

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The discrete random variable $X \sim \text{Po}(2.3)$. Find

- a $P(X = 4)$,
- b $P(X \geq 1)$,
- c $P(4 < X < 6)$.

Solution:

$$X \sim \text{Po}(2.3)$$

$$\text{a } P(X = 4) = \frac{e^{-2.3}(2.3)^4}{4!} = 0.11690\dots = 0.117 \text{ (3 s.f.)}$$

$$\begin{aligned} \text{b } P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - e^{-2.3} \\ &= 0.89974\dots = 0.900 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{c } P(4 < X < 6) \\ &= P(X = 5) \\ &= 0.05377 = 0.0538 \text{ (3 s.f.)} \end{aligned}$$

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Exercise A, Question 2

Question:

The discrete random variable $X \sim \text{Po}(5.7)$. Find

- a $P(X=7)$,
- b $P(X \leq 1)$,
- c $P(X > 2)$.

Solution:

$$X \sim \text{Po}(5.7)$$

$$\begin{aligned} \text{a } P(X=7) &= \frac{e^{-5.7}(5.7)^7}{7!} \\ &= 0.12978\dots = 0.130 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} P(X \leq 1) &= P(X=0) + P(X=1) \\ \text{b } &= e^{-5.7} + e^{-5.7} \times \frac{5.7}{1!} \\ &= 6.7 \times e^{-5.7} \\ &= 0.022417\dots = 0.0224 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - [P(X=0) + P(X=1) + P(X=2)] \\ \text{c } &= 1 - 0.022417 - \frac{e^{-5.7} \times 5.7^2}{2!} \\ &= 0.923226\dots = 0.923 \text{ (3 s.f.)} \end{aligned}$$

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Exercise A, Question 3

Question:

The random variable $Y \sim \text{Po}(0.35)$. Find

- a $P(Y=1)$,
- b $P(Y \geq 1)$,
- c $P(1 \leq Y < 3)$.

Solution:

$$Y \sim \text{Po}(0.35)$$

$$\begin{aligned} \text{a } P(Y=1) &= \frac{e^{-0.35} \times 0.35}{1!} \\ &= 0.24664\dots = 0.247 \end{aligned}$$

$$\begin{aligned} \text{b } P(Y \geq 1) &= 1 - P(Y=0) \\ &= 0.29531\dots = 0.295 \end{aligned}$$

$$\begin{aligned} \text{c } P(1 \leq Y < 3) &= P(Y=1) + P(Y=2) \\ &= 0.24664\dots + \frac{e^{-0.35} \times (0.35)^2}{2!} \\ &= 0.289802\dots = 0.290 \end{aligned}$$

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Exercise A, Question 4

Question:

The random variable $X \sim \text{Po}(3.6)$. Find

- a $P(X = 5)$,
- b $P(3 < X \leq 6)$,
- c $P(X < 2)$.

Solution:

$$X \sim \text{P}_0(3.6)$$

a

$$\begin{aligned} P(X = 5) &= \frac{e^{-3.6}(3.6)^5}{5!} \\ &= 0.13768\dots = 0.138 \end{aligned}$$

b

$$\begin{aligned} P(3 < X \leq 6) &= P(X = 4) + P(X = 5) + P(X = 6) \\ &= \frac{e^{-3.6}(3.6)^4}{4!} + \frac{e^{-3.6}(3.6)^5}{5!} + \frac{e^{-3.6}(3.6)^6}{6!} \\ &= 0.41151\dots = 0.412 \end{aligned}$$

c

$$\begin{aligned} P(X < 2) &= P(X = 0) + P(X = 1) \\ &= e^{-3.6} + e^{-3.6} \times 3.6 \\ &= 4.6 \times e^{-3.6} \\ &= 0.12568\dots = 0.126 \end{aligned}$$

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Exercise B, Question 1

Question:

The random variable $X \sim \text{Po}(2.5)$. Find

- a $P(X=1)$,
- b $P(X > 2)$,
- c $P(X \leq 5)$,
- d $P(3 \leq X \leq 5)$.

Solution:

$$X \sim \text{Po}(2.5)$$

$$\text{a } P(X=1) = e^{-2.5} \times 2.5 = 0.2052 \quad (4 \text{ d.p.})$$

$$\begin{aligned} \text{b } P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - 0.5438 \\ &= 0.4562 \end{aligned} \quad (\text{tables})$$

$$\text{c } P(X \leq 5) = 0.9580 \quad (\text{tables})$$

$$\begin{aligned} \text{d } P(3 \leq X \leq 5) &= P(X \leq 5) - P(X \leq 2) \\ &= 0.9580 - 0.5438 \\ &= 0.4142 \end{aligned} \quad (\text{tables})$$

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Exercise B, Question 2

Question:

The random variable $X \sim \text{Po}(6)$. Find

- a $P(X \leq 3)$,
- b $P(X > 4)$,
- c $P(X = 5)$,
- d $P(2 < X \leq 7)$.

