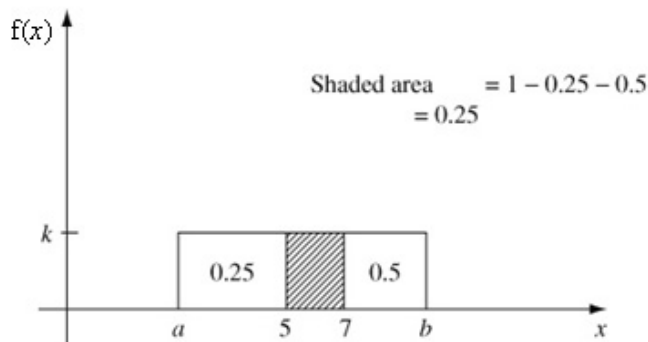
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Exercise A, Question 4

Question:

The continuous random variable $Y \sim U[a, b]$. Given that $P(Y < 5) = \frac{1}{4}$ and $P(Y > 7) = \frac{1}{2}$, find the value of a and the value of b .

Solution:



$$\text{shaded area} = \frac{1}{4}$$

$$2 \times k = \frac{1}{4}$$

$$k = \frac{1}{8}$$

$$(b-7) \times \frac{1}{8} = 0.5$$

$$(b-7) = 4$$

$$b = 11$$

$$(5-a) \times \frac{1}{8} = 0.25$$

$$(5-a) = 2$$

$$a = 3$$

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Exercise A, Question 5

Question:

The continuous random variable $X \sim U[2, 8]$.

- a Write down the distribution of $Y = 2X + 5$.
- b Find $P(12 < Y < 20)$.

Solution:

a

$$2 \times 2 + 5 = 9$$

$$2 \times 8 + 5 = 21$$

$$Y \sim U[9, 21]$$

b For Y , $\frac{1}{b-a} = \frac{1}{21-9} = \frac{1}{12}$

$$\begin{aligned} P(12 < Y < 20) &= (20 - 12) \times \frac{1}{12} \\ &= \frac{2}{3} \end{aligned}$$

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Exercise B, Question 1

Question:

The continuous variable Y is uniformly distributed over the interval $[-3, 5]$.

Find:

- $E(X)$,
- $\text{Var}(X)$,
- $E(X^2)$,
- the cumulative distribution function of X , for all x .

Solution:

$$\begin{aligned} \text{a } E(X) &= \frac{5+(-3)}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Var}(X) &= \frac{(5-(-3))^2}{12} \\ &= 5\frac{1}{3} \end{aligned}$$

$$\text{c } \text{Var}(X) = E(X^2) - (E(X))^2$$

$$5\frac{1}{3} = E(X^2) - 1$$

$$E(X^2) = 6\frac{1}{3}$$

$$\begin{aligned} \text{d } F(x) &= \int_{-3}^x \frac{1}{5-(-3)} dt \\ &= \left[\frac{t}{8} \right]_{-3}^x \\ &= \frac{x+3}{8} \end{aligned}$$

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{8} & -3 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

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Exercise B, Question 2

Question:

Find $E(X)$ and $\text{Var}(X)$ for the following probability density functions.

$$\mathbf{a} \quad f(x) = \begin{cases} \frac{1}{4}, & 1 \leq x \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbf{b} \quad f(x) = \begin{cases} \frac{1}{8}, & -2 \leq x \leq 6, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

$$\mathbf{a} \quad E(X) = \frac{5+1}{2} \\ = 3$$

$$\text{Var}(X) = \frac{(5-1)^2}{12} \\ = 1\frac{1}{3}$$

$$\mathbf{b} \quad E(X) = \frac{6+(-2)}{2} \\ = 2$$

$$\text{Var}(X) = \frac{(6-(-2))^2}{12} \\ = 5\frac{1}{3}$$

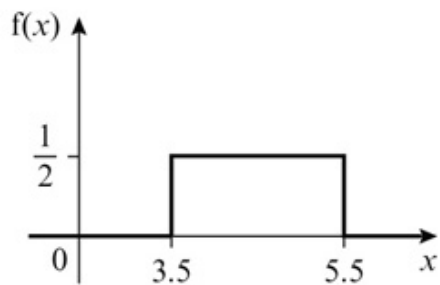
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Exercise B, Question 3

Question:

The continuous random variable X has p.d.f as shown in the diagram.



