

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

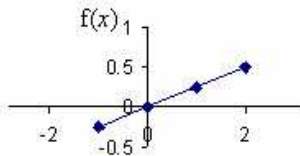
Question:

Give reasons why the following are not valid probability density functions.

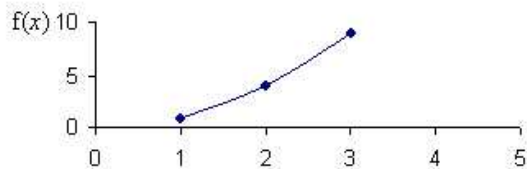
- a $f(x) = \begin{cases} \frac{1}{4}x, & -1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$
- b $f(x) = \begin{cases} x^2, & 1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$
- c $f(x) = \begin{cases} (x^3 - 2), & -1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

- a There are negative values for $f(x)$ when $x < 0$ so this is not a probability density function.



b



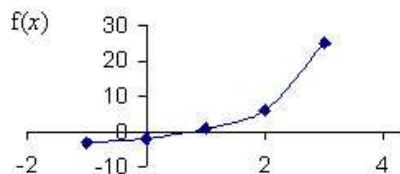
No negative values of $f(x)$

$$\text{Area} = \int_1^3 x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_1^3$$

$$= 8\frac{2}{3} \text{ not equal to 1 therefore it is not a valid probability density function.}$$

- c There are negative values for $f(x)$ so this is not a probability density function.



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Exercise A, Question 2

Question:

For what value of k is the following a valid probability density function?

$$f(x) = \begin{cases} k(1-x^2), & -4 \leq x \leq -2, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

$$\begin{aligned} \int_{-4}^{-2} k - kx^2 \, dx &= 1 \\ \left[kx - \frac{kx^3}{3} \right]_{-4}^{-2} &= 1 \\ \left[-2k + \frac{8k}{3} \right] - \left[-4k + \frac{64k}{3} \right] &= 1 \\ -\frac{50}{3}k &= 1 \\ k &= -\frac{3}{50} \end{aligned}$$

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Exercise A, Question 3

Question:

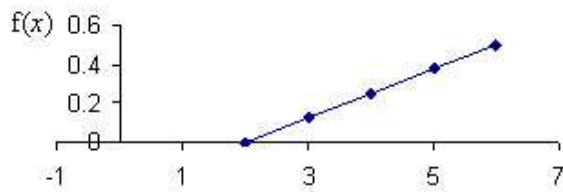
Sketch the following probability density functions.

$$\mathbf{a} \quad f(x) = \begin{cases} \frac{1}{8}(x-2), & 2 \leq x \leq 6, \\ 0, & \text{otherwise.} \end{cases}$$

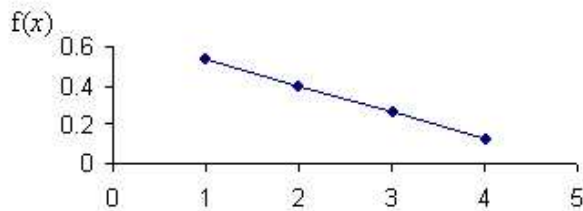
$$\mathbf{b} \quad f(x) = \begin{cases} \frac{2}{15}(5-x), & 1 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

a



b



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Exercise A, Question 4

Question:

Find the value of k so that each of the following are valid probability density functions.

$$\text{a } f(x) = \begin{cases} kx, & 1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{b } f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{c } f(x) = \begin{cases} k(1+x^2), & -1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

a

$$\int_1^3 kx \, dx = 1$$

$$\left[\frac{kx^2}{2} \right]_1^3 = 1$$

$$\frac{9k}{2} - \frac{k}{2} = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$\int_0^3 kx^2 \, dx = 1$$

$$\left[\frac{kx^3}{3} \right]_0^3 = 1$$

$$\frac{27k}{3} = 1$$

$$27k = 3$$

$$k = \frac{3}{27} = \frac{1}{9}$$

c

$$\int_{-1}^2 k(1+x^2) \, dx = 1$$

$$\left[kx + \frac{kx^3}{3} \right]_{-1}^2 = 1$$

$$\left(2k + \frac{8k}{3} \right) - \left(-k - \frac{k}{3} \right) = 1$$

$$\frac{14k}{3} - \left(-\frac{4k}{3} \right) = 1$$

$$\frac{14k}{3} + \frac{4k}{3} = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

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Exercise A, Question 5

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4-x), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Find the value of k .
- Sketch the probability density function for all values of x .

Solution:

a

$$\int_0^2 k(4-x) \, dx = 1$$

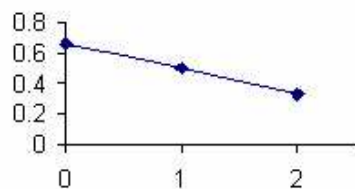
$$\left[4kx - \frac{kx^2}{2} \right]_0^2 = 1$$

$$8k - 2k = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

b



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Exercise A, Question 6

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^2(2-x), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k .

Solution:

$$\int_0^2 kx^2(2-x) dx = 1$$

$$\left[\frac{2kx^3}{3} - \frac{kx^4}{4} \right]_0^2 = 1$$

$$\left(\frac{16k}{3} - \frac{16k}{4} \right) - 0 = 1$$

$$\frac{16k}{12} = 1$$

$$16k = 12$$

$$k = \frac{3}{4} \text{ or } 0.75$$

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Exercise A, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^3, & 1 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k .

Solution:

$$\int_1^4 kx^3 dx = 1$$

$$\left[\frac{kx^4}{4} \right]_1^4 = 1$$

$$\frac{256k}{4} - \frac{k}{4} = 1$$

$$\frac{255k}{4} = 1$$

$$k = \frac{4}{255}$$

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Exercise A, Question 8

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & 0 < x < 2, \\ k(2x - 3), & 2 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

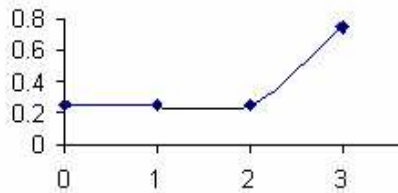
- Find the value of k .
- Sketch the probability density function for all values of x .

Solution:

a

$$\begin{aligned} \int_0^2 k \, dx + \int_2^3 k(2x - 3) \, dx &= 1 \\ [kx]_0^2 + \left[\frac{2kx^2}{2} - 3kx \right]_2^3 &= 1 \\ 2k + [(9k - 9k) - (4k - 6k)] &= 1 \\ 2k + 2k &= 1 \\ k &= \frac{1}{4} \text{ or } 0.25 \end{aligned}$$

b



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Exercise B, Question 1

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{3x^2}{8}, & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

<p>1 Method 1:</p> $F(x) = \int_0^x \frac{3t^2}{8} dt$ $= \left[\frac{3t^3}{24} \right]_0^x$ $= \frac{3x^3}{24} - 0$ $= \frac{3x^3}{24}$	<p>Method 2:</p> $F(x) = \int \frac{3x^2}{8} dx$ $= \frac{3x^3}{24} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x^3}{24} & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	

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Exercise B, Question 2

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{1}{4}(4-x), & 1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

<p>2 Method 1:</p> $F(x) = \int_1^x \frac{1}{4}(4-t) dt$ $= \left[t - \frac{t^2}{8} \right]_1^x$ $= \left(x - \frac{x^2}{8} \right) - \left(1 - \frac{1}{8} \right)$ $= x - \frac{x^2}{8} - \frac{7}{8}$	<p>Method 2:</p> $F(x) = \int \frac{1}{4}(4-x) dx$ $= x - \frac{x^2}{8} + C$ $F(3) = 1$ $3 - \frac{9}{8} + C = 1$ $C = -\frac{7}{8}$
$F(x) = \begin{cases} 0 & x < 1 \\ x - \frac{x^2}{8} - \frac{7}{8} & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$	

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Exercise B, Question 3

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{x}{9}, & 0 < x < 3, \\ \frac{1}{9}(6-x) & 3 \leq x \leq 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

<p>Method 1:</p> $F(x) = \int_0^x \frac{t}{9} dt$ $= \left[\frac{t^2}{18} \right]_0^x$ $= \frac{x^2}{18}$ $F(x) = \int_3^x \frac{1}{9}(6-t) dt + \int_0^3 \frac{x}{9} dx$ $= \left[\frac{2t}{3} - \frac{t^2}{18} \right]_3^x + \left[\frac{x^2}{18} \right]_0^3$ $= \left(\frac{2x}{3} - \frac{x^2}{18} \right) - \left(2 - \frac{9}{18} \right) + 0.5$ $= \frac{2x}{3} - \frac{x^2}{18} - 1$	<p>Method 2:</p> $F(x) = \int \frac{x}{9} dx$ $= \frac{x^2}{18} + C$ $F(0) = 0$ $0 + C = 0$ $C = 0$ $F(x) = \int \frac{1}{9}(6-x) dx$ $= \frac{2x}{3} - \frac{x^2}{18} + C$ $F(6) = 1$ $4 - 2 + C = 1$ $C = -1$
$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{18} & 0 < x < 3 \\ \frac{2x}{3} - \frac{x^2}{18} - 1 & 3 \leq x \leq 6 \\ 1 & x > 6 \end{cases}$	