Solution:

$$X \sim \text{Po}(6)$$

$$\text{a } P(X \leq 3) = 0.1512 \quad (\text{tables})$$

$$\begin{aligned} \text{b } P(X > 4) &= 1 - P(X \leq 4) \\ &= 1 - 0.2851 \\ &= 0.7149 \end{aligned} \quad (\text{tables})$$

$$\begin{aligned} \text{c } P(X = 5) &= P(X \leq 5) - P(X \leq 4) \\ &= 0.4457 - 0.2851 \\ &= 0.1606 \end{aligned} \quad (\text{tables})$$

$$\begin{aligned} \text{d } P(2 < X \leq 7) &= P(X \leq 7) - P(X \leq 2) \\ &= 0.7440 - 0.0620 \\ &= 0.6820 \end{aligned} \quad (\text{tables})$$

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Exercise B, Question 3

Question:

The random variable Y has a Poisson distribution with mean 4.5. Find

- a $P(Y = 2)$,
- b $P(Y \leq 1)$,
- c $P(Y > 4)$,
- d $P(2 \leq Y \leq 6)$.

Solution:

$$Y \sim P_0(4.5)$$

$$\begin{aligned} \text{a } P(Y = 2) &= P(Y \leq 2) - P(Y \leq 1) \\ &= 0.1736 - 0.0611 && \text{(tables)} \\ &= 0.1125 \end{aligned}$$

$$\text{b } P(Y \leq 1) = 0.0611 \quad \text{(tables)}$$

$$\begin{aligned} \text{c } P(Y > 4) &= 1 - P(Y \leq 4) \\ &= 1 - 0.5321 && \text{(tables)} \\ &= 0.4679 \end{aligned}$$

$$\begin{aligned} \text{d } P(2 \leq Y \leq 6) &= P(Y \leq 6) - P(Y \leq 1) \\ &= 0.8311 - 0.0611 && \text{(tables)} \\ &= 0.7700 \end{aligned}$$

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Exercise B, Question 4

Question:

The random variable $X \sim \text{Po}(8)$. Find the values of a, b, c and d such that

- a $P(X \leq a) = 0.3134$,
- b $P(X \leq b) = 0.7166$,
- c $P(X < c) = 0.0996$,
- d $P(X > d) = 0.8088$.

Solution:

$$X \sim \text{Po}(8)$$

$$\begin{aligned} \text{a } P(X \leq 6) &= 0.3134 && \text{(tables)} \\ \therefore a &= 6 \end{aligned}$$

$$\begin{aligned} \text{b } P(X \leq 9) &= 0.7166 && \text{(tables)} \\ \therefore b &= 9 \end{aligned}$$

$$\begin{aligned} \text{c } P(X \leq 4) &= 0.0996 \\ P(X < 5) &= 0.0996 && \text{(tables)} \\ \therefore c &= 5 \end{aligned}$$

$$\begin{aligned} \text{d } P(X \leq 5) &= 0.1912 \\ \therefore P(X > 5) &= 1 - 0.1912 && \text{(tables)} \\ &= 0.8088 \\ \therefore d &= 5 \end{aligned}$$

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Exercise B, Question 5

Question:

The random variable $X \sim \text{Po}(3.5)$. Find the values of a, b, c and d such that

- a $P(X \leq a) = 0.8576$,
- b $P(X > b) = 0.6792$,
- c $P(X \leq c) \geq 0.95$,
- d $P(X > d) \leq 0.005$.

Solution:

$$X \sim \text{Po}(3.5)$$

$$\begin{aligned} \text{a } P(X \leq 5) &= 0.8576 && \text{(tables)} \\ \therefore a &= 5 \end{aligned}$$

$$\begin{aligned} \text{b } 1 - 0.6792 &= 0.3208 \\ P(X \leq 2) &= 0.3208 && \text{(tables)} \\ P(X > 2) &= 0.6792 \\ \therefore b &= 2 \end{aligned}$$

$$\begin{aligned} \text{c } P(X \leq 6) &= 0.9347 \\ P(X \leq 7) &= 0.9733 && \text{(tables)} \\ \therefore c &\geq 7 \end{aligned}$$

$$\begin{aligned} \text{d } P(X \leq 8) &= 0.9901 \Rightarrow P(X > 8) = 0.0099 > 0.005 \\ P(X \leq 9) &= 0.9967 \Rightarrow P(X > 9) = 0.0033 < 0.005 \text{ (tables)} \\ \therefore d &\geq 9 \end{aligned}$$

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Exercise B, Question 6

Question:

The number of telephone calls received at an exchange during a weekday morning follows a Poisson distribution with a mean of 6 calls per 5-minute period. Find the probability that

- a there are no calls in the next 5 minutes,
- b 3 or fewer calls are received in the next 5 minutes,
- c fewer than 2 calls are received between 11:00 and 11:05,
- d no more than 2 calls are received between 11:30 and 11:35.