Find:

- $E(X)$,
- $\text{Var}(X)$,
- $E(X^2)$,
- the cumulative distribution function of X , for all x .

Solution:

$$\begin{aligned} \mathbf{a} \quad E(X) &= \frac{5.5+3.5}{2} \\ &= 4.5 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \text{Var}(X) &= \frac{(5.5-3.5)^2}{12} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad \text{Var}(X) &= E(X^2) - (E(X))^2 \\ \frac{1}{3} &= E(X^2) - 20.25 \\ E(X^2) &= 20\frac{7}{12} = 20.6 \text{ (3 s.f.)} \end{aligned}$$

$$\mathbf{d} \quad F(x) = \int_{3.5}^x \frac{1}{5.5-3.5} dt$$

$$= \left[\frac{t}{2} \right]_{3.5}^x$$

$$= \frac{x}{2} - \frac{3.5}{2}$$

$$= \frac{x}{2} - 1.75$$

$$F(x) = \begin{cases} 0 & x < 3.5 \\ \frac{x}{2} - 1.75 & 3.5 \leq x \leq 5.5 \\ 1 & x > 5.5 \end{cases}$$

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Exercise B, Question 4

Question:

The continuous random variable $Y \sim U[a, b]$. Given $E(Y) = 1$ and $\text{Var}(Y) = \frac{4}{3}$, find the value of a and the value of b .

Solution:

$$\begin{aligned} E(Y) \\ \frac{a+b}{2} &= 1 \\ a+b &= 2 \end{aligned} \quad (1)$$

$$\begin{aligned} \text{Var}(Y) \\ \frac{(b-a)^2}{12} &= \frac{4}{3} \\ (b-a)^2 &= 16 \end{aligned} \quad (2)$$

Solving equations (1) and (2) simultaneously

$$\begin{aligned} b &= 2 - a \\ (2 - a - a)^2 &= 16 \\ (2 - 2a) &= \pm 4 \\ \begin{array}{l} 2 - 2a = 4 \\ a = -1 \\ b = 2 - (-1) \\ \quad = 3 \end{array} & \quad \begin{array}{l} 2 - 2a = -4 \\ a = 3 \\ b = 2 - 3 \\ \quad = -1 \end{array} \end{aligned}$$

Since $a < b$ $a = -1$ and $b = 3$

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Exercise B, Question 5

Question:

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{6}, & -1 \leq x \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

Given that $Y = 4X - 6$, find $E(Y)$ and $\text{Var}(Y)$.

Solution:

$$\begin{aligned} E(X) &= \frac{5 + (-1)}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= \frac{(5 - (-1))^2}{12} \\ &= 3 \end{aligned}$$

$$\begin{aligned} E(Y) &= 4E(X) - 6 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y) &= 16 \text{Var}(X) \\ &= 48 \end{aligned}$$

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Exercise B, Question 6

Question:

The random variable X is the length of a side of a square. $X \sim U[4.5, 5.5]$.
The random variable Y is the area of the square.
Find $E(Y)$.