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Exercise B, Question 4

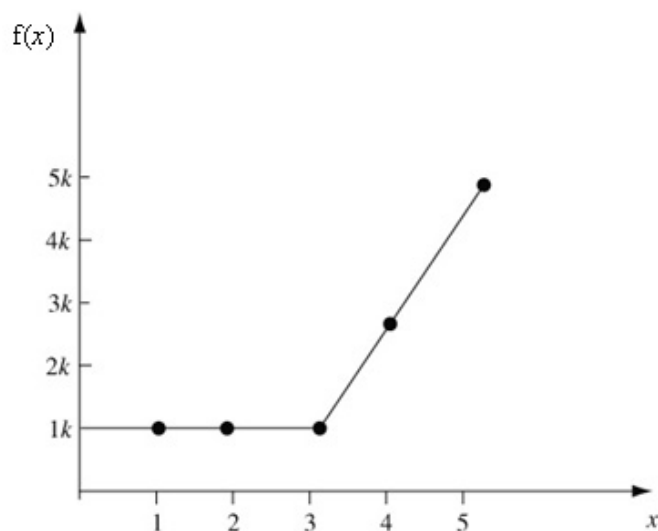
Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & 0 \leq x \leq 3, \\ k(2x-5), & 3 \leq x \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

- a Sketch $f(x)$.
- b Find the value of k .
- c Find $F(x)$.

Solution:

a**b**

$$\int_0^3 k \, dx + \int_3^5 k(2x-5) \, dx = 1$$

$$[kx]_0^3 + [k(x^2 - 5x)]_3^5 = 1$$

$$3k + [k(25 - 25) - k(9 - 15)] = 1$$

$$9k = 1$$

$$k = \frac{1}{9}$$

c Method 1

$$\int_0^x \frac{1}{9} \, dt = \left[\frac{1}{9}t \right]_0^x$$

$$= \frac{1}{9}x$$

$$\int_3^x \frac{1}{9}(2t-5) \, dt + \int_0^3 \frac{1}{9} \, dx = \left[\frac{1}{9}(t^2 - 5t) \right]_3^x + \left[\frac{x}{9} \right]_0^3$$

$$= \left[\frac{1}{9}(x^2 - 5x) - \frac{1}{9}(9 - 15) \right] + \left[\frac{3}{9} \right]$$

$$= \frac{1}{9}x^2 - \frac{5}{9}x + 1$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x}{9} & 0 < x < 3 \\ \frac{x^2}{9} - \frac{5x}{9} + 1 & 3 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

Method 2

$$\int \frac{1}{9} \, dx = \frac{1}{9}x + C$$

$$F(0) = 0$$

$$C = 0$$

$$\int \frac{1}{9}(2x-5) \, dx = \frac{1}{9}(x^2 - 5x) + C$$

$$F(5) = 1$$

$$\frac{1}{9}(25 - 25) + C = 1$$

$$C = 1$$

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Exercise B, Question 5

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \leq x \leq 3, \\ 1, & x > 3. \end{cases}$$

Find the probability density function, $f(x)$.

Solution:

$$\frac{d}{dx} F(x) = \frac{2x}{5}$$

$$f(x) = \begin{cases} \frac{2x}{5} & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

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Exercise B, Question 6

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{2}(x-1), & 1 \leq x \leq 3, \\ 1, & x > 3. \end{cases}$$

- a Find $P(X \leq 2.5)$.
- b Find $P(X > 1.5)$.
- c Find $P(1.5 \leq X \leq 2.5)$.

Solution:

<p>a $P(X \leq 2.5) = F(2.5)$ $= \frac{1}{2}(2.5-1)$ $= 0.75$</p> <p>b $P(X > 1.5) = 1 - F(1.5)$ $= 1 - \frac{1}{2}(1.5-1)$ $= 0.75$</p> <p>c $P(1.5 \leq X \leq 2.5) = F(2.5) - F(1.5)$ $= 0.75 - 0.25$ $= 0.5$</p>	
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Exercise B, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{3x^2}{8} & 0 \leq x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

- a Find the cumulative distribution function of X .
b Find $P(X \leq 1)$.

Solution:

<p>a Method 1:</p> $F(X) = \int_0^x \frac{3x^2}{8} dx$ $= \left[\frac{x^3}{8} \right]_0^x$ $= \frac{x^3}{8}$	<p>Method 2:</p> $F(X) = \int \frac{3x^2}{8} dx$ $= \frac{x^3}{8} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$	
<p>b $P(x \leq 1) = F(1) = \frac{1}{8}$</p>	

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Exercise B, Question 8

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{2}(x^3 - 2x^2 + x), & 1 \leq x \leq 2, \\ 1, & x > 2. \end{cases}$$

- Find the probability density function $f(x)$.
- Sketch the probability density function.
- Find $P(X < 1.5)$.

Solution:

<p>a</p> $\frac{d}{dx} F(x) = \frac{3x^2}{2} - 2x + \frac{1}{2}$ $f(x) = \begin{cases} \frac{3}{2}x^2 - 2x + \frac{1}{2} & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$ <p>b</p> <p>c</p> $\begin{aligned} P(X < 1.5) &= F(1.5) \\ &= \frac{1}{2}(1.5^3 - 2 \times 1.5^2 + 1.5) \\ &= 0.1875 \end{aligned}$	
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Exercise B, Question 9

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4 - x^2), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Show that $k = \frac{3}{16}$.
- Find the cumulative distribution function of X .
- Find $P(0.69 < X < 0.70)$. Give your answer correct to one significant figure.

Solution:

<p>a $\int_0^2 k(4 - x^2) dx = 1$</p> $\left[k \left(4x - \frac{x^3}{3} \right) \right]_0^2 = 1$ $k \left(8 - \frac{8}{3} \right) = 1$ $\frac{16k}{3} = 1$ $k = \frac{3}{16}$	
<p>b Method 1:</p> $F(x) = \int_0^x \frac{3}{16} (4 - t^2) dt$ $= \left[\frac{3}{16} \left(4t - \frac{t^3}{3} \right) \right]_0^x$ $= \frac{3}{16} \left(4x - \frac{x^3}{3} \right)$ $F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{16} \left(4x - \frac{x^3}{3} \right) & 0 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$ <p>c $P(0.69 < X < 0.70) = F(0.70) - F(0.69)$</p> $= \frac{3}{16} \left(2.8 - \frac{0.343}{3} \right) - \frac{3}{16} \left(2.76 - \frac{0.328509}{3} \right)$ $= 0.00659$ $= 0.007 \text{ (1 s.f.)}$	<p>Method 2:</p> $F(x) = \int \frac{3}{16} (4 - x^2) dx$ $= \frac{3}{16} \left(4x - \frac{x^3}{3} \right) + C$ $F(2) = 1$ $\frac{3}{16} \left(8 - \frac{8}{3} \right) + C = 1$ $C = 0$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

- a k ,
- b $E(X)$,
- c $\text{Var}(X)$.

Solution:

a

$$\int_0^2 kx^2 = 1$$

$$\left[\frac{kx^3}{3} \right]_0^2 = 1$$

$$\frac{8k}{3} - 0 = 1$$

$$8k = 3$$

$$k = \frac{3}{8}$$

b

$$E(X) = \int_0^2 \frac{3x^3}{8} dx$$

$$= \left[\frac{3x^4}{32} \right]_0^2$$

$$= \frac{48}{32} - 0$$

$$= 1.5$$

c

$$\text{Var}(X) = \int_0^2 \frac{3x^4}{8} dx - 1.5^2$$

$$= \left[\frac{3x^5}{40} \right]_0^2 - 1.5^2$$

$$= \left(\frac{96}{40} - 0 \right) - 2.25$$

$$= 2.4 - 2.25$$

$$= 0.15$$

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Exercise C, Question 2

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y^2}{9}, & 0 \leq y \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

- Find $E(Y)$.
- Find $\text{Var}(Y)$.
- Find the standard deviation of Y .

Solution:

a

$$\begin{aligned} E(Y) &= \int_0^3 \frac{y^3}{9} \, dy \\ &= \left[\frac{y^4}{36} \right]_0^3 \\ &= \frac{81}{36} - 0 \\ &= 2.25 \end{aligned}$$

b

$$\begin{aligned} \text{Var}(Y) &= \int_0^3 \frac{y^4}{9} \, dy - 2.25^2 \\ &= \left[\frac{y^5}{45} \right]_0^3 - 2.25^2 \\ &= \left(\frac{243}{45} - 0 \right) - 5.0625 \\ &= 5.4 - 5.0625 \\ &= 0.3375 \end{aligned}$$

$$\text{c } \sigma = \sqrt{0.3375} = 0.581$$

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Exercise C, Question 3

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y}{8}, & 0 \leq y \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

- a Find $E(Y)$.
- b Find $\text{Var}(Y)$.
- c Find the standard deviation of Y .
- d Find $P(Y > \mu)$.
- e Find $\text{Var}(3Y + 2)$.
- f Find $E(Y + 2)$.