Solution:

$$\begin{aligned} E(Y) &= E(X^2) & \text{or } \int_{4.5}^{5.5} x^2 dx &= \left[\frac{x^3}{3} \right]_{4.5}^{5.5} \\ E(X) &= \frac{4.5+5.5}{2} & &= \left[\frac{5.5^3}{3} \right] - \left[\frac{4.5^3}{3} \right] \\ &= 5 & &= 25 \frac{1}{12} \\ \text{Var}(X) &= \frac{(5.5-4.5)^2}{12} \\ &= \frac{1}{12} \end{aligned}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\frac{1}{12} = E(X^2) - 25$$

$$E(X^2) = 25 \frac{1}{12}$$

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Exercise B, Question 7

Question:

In a computer game an alien appears every 2 seconds. The player stops the alien by pressing a key. The object of the game is to stop the alien as soon as it appears. Given that the player actually presses the key T s after the alien first appears, a simple model of the game assumes that T is a continuous uniform random variable defined over the interval $[0, 1]$.

- Write down $P(T < 0.2)$.
- Write down $E(T)$.
- Use integration to find $\text{Var}(T)$.

Solution:

$$\frac{1}{b-a} = \frac{1}{1-0} = 1$$

$$\begin{aligned} \mathbf{a} \quad P(T < 0.2) &= (0.2 - 0) \times 1 \\ &= 0.2 \end{aligned}$$

$$\mathbf{b} \quad E(T) = 0.5$$

$$\begin{aligned} \mathbf{c} \quad \text{Var}(T) &= \int_0^1 t^2 dt - 0.5^2 \\ &= \left[\frac{t^3}{3} \right]_0^1 - 0.25 \\ &= \frac{1}{12} \end{aligned}$$

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Exercise C, Question 1

Question:

The continuous random variable X is uniformly distributed over the interval $[-2, 5]$.

a Sketch the probability density function $f(x)$ of X .

Find

b $E(X)$,

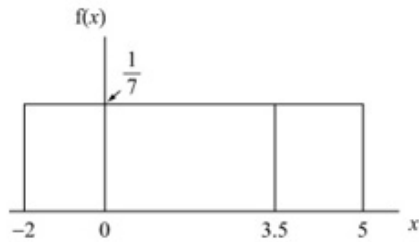
c $\text{Var}(X)$,

d the cumulative distribution function of X , for all x ,

e $P(3.5 < X < 5.5)$,

f $P(X = 4)$.

Solution:

a

$$\mathbf{b} \quad E(X) = \frac{5 + (-2)}{2} = 1.5$$

$$\mathbf{c} \quad \text{Var}(X) = \frac{(5 - (-2))^2}{12} = 4 \frac{1}{12}$$

$$\mathbf{d} \quad F(x) = \int_{-2}^x \frac{1}{5 - (-2)} dt$$

$$= \left[\frac{t}{7} \right]_{-2}^x$$

$$= \frac{x+2}{7}$$

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{x+2}{7} & -2 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

$$\begin{aligned} \mathbf{e} \quad P(3.5 < X < 5.5) &= P(3.5 < X < 5) \\ &= ((5 - 3.5) \times \frac{1}{7}) \\ &= \frac{3}{14} \end{aligned}$$

$$\mathbf{f} \quad P(X = 4) = 0$$

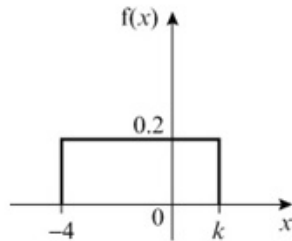
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Exercise C, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

- the value of k ,
- $P(-2 < X < -1)$,
- $E(X)$,
- $\text{Var}(X)$,
- the cumulative distribution function of X , for all x .

Solution:

a Area = 1 so

$$(k + 4) \times 0.2 = 1$$

$$0.2k + 0.8 = 1$$

$$k = 1$$

b $P(-2 < X < -1) = 1 \times 0.2 = 0.2$

c $E(X) = \frac{-4+1}{2} = -1.5$

d $\text{Var}(X) = \frac{(b-a)^2}{12} = \frac{(1-(-4))^2}{12} = 2\frac{1}{12}$

e

$$F(x) = \int_{-4}^x \frac{1}{1-(-4)} dt$$

$$= \left[\frac{t}{5} \right]_{-4}^x$$

$$= \frac{x+4}{5}$$

$$F(x) = \begin{cases} 0 & x < -4 \\ \frac{x+4}{5} & -4 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

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Exercise C, Question 3

Question:

The continuous random variable Y is uniformly distributed on the interval $a \leq Y \leq b$.
Given $E(Y) = 2$ and $\text{Var}(Y) = 3$.