Solution:

a

$$\begin{aligned}
 E(Y) &= \int_0^4 \frac{y^2}{8} dy \\
 &= \left[\frac{y^3}{24} \right]_0^4 \\
 &= \frac{64}{24} - 0 \\
 &= \frac{8}{3}
 \end{aligned}$$

b

$$\begin{aligned}
 \text{Var}(Y) &= \int_0^4 \frac{y^3}{8} dy - \left(\frac{8}{3} \right)^2 \\
 &= \left[\frac{y^4}{32} \right]_0^4 - \left(\frac{64}{9} \right) \\
 &= \left(\frac{256}{32} - 0 \right) - \left(\frac{64}{9} \right) \\
 &= 8 - \left(\frac{64}{9} \right) \\
 &= \frac{8}{9}
 \end{aligned}$$

$$\text{c } \sigma = \sqrt{\frac{8}{9}} = 0.943$$

d

$$\begin{aligned}
 P(Y > \mu) &= P\left(Y > \frac{8}{3}\right) \\
 &= \int_{\frac{8}{3}}^4 \frac{y}{8} dy \\
 &= \left[\frac{y^2}{16} \right]_{\frac{8}{3}}^4 \\
 &= 1 - 0.4444 \\
 &= 0.556
 \end{aligned}$$

e

$$\begin{aligned}
 \text{Var}(3Y+2) &= 9 \text{Var}(Y) \\
 &= 9 \times \frac{8}{9} \\
 &= 8
 \end{aligned}$$

f

$$\begin{aligned}
 E(Y+2) &= E(Y) + 2 \\
 &= \frac{8}{3} + 2 \\
 &= 4 \frac{2}{3}
 \end{aligned}$$

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Exercise C, Question 4

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k(1-x), & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- a Find k .
- b Find $E(X)$.
- c Show that $\text{Var}(X) = \frac{1}{18}$.
- d Find $P(X > \mu)$.

Solution:

a

$$\int_0^1 k(1-x) \, dx = 1$$

$$\left[kx - \frac{kx^2}{2} \right]_0^1 = 1$$

$$k - \frac{1}{2}k = 1$$

$$k = 2$$

b

$$E(X) = \int_0^1 (2x - 2x^2) \, dx$$

$$= \left[\frac{2x^2}{2} - \frac{2x^3}{3} \right]_0^1$$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

c

$$\text{Var}(X) = \int_0^1 (2x^2 - 2x^3) \, dx - \left(\frac{1}{3}\right)^2$$

$$= \left[\frac{2x^3}{3} - \frac{2x^4}{4} \right]_0^1 - \frac{1}{9}$$

$$= \left(\frac{2}{3} - \frac{1}{2}\right) - 0 - \frac{1}{9}$$

$$= \frac{1}{18}$$

d

$$P\left(X > \frac{1}{3}\right) = \int_{\frac{1}{3}}^1 2(1-x) \, dx$$

$$= \left[2x - x^2 \right]_{\frac{1}{3}}^1$$

$$= (2 - 1) - \left(\frac{2}{3} - \frac{1}{9}\right)$$

$$= \frac{4}{9}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 5

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- a** Find $P(X < 0.5)$.
b Find $E(X)$.

Solution:

a

$$\begin{aligned} P(X < 0.5) &= \int_0^{0.5} 12x^2 - 12x^3 \, dx \\ &= \left[4x^3 - 3x^4 \right]_0^{0.5} \\ &= \frac{1}{2} - \frac{3}{16} \\ &= \frac{5}{16} \text{ or } 0.3125 \end{aligned}$$

b

$$\begin{aligned} E(X) &= \int_0^1 12x^3 - 12x^4 \, dx \\ &= \left[3x^4 - \frac{12x^5}{5} \right]_0^1 \\ &= (3 - 2.4) - 0 \\ &= 0.6 \text{ or } \frac{3}{5} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 6

Question:

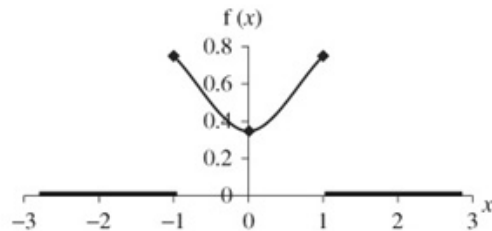
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{3}{8}(1+x^2), & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- Write down $E(X)$.
- Show that $\sigma^2 = 0.4$.
- Find $P(-\sigma < X < \sigma)$.

Solution:

a



b

$$E(X) = 0 \text{ (by symmetry)}$$

c

$$\begin{aligned} \sigma^2 &= \text{Var}(X) \\ &= \int_{-1}^1 \frac{3x^2}{8} + \frac{3x^4}{8} dx - 0^2 \\ &= \left[\frac{3x^3}{24} + \frac{3x^5}{40} \right]_{-1}^1 \\ &= \left(\frac{3}{24} + \frac{3}{40} \right) - \left(-\frac{3}{24} - \frac{3}{40} \right) \\ &= 0.4 \end{aligned}$$

d

$$\begin{aligned} P(-\sqrt{0.4} < X < \sqrt{0.4}) &= \int_{-\sqrt{0.4}}^{\sqrt{0.4}} \frac{3}{8} + \frac{3x^2}{8} dx \\ &= \left[\frac{3x}{8} + \frac{3x^3}{24} \right]_{-\sqrt{0.4}}^{\sqrt{0.4}} \\ &= \left(\frac{3}{8} \times \sqrt{0.4} + \frac{3}{24} \times (\sqrt{0.4})^3 \right) - \left(\frac{3}{8} \times (-\sqrt{0.4}) + \frac{3}{24} \times (-\sqrt{0.4})^3 \right) \\ &= 0.538 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 7

Question:

The continuous random variable T has c.d.f. given by

$$f(t) = \begin{cases} kt^3, & 0 \leq t \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

where k is a positive constant.

- Find k .
- Show that $E(T)$ is 1.6.
- Find $E(2T+3)$.
- Find $\text{Var}(T)$.
- Find $\text{Var}(2T+3)$.
- Find $P(T < 1)$.

Solution:

a

$$\int_0^2 kx^3 dx = 1$$

$$\left[\frac{kx^4}{4} \right]_0^2 = 1$$

$$4k - 0 = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$E(T) = \int_0^2 \frac{t^4}{4} dx$$

$$= \left[\frac{t^5}{20} \right]_0^2$$

$$= \frac{32}{20} - 0$$

$$= 1.6$$

c

$$E(2T + 3) = 2E(T) + 3$$

$$= 2 \times 1.6 + 3$$

$$= 6.2$$

d

$$\text{Var}(T) = \int_0^2 \frac{t^5}{4} dt - \left(\frac{8}{5} \right)^2$$

$$= \left[\frac{t^6}{24} \right]_0^2 - \left(\frac{8}{5} \right)^2$$

$$= \left(\frac{64}{24} - 0 \right) - \left(\frac{64}{25} \right)$$

$$= \frac{8}{75}$$

e

$$\text{Var}(2T + 3) = 4 \text{Var}(T)$$

$$= \frac{32}{75}$$

f

$$P(T < 1) = \int_0^1 \frac{t^3}{4} dt$$

$$= \left[\frac{t^4}{16} \right]_0^1$$

$$= \frac{1}{16}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 8

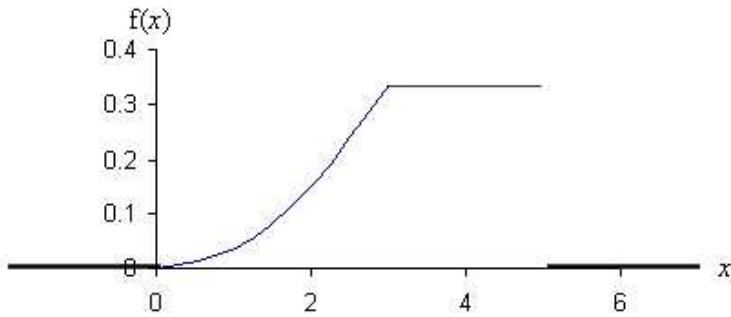
Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{x^2}{27}, & 0 \leq x < 3, \\ \frac{1}{3}, & 3 \leq x \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

- Draw a rough sketch of $f(x)$.
- Find $E(X)$.
- Find $\text{Var}(X)$.
- Find the standard deviation, σ , of X .