Find

- the value of a and the value of b ,
- $P(X > 1.8)$.

Solution:

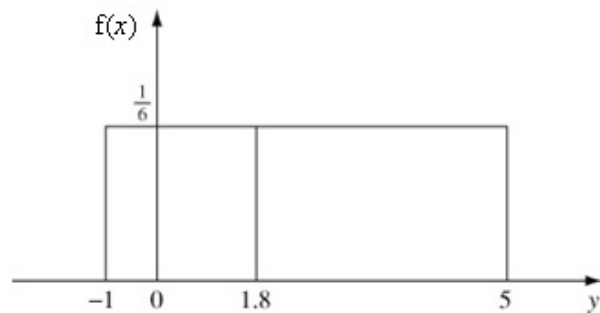
$$\text{a} \quad E(Y) = \frac{a+b}{2} = 2 \quad \text{so } a+b=4 \quad \text{so } a=4-b$$

$$\text{Var}(Y) = \frac{(b-a)^2}{12} = 3$$

Substituting for a gives $(2b-4)^2 = 36$
 $(2b-4) = \pm 6$
 $b = 5 \quad \text{or } b = -1$
 $a = -1 \quad \quad a = 5$

but $b > a$
 $b = 5 \quad a = -1$

b



$$P(X > 1.8) = (5 - 1.8) \times \frac{1}{6} = 0.533 \text{ (3 s.f.)}$$

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Exercise C, Question 4

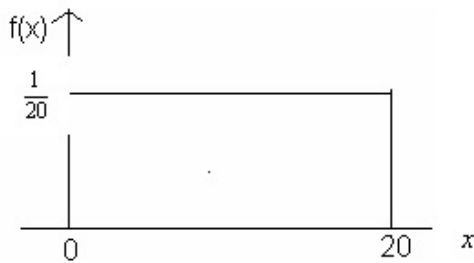
Question:

A child has a pair of scissors and a piece of string 20 cm long, which has a mark on one end. The child cuts the string, at a randomly chosen point, into two pieces. Let X represent the length of the piece of string with the mark on it.

- Write down the name of the probability distribution of X and sketch the graph of its probability density function.
- Find the values of $E(X)$ and $\text{Var}(X)$.
- Using your model, calculate the probability that the shorter piece of string is at least 8 cm long.

Solution:

a $X \sim U(0, 20)$



b

$$E(X) = \frac{20+0}{2} = 10$$

$$\text{Var}(X) = \frac{(20-0)^2}{12} = \frac{400}{12} = 33\frac{1}{3}$$

c $P(8 \leq X \leq 12) = (12 - 8) \times \frac{1}{20} = 0.2$

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Exercise C, Question 5

Question:

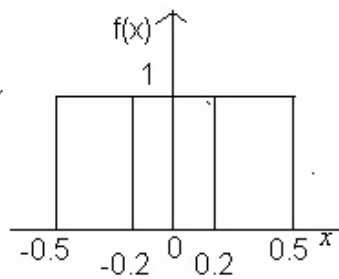
Joan records the temperature every day. The highest temperature she recorded was 29°C to the nearest degree. Let X represent the error in the measured temperature.

- Suggest a suitable model for the distribution of X .
- Using your model calculate the probability that the error will be less than 0.2°C .
- Find the variance of the error in the measured temperature.