Solution:

a**b**

$$\begin{aligned}
 E(X) &= \int_0^3 \frac{x^3}{27} dx + \int_3^5 \frac{x}{3} dx \\
 &= \left[\frac{x^4}{108} \right]_0^3 + \left[\frac{x^2}{6} \right]_3^5 \\
 &= \left(\frac{81}{108} - 0 \right) + \left(\frac{25}{6} - \frac{9}{6} \right) \\
 &= \frac{41}{12} \\
 &= 3.417
 \end{aligned}$$

c

$$\begin{aligned}
 \text{Var}(X) &= \left(\int_0^3 \frac{x^4}{27} dx + \int_3^5 \frac{x^2}{3} dx \right) - \left(\frac{41}{12} \right)^2 \\
 &= \left(\left[\frac{x^5}{135} \right]_0^3 + \left[\frac{x^3}{9} \right]_3^5 \right) - \left(\frac{1681}{144} \right) \\
 &= \left(\frac{243}{135} - 0 \right) + \left(\frac{125}{9} - \frac{27}{9} \right) - \left(\frac{1681}{144} \right) \\
 &= 1.0152
 \end{aligned}$$

d

$$\sigma = \sqrt{1.0152} = 1.01$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

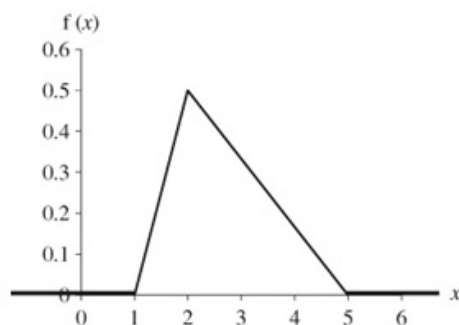
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}(x-1), & 1 \leq x < 2, \\ \frac{1}{6}(5-x), & 2 \leq x \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch $f(x)$.
- Find $E(X)$.
- Find $\text{Var}(X)$.

Solution:

a



b

$$\begin{aligned} E(X) &= \int_1^2 \left(\frac{x^2}{2} - \frac{x}{2} \right) dx + \int_2^5 \left(\frac{5x}{6} - \frac{x^2}{6} \right) dx \\ &= \left[\frac{x^3}{6} - \frac{x^2}{4} \right]_1^2 + \left[\frac{5x^2}{12} - \frac{x^3}{18} \right]_2^5 \\ &= \left[\left(\frac{8}{6} - 1 \right) - \left(\frac{1}{6} - \frac{1}{4} \right) \right] + \left[\left(\frac{125}{12} - \frac{125}{18} \right) - \left(\frac{20}{12} - \frac{8}{18} \right) \right] \\ &= 2\frac{2}{3} \end{aligned}$$

c

$$\begin{aligned} \text{Var}(X) &= \int_1^2 \left(\frac{x^3}{2} - \frac{x^2}{2} \right) dx + \int_2^5 \left(\frac{5x^2}{6} - \frac{x^3}{6} \right) dx - \left(2\frac{2}{3} \right)^2 \\ &= \left[\frac{x^4}{8} - \frac{x^3}{6} \right]_1^2 + \left[\frac{5x^3}{18} - \frac{x^4}{24} \right]_2^5 - \left(2\frac{2}{3} \right)^2 \\ &= \left[\left(\frac{16}{8} - \frac{8}{6} \right) - \left(\frac{1}{8} - \frac{1}{6} \right) \right] + \left[\left(\frac{625}{18} - \frac{625}{24} \right) - \left(\frac{40}{18} - \frac{16}{24} \right) \right] - \left(2\frac{2}{3} \right)^2 \\ &= \frac{13}{18} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 10

Question:

Telephone calls arriving at a company are referred immediately by the telephonist to other people working in the company. The time a call takes, in minutes, is modelled by a continuous random variable T , having a p.d.f. given by

$$f(t) = \begin{cases} kt^2, & 0 \leq t \leq 10, \\ 0, & \text{otherwise.} \end{cases}$$

- Show that $k = 0.003$.
- Find $E(T)$.
- Find $\text{Var}(T)$.
- Find the probability of a call lasting between 7 and 9 minutes.
- Sketch the p.d.f.

Solution:

a

$$\int_0^{10} kt^2 dt = 1$$

$$\left[\frac{kt^3}{3} \right]_0^{10} = 1$$

$$\frac{1000k}{3} - 0 = 1$$

$$1000k = 3$$

$$k = 0.003$$

b

$$E(T) = \int_0^{10} 0.003t^3 dt$$

$$= \left[\frac{0.003t^4}{4} \right]_0^{10}$$

$$= \frac{30}{4} - 0$$

$$= 7.5$$

c

$$\text{Var}(x) = \int_0^{10} 0.003t^4 dt - 7.5^2$$

$$= \left[\frac{0.003t^5}{5} \right]_0^{10} - 7.5^2$$

$$= (60 - 0) - 56.25$$

$$= 3.75$$

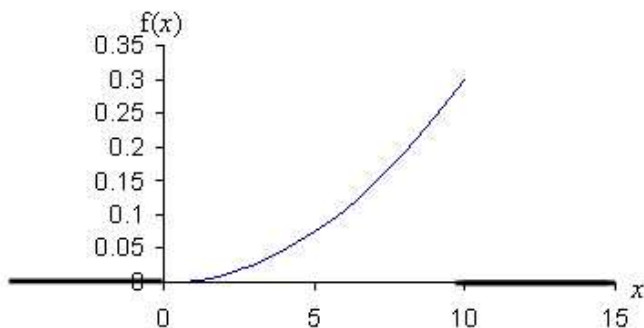
d

$$P(7 < T < 9) = \int_7^9 0.003t^2 dt$$

$$= \left[\frac{0.003t^3}{3} \right]_7^9$$

$$= 0.729 - 0.343$$

$$= 0.386$$

e

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

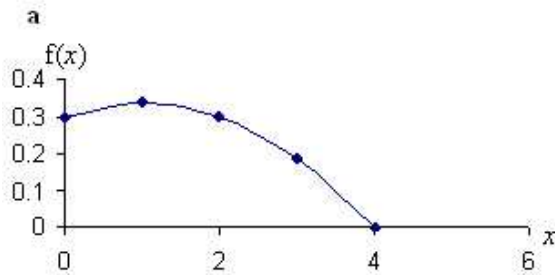
Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{80}(8 + 2x - x^2), & 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- Find the mode of X .

Solution:



b Differentiating $\frac{d}{dx} \frac{3}{80}(8 + 2x - x^2) = 0$

$$\frac{3}{80}(2 - 2x) = 0$$

This = 0 when $(2 - 2x) = 0$

$$x = 1$$

The mode is 1.

(Note: To check this is a maximum you could differentiate again and see if f'' is negative for all values of x .)

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

The continuous random variable X has p.d.f. given by

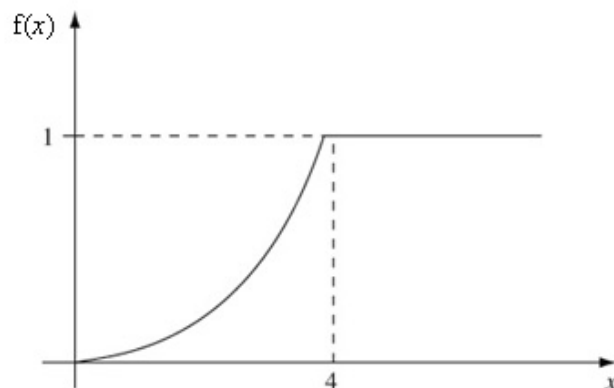
$$f(x) = \begin{cases} \frac{1}{8}x, & 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the c.d.f. of X .
- Find the median of X .

Solution:

a

Method 1	Method 2
$\int_0^x \frac{1}{8}t \, dx = \left[\frac{t^2}{16} \right]_0^x$ $= \frac{x^2}{16}$	$F(x) = \int \frac{1}{8}x \, dx$ $= \frac{x^2}{16} + C$ $F(4) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$	



$$\text{b} \quad \frac{m^2}{16} = 0.5 \text{ so } m^2 = 8$$

$$m = \sqrt{8} = 2.83 \text{ or } -2.83$$

Median = 2.83 since -2.83 is not in the range.

Solutionbank S2

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Exercise D, Question 3

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{6}, & 0 \leq x \leq 2, \\ -\frac{x^2}{3} + 2x - 2, & 2 \leq x \leq 3 \\ 1, & x \geq 3. \end{cases}$$

- a Find the median value of X . Give your answer to 3 decimal places.
 b Find the quartiles and the inter-quartile range of X . Give your answer to 3 decimal places.

Solution:

- a $F(m) = 0.5$ where m is the median.

Since $F(2) = \frac{2}{3}$ the median must lie in the range $0 \leq x \leq 2$

$$\text{So } F(m) = \frac{m^2}{6} = 0.5$$

$$m^2 = 2$$

$$m = 1.73 \text{ or } -1.73$$

Median = 1.73 since -1.73 is not in the range.

- b Lower quartile lies in the range $0 \leq x \leq 2$

$$\frac{Q_1^2}{6} = 0.25$$

$$Q_1 = \sqrt{1.5} = 1.225$$

Upper quartile lies in the range $2 \leq x \leq 3$

$$-\frac{Q_3^2}{3} + 2Q_3 - 2 = 0.75$$

$$-Q_3^2 + 6Q_3 - 6 = 2.25$$

$$-Q_3^2 + 6Q_3 - 8.25 = 0$$

$$Q_3 = \frac{-6 \pm \sqrt{36 - 33}}{-2}$$

$$= 2.134 \text{ or } 3.87$$

$Q_3 = 2.134$ as 3.87 does not lie in the range

$$\text{IQR} = 2.134 - 1.225 = 0.909$$

Solutionbank S2

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Exercise D, Question 4

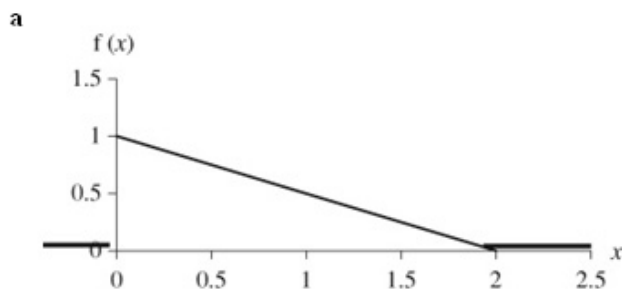
Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x, & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- Write down the mode of X .
- Find the c.d.f. of X .
- Find the median value of X .