Solution:

a
 $X \sim U(-0.5, 0.5)$

b



$$P(-0.2 < X < 0.2) = 0.4 \times 1 = 0.4$$

c
$$\text{Var}(X) = \frac{(b-a)^2}{12} = \frac{(0.5 - (-0.5))^2}{12} = \frac{1}{12}$$

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Exercise C, Question 6

Question:

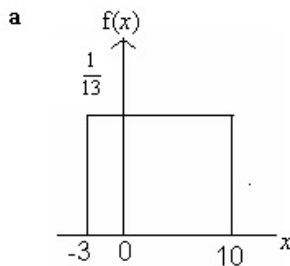
Jameil catches a bus to work every morning. According to the timetable the bus is due at 9 a.m., but Jameil knows that the bus can arrive at a random time between three minutes early and ten minutes late. The random variable X represents the time, in minutes, after 9 a.m. when the bus arrives.

- Suggest a suitable model for the distribution of X and specify it fully.
- Calculate the mean value of X .
- Find the cumulative distribution function of X .

Jameil will be late for work if the bus arrives after 9.05 a.m.

- Find the probability that Jameil is late for work.

Solution:



$$X \sim U(-3, 10)$$

$$f(x) = \begin{cases} \frac{1}{13} & -3 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

b Mean = $E(X) = \frac{-3+10}{2} = 3.5$ minutes

c

$$F(x) = \int_{-3}^x \frac{1}{13} dt$$

$$= \left[\frac{t}{13} \right]_{-3}^x$$

$$= \frac{x+3}{13}$$

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{13} & -3 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$

d $P(5 < X < 10) = (10 - 5) \times \frac{1}{13} = \frac{5}{13}$

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Exercise C, Question 7

Question:

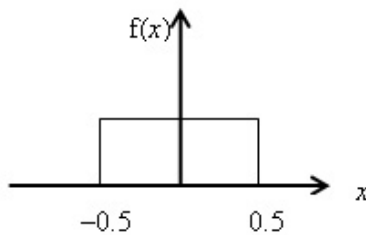
A plumber measures, to the nearest cm, the lengths of pipes.

- Suggest a suitable model to represent the difference between the true lengths and the measured lengths.
- Find the probability that for a randomly chosen rod the measured length will be within 0.2 cm of the true length.
- Three pipes are selected at random. Find the probability that all three pipes will be within 0.2 cm of the true length.

Solution:

a $U(-0.5, 0.5)$

b



$$P(-0.2 \leq X \leq 0.2) = (0.2 - (-0.2)) \times 1 = 0.4$$

c $P(3 \text{ pipes between } -0.2 \text{ and } 0.2) = 0.4^3 = 0.064$

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Exercise C, Question 8

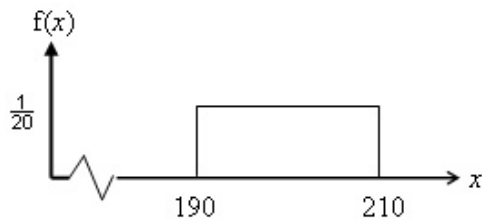
Question:

A coffee machine dispenses coffee into cups. It is electronically controlled to cut off the flow of coffee randomly between 190 ml and 210 ml. The random variable X is the volume of coffee dispensed into a cup.

- Specify the probability density function of X and sketch its graph.
- Find the probability that the machine dispenses
 - less than 198 ml,
 - exactly 198 ml.
- Calculate the inter-quartile range of X .

Solution:

a



$$f(x) = \begin{cases} \frac{1}{20} & 190 \leq x \leq 210 \\ 0 & \text{otherwise} \end{cases}$$

b i $P(X < 198) = (198 - 190) \times \frac{1}{20} = \frac{2}{5}$

ii 0

c $\frac{210 - 190}{4} = 5$ is one quarter of the range

$$Q_1 = 190 + 5 = 195 \quad Q_3 = 210 - 5 = 205$$

$$\text{IQR} = 205 - 195 = 10$$

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Exercise C, Question 9

Question:

Write down the name of the distribution you would recommend as a suitable model for each of the following situations.

- a the difference between the true height and the height measured, to the nearest cm, of randomly chosen people.
- b the heights of randomly selected 18-year-old females.

Solution:

a Uniform

b Normal

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