Solution:



b 0

c

Method 1	Method 2
$\int_0^x \left(1 - \frac{1}{2}t\right) dt = \left[t - \frac{1}{4}t^2\right]_0^x$ $= x - \frac{1}{4}x^2$	$F(x) = \int 1 - \frac{1}{2}x \, dx$ $= x - \frac{1}{4}x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ x - \frac{1}{4}x^2 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	

d $m - \frac{1}{4}m^2 = 0.5$

$$m^2 - 4m + 2 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$m = 2 - \sqrt{2}$ or $2 + \sqrt{2}$ therefore median = $2 - \sqrt{2}$ as $2 + \sqrt{2}$ is not in range.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 5

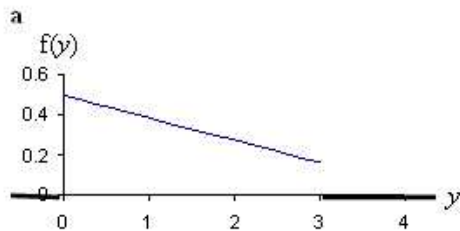
Question:

The continuous random variable Y has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{9}y, & 0 \leq y \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of Y .
- Write down the mode of Y .
- Find the c.d.f. of Y .
- Find the median value of Y .

Solution:



b 0

c

Method 1	Method 2
$\int_0^y \left(\frac{1}{2} - \frac{1}{9}t \right) dt = \left[\frac{t}{2} - \frac{1}{18}t^2 \right]_0^y$ $= \frac{y}{2} - \frac{1}{18}y^2$	$F(x) = \int \left(\frac{y}{2} - \frac{1}{9}y \right) dy$ $= \frac{y}{2} - \frac{1}{18}y^2 + C$ $F(3) = 1 \quad 1 = \frac{3}{2} - \frac{9}{18} + C$ $C = 0$
$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y}{2} - \frac{1}{18}y^2 & 0 \leq y \leq 3 \\ 1 & y > 3 \end{cases}$	

d

$$\frac{m}{2} - \frac{1}{18}m^2 = 0.5$$

$$m^2 - 9m + 9 = 0$$

$$m = \frac{9 \pm \sqrt{81 - 36}}{2}$$

$$\text{median} = \frac{9 - 3\sqrt{5}}{2} = 1.15$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 6

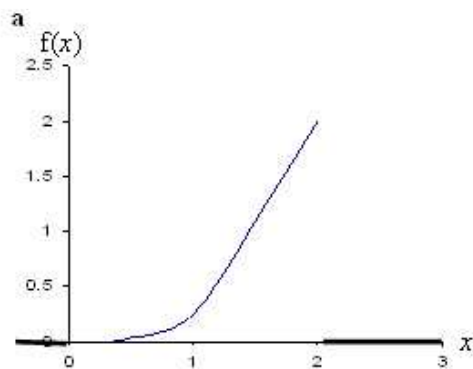
Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{4}x^3, & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- Write down the mode of X .
- Find the c.d.f. of X .
- Find the median value of X .

Solution:



b 2

c

Method 1	Method 2
$\int_0^x \frac{1}{4}t^3 dt = \left[\frac{1}{16}t^4 \right]_0^x$ $= \frac{1}{16}x^4$	$F(x) = \int \frac{1}{4}x^3 dx$ $= \frac{1}{16}x^4 + C$ $F(2) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16}x^4 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	

d $\frac{1}{16}m^4 = 0.5$
 $m^4 = 8$
 $m = \pm\sqrt[4]{8}$
 median = 1.68

Solutionbank S2

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Exercise D, Question 7

Question:

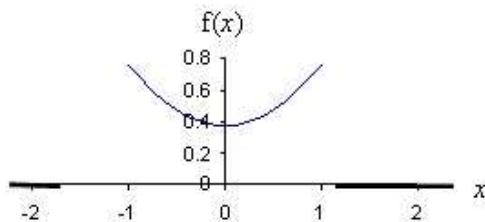
The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{8}(x^2 + 1), & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- What can you say about the mode of X ?
- Write down the median value of X .
- Find the c.d.f. of X .

Solution:

a



b bimodal -1 and 1

c median = 0

d

Method 1	Method 2
$\int_{-1}^x \frac{3}{8}x^2 + \frac{3}{8} dx = \left[\frac{1}{8}x^3 + \frac{3}{8}x \right]_{-1}^x$ $= \left[\frac{1}{8}x^3 + \frac{3}{8}x \right] - \left[-\frac{1}{8} - \frac{3}{8} \right]$ $= \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2}$	$F(x) = \int \frac{3}{8}x^2 + \frac{3}{8} dx$ $= \frac{1}{8}x^3 + \frac{3}{8}x + C$ $F(1) = 1 \quad 1 = \frac{1}{8} + \frac{3}{8} + C$ $C = \frac{1}{2}$
$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2} & -1 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$	

Solutionbank S2

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Exercise D, Question 8

Question:

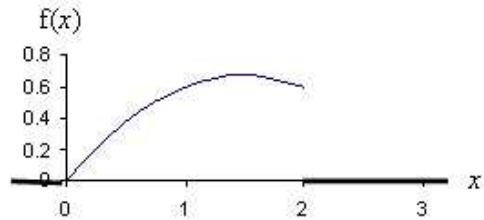
The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the p.d.f. of X .
- Find the mode of X .
- Find the c.d.f. of X .
- Show that the median value of X lies between 1.23 and 1.24.

Solution:

a



b Find maximum by differentiating

$$\frac{d}{dx} \left(\frac{9}{10}x - \frac{3}{10}x^2 \right) = \frac{9}{10} - \frac{6}{10}x$$

$$\frac{9}{10} - \frac{6}{10}x = 0$$

$$x = \frac{3}{2} \quad \text{mode} = 1.5$$

c

Method 1	Method 2
$\int_0^x \left(\frac{9}{10}t - \frac{3}{10}t^2 \right) dt = \left[\frac{9}{20}t^2 - \frac{1}{10}t^3 \right]_0^x$ $= \frac{9}{20}x^2 - \frac{1}{10}x^3$	$F(x) = \int \frac{9}{10}x - \frac{3}{10}x^2 dx$ $= \frac{9}{20}x^2 - \frac{1}{10}x^3 + C$ $F(2) = 1 \quad 1 = \frac{36}{20} - \frac{8}{10} + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{9}{20}x^2 - \frac{1}{10}x^3 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	

$$\text{d } F(1.23) = \frac{9}{20} \times 1.23^2 - \frac{1}{10} \times 1.23^3 = 0.495$$

$$F(1.24) = \frac{9}{20} \times 1.24^2 - \frac{1}{10} \times 1.24^3 = 0.501$$

Since 0.5 is in between the median lies between 1.23 and 1.24.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 9

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{8}(x^2 - 1), & 1 \leq x \leq 3, \\ 1, & x > 3. \end{cases}$$

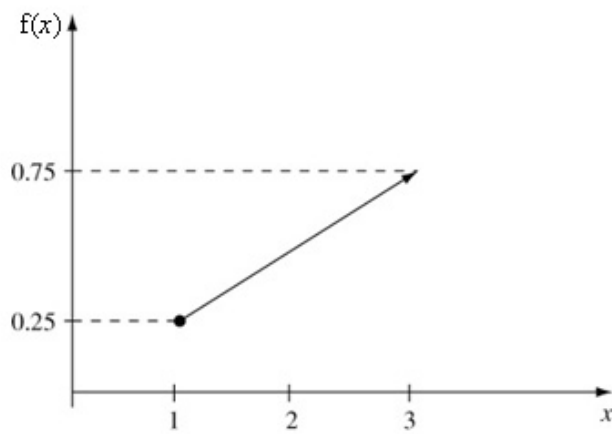
- Find the p.d.f. of the random variable X .
- Find the mode of X .
- Find the median of X .
- Find the quartiles of X .

Solution:

a Differentiating $\frac{d}{dx}\left(\frac{1}{8}x^2 - \frac{1}{8}\right) = \frac{1}{4}x$

$$f(x) = \begin{cases} \frac{1}{4}x, & 1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

b mode = 3



c $\frac{1}{8}m^2 - \frac{1}{8} = 0.5$

$$\frac{1}{8}m^2 = \frac{5}{8}$$

$$m = \sqrt{5}$$

$$\text{median} = \sqrt{5}$$

d $\frac{1}{8}Q_1^2 - \frac{1}{8} = 0.25$

$$\frac{1}{8}Q_1^2 = \frac{3}{8}$$

$$Q_1 = \sqrt{3}$$

$$\text{lower quartile} = \sqrt{3}$$

$$\frac{1}{8}Q_3^2 - \frac{1}{8} = 0.75$$

$$\frac{1}{8}Q_3^2 = \frac{7}{8}$$

$$Q_3 = \sqrt{7}$$

$$\text{upper quartile} = \sqrt{7}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 10

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 0, \\ 4x^3 - 3x^4, & 0 \leq x \leq 1, \\ 1, & x > 1. \end{cases}$$

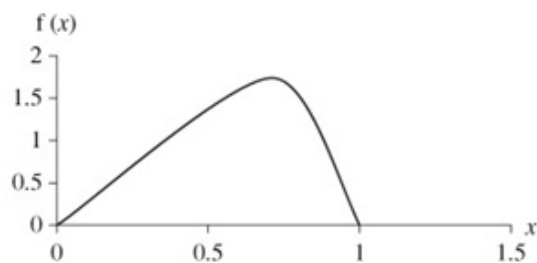
- Find the p.d.f. of the random variable X .
- Find the mode of X .
- Find $P(0.2 < X < 0.5)$.

Solution:

$$\text{a } \frac{d}{dx}(4x^3 - 3x^4) = 12x^2 - 12x^3$$

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

b



$$\text{maximum } \frac{d}{dx}(12x^2 - 12x^3) = 24x - 36x^2$$

$$24x - 36x^2 = 0$$

$$12x(2 - 3x) = 0$$

$$x = 0 \text{ or } \frac{2}{3} \text{ mode} = \frac{2}{3}$$

$$\begin{aligned} \text{c } P(0.2 < X < 0.5) &= F(0.5) - F(0.2) \\ &= (4 \times 0.5^3 - 3 \times 0.5^4) - (4 \times 0.2^3 - 3 \times 0.2^4) \\ &= 0.2853 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 11

Question:

The amount of vegetables eaten by a family in a week is a continuous random variable W kg. The continuous random variable W has p.d.f. given by

$$f(w) = \begin{cases} \frac{20}{5^5} w^3 (5-w), & 0 \leq w \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

- a Find the c.d.f. of the random variable W .
 b Find, to 3 decimal places, the probability that the family eat between 2 kg and 4 kg of vegetables in one week. **E**

Solution:

a

Method 1	Method 2
$\int_0^w \frac{20}{5^5} t^3 (5-t) dt = \left[\frac{100}{4 \times 5^5} t^4 - \frac{20}{5 \times 5^5} t^5 \right]_0^w$ $= \frac{25}{5^5} w^4 - \frac{4}{5^5} w^5$ $= \frac{w^4}{5^5} (25 - 4w)$	$F(x) = \int \frac{20}{5^5} w^3 (5-w) dx$ $= \frac{25}{5^5} w^4 - \frac{4}{5^5} w^5 + C$ $F(5) = 1 \quad 1 = 5 - 4 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{w^4}{5^5} (25 - 4w) & 0 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$	

b $P(2 < w < 4) = F(4) - F(2)$

$$= \left[\frac{4^4}{5^5} (25 - 16) \right] - \left[\frac{2^4}{5^5} (25 - 8) \right]$$

$$= 0.650$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 12

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}, & 0 \leq x < 1, \\ \frac{x^3}{5}, & 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Find the cumulative distribution function.
- Find, to 3 decimal places, the median and the inter-quartile range of the distribution. *E*

Solution:

a Method 1

$$\begin{aligned} F(x) &= \int_0^x \frac{1}{4} dt \\ &= \left[\frac{t}{4} \right]_0^x \\ &= \frac{x}{4} \end{aligned}$$

$$\begin{aligned} \int_0^1 \frac{1}{4} dx + \int_1^x \frac{t^3}{5} dt &= \left[\frac{x}{4} \right]_0^1 + \left[\frac{t^4}{20} \right]_1^x \\ &= \frac{1}{4} + \frac{x^4}{20} - \frac{1}{20} \end{aligned}$$

$$1 = \frac{16}{20} + C$$

$$= \frac{x^4}{20} + \frac{1}{5}$$

Method 2

$$\begin{aligned} F(x) &= \int \frac{1}{4} dx \\ &= \frac{x}{4} + C \end{aligned}$$

$$F(0) = 0 \text{ therefore } C = 0$$

$$\int \frac{x^3}{5} dx = \frac{x^4}{20} + C$$

$$F(2) = 1 \text{ therefore}$$

$$C = \frac{1}{5}$$

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{4}, & 0 \leq x < 1, \\ \frac{x^4}{20} + \frac{1}{5}, & 1 \leq x \leq 2, \\ 1, & x > 2 \end{cases}$$

$$\mathbf{b} \quad \frac{Q_1^4}{20} + \frac{1}{5} = 0.25$$

$$Q_1^4 + 4 - 5 = 0$$

$$Q_1 = 1$$

$$\frac{Q_3^4}{20} + \frac{1}{5} = 0.75$$

$$Q_3^4 + 4 = 15$$

$$Q_3^4 = 11$$

$$Q_3 = 1.821 \text{ or } -1.821$$

Therefore upper quartile = 1.821 as -1.821 is not in range

$$\text{IQR} = 1.82 - 1 = 0.821$$

$$\frac{m^4}{20} + \frac{1}{5} = 0.5$$

$$m^4 + 4 - 10 = 0$$

$$m^4 = 6$$

$$m = 1.57$$

Therefore median = 1.57

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{3} \left(1 + \frac{x}{2}\right), & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

- a $E(X)$ and $E(3X+2)$,
- b $\text{Var}(X)$ and $\text{Var}(3X+2)$,
- c $P(X < 1)$,
- d $P(X > \mu)$,
- e $P(0.5 < X < 1.5)$.

Solution:

$$\begin{aligned}
 \text{a } E(X) &= \int_0^2 \frac{x}{3} \left(1 + \frac{x}{2}\right) dx \\
 &= \int_0^2 \frac{x}{3} + \frac{x^2}{6} dx \\
 &= \left[\frac{x^2}{6} + \frac{x^3}{18} \right]_0^2 \\
 &= \left[\frac{2^2}{6} + \frac{2^3}{18} \right] \\
 &= \frac{10}{9}
 \end{aligned}$$

$$\begin{aligned}
 E(3X+2) &= 3E(X) + 2 \\
 &= 3 \times \frac{10}{9} + 2 \\
 &= 5\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \text{Var}(X) &= \int_0^2 \frac{x^2}{3} \left(1 + \frac{x}{2}\right) dx - \left(\frac{10}{9}\right)^2 \\
 &= \int_0^2 \frac{x^2}{3} + \frac{x^3}{6} dx - \left(\frac{100}{81}\right) \\
 &= \left[\frac{x^3}{9} + \frac{x^4}{24} \right]_0^2 - \left(\frac{100}{81}\right) \\
 &= \left[\frac{2^3}{9} + \frac{2^4}{24} \right] - \left(\frac{100}{81}\right) \\
 &= 0.321
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(3X+2) &= 9\text{Var}(X) \\
 &= 2.89
 \end{aligned}$$

$$\begin{aligned}
 \text{c } P(X < 1) &= \int_0^1 \frac{1}{3} \left(1 + \frac{x}{2}\right) dx \\
 &= \int_0^1 \frac{1}{3} + \frac{x}{6} dx \\
 &= \left[\frac{x}{3} + \frac{x^2}{12} \right]_0^1 \\
 &= \left[\frac{1}{3} + \frac{1}{12} \right] \\
 &= \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{d } P(X > \mu) &= P\left(X > \frac{10}{9}\right) \\
 &= 1 - \int_0^{\frac{10}{9}} \frac{1}{3} \left(1 + \frac{x}{2}\right) dx \quad \text{or} \quad \int_{\frac{10}{9}}^2 \frac{1}{3} \left(1 + \frac{x}{2}\right) dx = \left[\frac{x}{3} + \frac{x^2}{12} \right]_{\frac{10}{9}}^2 \\
 &= 1 - \int_0^{\frac{10}{9}} \frac{1}{3} + \frac{x}{6} dx = \left[\frac{2}{3} + \frac{4}{12} \right] - \left[\frac{10}{27} + \frac{100}{972} \right] \\
 &= 1 - \left[\frac{x}{3} + \frac{x^2}{12} \right]_0^{\frac{10}{9}} = \frac{128}{243} \\
 &= 1 - \left[\frac{10}{27} + \frac{100}{972} \right] \\
 &= 1 - \frac{115}{243} \\
 &= \frac{128}{243}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } P(0.5 < X < 1.5) &= P(X < 1.5) - P(X < 0.5) \\
 &= \int_{0.5}^{1.5} \left(\frac{1}{3} + \frac{x}{6} \right) dx \\
 &= \left[\frac{x}{3} + \frac{x^2}{12} \right]_{0.5}^{1.5} \\
 &= \left[\frac{1.5}{3} + \frac{1.5^2}{12} \right] - \left[\frac{0.5}{3} + \frac{0.5^2}{12} \right] \\
 &= 0.5
 \end{aligned}$$

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Solutionbank S2

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Exercise E, Question 2

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 2 - 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- Evaluate $E(X)$.
- Evaluate $\text{Var}(X)$.
- Write down the values of $E(2X+1)$ and $\text{Var}(2X+1)$.
- Specify fully the cumulative distribution function of X .
- Work out the median value of X .

Solution:

$$\begin{aligned} \text{a } E(X) &= \int_0^1 2x - 2x^2 dx \\ &= \left[x^2 - \frac{2}{3}x^3 \right]_0^1 \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{b } \text{Var}(X) &= \int_0^1 2x^2 - 2x^3 dx - \left(\frac{1}{3}\right)^2 \\ &= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4 \right]_0^1 - \left(\frac{1}{9}\right) \\ &= \frac{1}{18} \end{aligned}$$

$$\begin{aligned} \text{c } E(2X + 1) &= 2E(X) + 1 \\ &= 2 \times \frac{1}{3} + 1 \\ &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{Var}(2X + 1) &= 4\text{Var}(X) \\ &= \frac{4}{18} = \frac{2}{9} \end{aligned}$$

d

Method 1	Method 2
$\int_0^x (2 - 2t) dt = \left[2x - x^2 \right]_0^x$ $= 2x - x^2$	$\int 2 - 2x dx = 2x - x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ 2x - x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$	

$$\begin{aligned} \text{e } 2x - x^2 &= 0.5 \\ x^2 - 2x + 0.5 &= 0 \end{aligned}$$

$$x = \frac{2 \pm \sqrt{4 - 2}}{2}$$

$$x = 1.71 \text{ or } 0.293$$

median = 0.293 as 1.71 is not in the range

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 3

Question:

The continuous random variable Y has cumulative distribution function given by

$$F(y) = \begin{cases} 0, & y < 1, \\ k(y^2 - y), & 1 \leq y \leq 2, \\ 1, & y > 2, \end{cases}$$

where k is a positive constant.

- Show that $k = \frac{1}{2}$.
- Find $P(Y < 1.5)$.
- Find the value of the median.
- Specify fully the probability density function $f(y)$.

Solution:

a $F(2) = 1$

$$F(y) = k(y^2 - y)$$

$$k(4 - 2) = 1$$

$$k = \frac{1}{2}$$

b $P(Y < 1.5) = F(1.5)$

$$= \frac{1}{2} \times (1.5^2 - 1.5)$$

$$= 0.375$$

c $\frac{1}{2}(y^2 - y) = 0.5$

$$y^2 - y - 1 = 0$$

$$y = \frac{1 \pm \sqrt{1+4}}{2}$$

$$y = 1.62 \text{ or } -0.618$$

median = 1.62 as -0.618 is not in the range

d $\frac{d}{dx} \left[\frac{1}{2}(y^2 - y) \right] = y - \frac{1}{2}$

$$f(x) = \begin{cases} y - \frac{1}{2} & 1 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

Question:

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \leq x \leq 3, \\ 1, & x > 3. \end{cases}$$

- Find $P(X > 2.4)$.
- Find the median.
- Find the probability density function, $f(x)$.
- Evaluate $E(X)$.
- Find the mode of X .

Solution:

$$\begin{aligned} \text{a } P(X > 2.4) &= F(3) - F(2.4) \\ &= \frac{1}{5}(3^2 - 4) - \frac{1}{5}(2.4^2 - 4) \\ &= 0.648 \end{aligned}$$

or

$$\begin{aligned} P(X > 2.4) &= 1 - F(2.4) \\ &= 1 - \frac{1}{5}(2.4^2 - 4) \\ &= 0.648 \end{aligned}$$

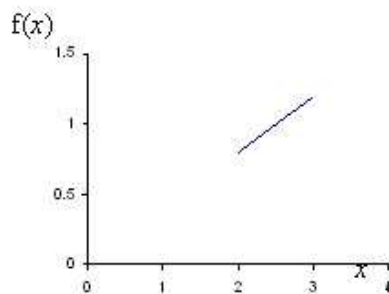
$$\begin{aligned} \text{b } \frac{1}{5}(x^2 - 4) &= 0.5 \\ 2x^2 - 8 &= 5 \\ 2x^2 &= 13 \\ x^2 &= 6.5 \\ x &= 2.55 \text{ or } -2.55 \end{aligned}$$

median = 2.55 as -2.55 is not in the range

$$\begin{aligned} \text{c } \frac{d}{dx} \left[\frac{1}{5}(x^2 - 4) \right] &= \frac{2x}{5} \\ f(x) &= \begin{cases} \frac{2x}{5} & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{aligned} \text{d } E(X) &= \int_2^3 \frac{2x^2}{5} dx \\ &= \left[\frac{2x^3}{15} \right]_2^3 \\ &= \frac{54}{15} - \frac{16}{15} \\ &= \frac{38}{15} \end{aligned}$$

e



mode = 3

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 5

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

- Show that $k = \frac{3}{8}$.
- Calculate $E(X)$.
- Specify fully the cumulative distribution function of X .
- Find the value of the median.
- Find the value of the mode.

Solution:

$$\text{a } \int_0^2 kx^2 dx = 1$$

$$\left[\frac{kx^3}{3} \right]_0^2 = 1$$

$$\frac{8k}{3} = 1$$

$$k = \frac{3}{8}$$

$$\text{b } E(X) = \int_0^2 \frac{3x^3}{8} dx$$

$$= \left[\frac{3x^4}{32} \right]_0^2$$

$$= 1.5$$

c Method 1

$$F(x) = \int_0^x \frac{3t^2}{8} dt$$

$$= \left[\frac{t^3}{8} \right]_0^x$$

$$= \frac{x^3}{8}$$

Method 2

$$F(x) = \int \frac{3x^2}{8} dx$$

$$= \frac{x^3}{8} + C$$

$$F(2) = 1 \text{ therefore } \frac{8}{8} + C = 1$$

$$C = 0$$

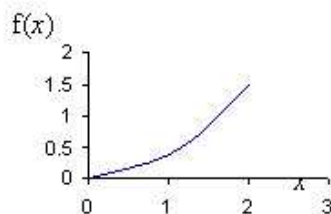
$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

$$\text{d } \frac{m^3}{8} = 0.5$$

$$m^3 = 4$$

$$m = 1.59$$

e



$$\text{mode} = 2$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 6

Question:

The random variable Y has probability density function $f(y)$ given by

$$f(x) = \begin{cases} k(y^2 + 2y + 2), & 1 \leq y \leq 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

- Show that $k = \frac{3}{62}$.
- Specify fully the cumulative distribution function of Y .
- Evaluate $P(Y \leq 2)$.

Solution:

$$\begin{aligned}
 \text{a} \quad \int_1^3 k(y^2 + 2y + 2) dy &= 1 \\
 \left[k \left(\frac{y^3}{3} + y^2 + 2y \right) \right]_1^3 &= 1 \\
 k \left(\frac{3^3}{3} + 3^2 + 6 \right) - k \left(\frac{1}{3} + 1 + 2 \right) &= 1 \\
 \frac{62}{3} k &= 1 \\
 k &= \frac{3}{62}
 \end{aligned}$$

b Method 1

$$\begin{aligned}
 F(y) &= \int_1^y \frac{3}{62} (t^2 + 2t + 2) dt \\
 &= \left[\frac{3}{62} \left(\frac{t^3}{3} + t^2 + 2t \right) \right]_1^y \\
 &= \frac{3}{62} \left(\frac{y^3}{3} + y^2 + 2y \right) - \frac{3}{62} \left(\frac{1}{3} + 1 + 2 \right) \\
 &= \frac{y^3}{62} + \frac{3y^2}{62} + \frac{3y}{31} - \frac{5}{31}
 \end{aligned}$$

Method 2

$$\begin{aligned}
 F(x) &= \int \frac{3}{62} (y^2 + 2y + 2) dx \\
 &= \frac{3}{62} \left(\frac{y^3}{3} + y^2 + 2y \right) + C \\
 F(3) &= 1 \quad \text{therefore} \\
 \frac{3}{62} \left(\frac{3^3}{3} + 3^2 + 6 \right) + C &= 1 \\
 C &= -\frac{5}{31}
 \end{aligned}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{y^3}{62} + \frac{3y^2}{62} + \frac{3y}{31} - \frac{5}{31} & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

$$\begin{aligned}
 \text{c} \quad P(Y < 2) &= \int_1^2 \frac{3}{62} (y^2 + 2y + 2) dy && \text{or } F(2) = \\
 \frac{2^3}{62} + \frac{3 \times 2^2}{62} + \frac{6}{31} - \frac{5}{31} &&& \\
 &= \left[\frac{3}{62} \left(\frac{y^3}{3} + y^2 + 2y \right) \right]_1^2 && = \frac{11}{31} \\
 &= \frac{3}{62} \left(\frac{2^3}{3} + 2^2 + 4 \right) - \frac{3}{62} \left(\frac{1}{3} + 1 + 2 \right) \\
 &= \frac{11}{31}
 \end{aligned}$$

Solutionbank S2

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Exercise E, Question 7

Question:

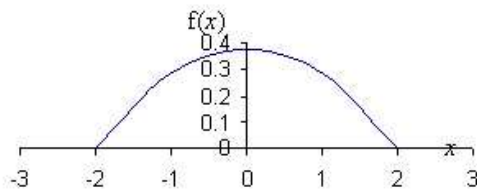
A random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{3}{32}(4-x^2), & -2 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the probability density function of X .
- Write down the mode of X .
- Specify fully the cumulative distribution function of X .
- Find $P(0.5 < X < 1.5)$.

Solution:

a



b mode = 0

c Method 1

$$\begin{aligned} F(x) &= \int_{-2}^x \frac{3}{32}(4-t^2) dt \\ &= \left[\frac{12t}{32} - \frac{t^3}{32} \right]_{-2}^x \\ &= \left(\frac{12x}{32} - \frac{x^3}{32} \right) - \left(-\frac{24}{32} + \frac{8}{32} \right) \end{aligned}$$

=1

$$= \frac{12x}{32} - \frac{x^3}{32} + \frac{1}{2}$$

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{12x}{32} - \frac{x^3}{32} + \frac{1}{2} & -2 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

Method 2

$$\begin{aligned} F(x) &= \int \frac{3}{32}(4-x^2) dx \\ &= \frac{12x}{32} - \frac{x^3}{32} + C \\ F(2) &= 1 \quad \text{therefore} \quad \frac{24}{32} - \frac{8}{32} + C \end{aligned}$$

$$C = 0.5$$

d $P(0.5 < X < 1.5) = F(1.5) - F(0.5)$

$$\begin{aligned} &= \left(\frac{18}{32} - \frac{1.5^3}{32} + \frac{1}{2} \right) - \left(\frac{6}{32} - \frac{0.5^3}{32} + \frac{1}{2} \right) \\ &= \frac{35}{128} \text{ or } 0.273 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 8

Question:

A random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{3}, & 0 \leq x < 1, \\ \frac{2}{7}x^2, & 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Find $E(X)$.
- Specify fully the cumulative distribution function of X .
- Find the median of X .

Solution:

$$\begin{aligned}
 \text{a } E(X) &= \int_0^1 \frac{x}{3} dx + \int_1^2 \frac{2x^3}{7} dx \\
 &= \left[\frac{x^2}{6} \right]_0^1 + \left[\frac{2x^4}{28} \right]_1^2 \\
 &= \frac{1}{6} + \left(\frac{32}{28} - \frac{2}{28} \right) \\
 &= \frac{26}{21}
 \end{aligned}$$

b Method 1

$$\begin{aligned}
 F(x) &= \int_0^x \frac{1}{3} dt \\
 &= \left[\frac{t}{3} \right]_0^x \\
 &= \frac{x}{3}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^1 \frac{1}{3} dx + \int_1^x \frac{2t^2}{7} dt &= \left[\frac{x}{3} \right]_0^1 + \left[\frac{2t^3}{21} \right]_1^x \\
 &= \frac{1}{3} + \frac{2x^3}{21} - \frac{2}{21}
 \end{aligned}$$

$$1 = \frac{16}{21} + C$$

$$= \frac{2x^3}{21} + \frac{5}{21}$$

Method 2

$$\begin{aligned}
 F(x) &= \int \frac{1}{3} dx \\
 &= \frac{x}{3} + C
 \end{aligned}$$

$$F(0) = 0 \text{ therefore } C = 0$$

$$\int \frac{2x^2}{7} dx = \frac{2x^3}{21} + C$$

$$F(2) = 1 \text{ therefore}$$

$$C = \frac{5}{21}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{3} & 0 \leq x < 1 \\ \frac{2x^3}{21} + \frac{5}{21} & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

c $F(1) = \frac{1}{3}$ therefore median lies in interval $1 \leq x \leq 2$

$$\frac{2x^3}{21} + \frac{5}{21} = 0.5$$

$$2x^3 + 5 = 10.5$$

$$2x^3 = 5.5$$

$$x^3 = 2.75$$

$$x = 1.40$$

$$\text{median} = 1.40$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 9

Question:

A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx - k, & 1 \leq x \leq 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

- Show that $k = \frac{1}{2}$.
- Find $E(X)$.
- Work out the cumulative distribution function, $F(x)$.
- Show that the median value lies between 2.4 and 2.5.

Solution:

$$\begin{aligned}
 \mathbf{a} \quad \int_1^3 kx - k dx &= 1 \\
 \left[\frac{kx^2}{2} - kx \right]_1^3 &= 1 \\
 \left(\frac{9k}{2} - 3k \right) - \left(\frac{k}{2} - k \right) &= 1 \\
 2k &= 1 \\
 k &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad E(X) &= \int_1^3 \frac{x^2}{2} - \frac{x}{2} dx \\
 &= \left[\frac{x^3}{6} - \frac{x^2}{4} \right]_1^3 \\
 &= \left(\frac{9}{4} \right) - \left(-\frac{1}{12} \right) \\
 &= \frac{7}{3}
 \end{aligned}$$

c Method 1

$$\begin{aligned}
 F(x) &= \int_1^x \frac{t}{2} - \frac{1}{2} dt \\
 &= \left[\frac{t^2}{4} - \frac{t}{2} \right]_1^x \\
 &= \left(\frac{x^2}{4} - \frac{x}{2} \right) - \left(\frac{1}{4} - \frac{1}{2} \right) \\
 &= \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4}
 \end{aligned}$$

Method 2

$$\begin{aligned}
 F(x) &= \int \frac{x}{2} - \frac{1}{2} dx \\
 &= \frac{x^2}{4} - \frac{x}{2} + C \\
 F(3) &= 1 \text{ therefore } \frac{9}{4} - \frac{3}{2} + C = 1 \\
 C &= 0.25
 \end{aligned}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4} & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

$$\begin{aligned}
 \mathbf{d} \quad F(2.4) &= \frac{2.4^2}{4} - \frac{2.4}{2} + \frac{1}{4} = 0.5 \\
 F(2.5) &= \frac{2.5^2}{4} - \frac{2.5}{2} + \frac{1}{4} = 0.5
 \end{aligned}$$

Since 0.5 lies in between, the median is between 2.4 and 2.5.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 10

Question:

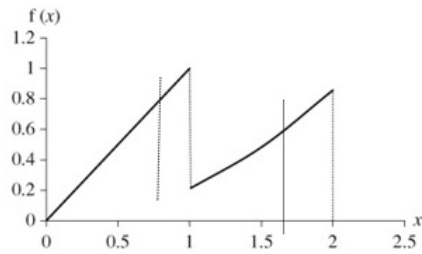
The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} x, & 0 \leq x < 1, \\ \frac{3x^2}{14}, & 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- Sketch the probability density function of X .
- Find the mode of X .
- Find $E(2X)$.
- Find $\text{Var}(2X+1)$.
- Specify fully the cumulative distribution function of X .
- Using your answer to part e find the median of X .

Solution:

a



b mode = 1

$$\begin{aligned} \text{c } E(X) &= \int_0^1 x^2 dx + \int_1^2 \frac{3x^3}{14} dx \\ &= \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{3x^4}{56} \right]_1^2 \\ &= \frac{1}{3} + \left(\frac{48}{56} - \frac{3}{56} \right) \\ &= \frac{191}{168} \end{aligned}$$

$$\begin{aligned} E(2X) &= 2 \times \frac{191}{168} \\ &= \frac{191}{84} \end{aligned}$$

$$\begin{aligned} \text{d } \text{Var}(X) &= \int_0^1 x^3 dx + \int_1^2 \frac{3x^4}{14} dx - \left(\frac{191}{168} \right)^2 \\ &= \left[\frac{x^4}{4} \right]_0^1 + \left[\frac{3x^5}{70} \right]_1^2 - \left(\frac{191}{168} \right)^2 \\ &= \frac{1}{4} + \left(\frac{48}{35} - \frac{3}{70} \right) - \left(\frac{191}{168} \right)^2 \\ &= 0.28601 \\ \text{Var}(2X+1) &= 4 \times 0.286 \\ &= 1.14 \end{aligned}$$

e Method 1

$$\begin{aligned} F(x) &= \int_0^x t dt \\ &= \left[\frac{t^2}{2} \right]_0^x \\ &= \frac{x^2}{2} \end{aligned}$$

$$\begin{aligned} \int_0^1 t dt + \int_1^x \frac{3t^2}{14} dt &= \left[\frac{t^2}{2} \right]_0^1 + \left[\frac{t^3}{14} \right]_1^x \\ &= \frac{1}{2} + \frac{x^3}{14} - \frac{1}{14} \end{aligned}$$

$$1 = \frac{8}{14} + C$$

$$= \frac{x^3}{14} + \frac{3}{7}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \leq x < 1 \\ \frac{x^3}{14} + \frac{3}{7} & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

Method 2

$$\begin{aligned} F(x) &= \int x dx \\ &= \frac{x^2}{2} + C \end{aligned}$$

$$F(0) = 0 \text{ therefore } C = 0$$

$$\int \frac{3x^2}{14} dx = \frac{x^3}{14} + C$$

$$F(2) = 1 \text{ therefore}$$

$$C = \frac{3}{7}$$

$$\text{f } \frac{x^2}{2} = 0.5 \text{ median} = 1$$

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