Solution:

Let X = number of telephone calls in a week day morning in 5 minutes

$$X \sim P_0(6)$$

- a $P(X = 0) = e^{-6} = 0.002478$
 $= 0.00248$ (3 s.f.)
or 0.0025 (tables)
- b $P(X \leq 3) = 0.1512$ (tables)
- c $P(X < 2) = P(X \leq 1)$ (tables)
 $= 0.0174$
- d $P(X \leq 2) = 0.0620$ (tables)

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Exercise B, Question 7

Question:

The random variable $X \sim \text{Po}(9)$. Find

- a $\mu = E(X)$,
- b $\sigma =$ standard deviation of X ,
- c $P(\mu \leq X < \mu + \sigma)$,
- d $P(X \leq \mu - \sigma)$.

Solution:

$$X \sim \text{P}_0(9)$$

a $\mu = E(X) = 9$

b $\sigma^2 = \text{Var}(X) = 9 \quad \sigma = 3$

c $P(9 \leq X < 12) = P(X \leq 11) - P(X \leq 8)$
 $= 0.8030 - 0.4557$
 $= 0.3473$

d $P(X \leq 9 - 3) = P(X \leq 6)$ (tables)
 $= 0.2068$

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Exercise B, Question 8

Question:

The mean number of faults in 2 m^2 of cloth produced by a factory is 1.5.

- a Find the probability of a 2 m^2 piece of cloth containing no faults.
- b Find the probability that a 2 m^2 piece of cloth contains no more than 2 faults.

Solution:

Let X = number of faults in 2 m^2 of cloth

$$X \sim P_0(1.5)$$

a $P(X = 0) = 0.2231$ (tables)

b $P(X \leq 2) = 0.8088$ (tables)

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Exercise C, Question 1

Question:

A technician is responsible for a large number of machines. Minor adjustments have to be made to these machines and these occur at random and at a constant rate of 7 per hour. Find the probability that

- a in a particular hour the technician makes 4 or fewer adjustments,
- b during a half-hour break no adjustments will be required.

Solution:

Let X = number of adjustments in an hour

$$X \sim P_0(7)$$

a $P(X \leq 4) = 0.1730$ (tables)

b Y = number of adjustments in a half-hour

$$Y \sim P_0(3.5)$$

$$P(Y = 0) = 0.0302$$
 (tables)

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Exercise C, Question 2

Question:

A textile firm produces rolls of cloth but slight defects sometimes occur. The average number of defects per square metre is 2.5. Use a Poisson distribution to calculate the probability that

- a 1.5 m^2 portion of cloth bought to make a skirt contains no defects,
- a 4 m^2 portion of cloth contains fewer than 5 defects.
- State briefly what assumptions have to be made before a Poisson distribution can be accepted as a suitable model in this situation.

Solution:

$$\begin{aligned} \text{a Let } X &= \text{number of defects in } 1.5 \text{ m}^2 \\ \lambda &= 1.5 \times 2.5 = 3.75 \\ \therefore X &\sim P_0(3.75) \\ P(X=0) &= e^{-3.75} = 0.0235177\dots \\ &= 0.0235 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{b Let } Y &= \text{number of defects in } 4 \text{ m}^2 \\ \lambda &= 4 \times 2.5 = 10 \\ \therefore Y &\sim P_0(10) && \text{(tables)} \\ P(Y < 5) &= P(Y \leq 4) = 0.0293 \end{aligned}$$

- Assume that defects occur independently and at random in the cloth and defects occur at a constant rate.

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Exercise C, Question 3

Question:

State which of the following could be modelled by a Poisson distribution and which can not. Give reasons for your answers.

- a The number of misprints on this page in the first draft of this book.
- b The number of pigs in a particular 5 m square of their field 1 hour after their food was placed in a central trough.
- c The number of pigs in a particular 5 m square of their field 1 minute after their food was placed in a central trough.
- d The amount of salt, in mg, contained in 1 cm^3 of water taken from a bucket immediately after a teaspoon of salt was added to the bucket.
- e The number of marathon runners passing the finishing post between 20 and 21 minutes after the winner of the race.

Solution:

- a If the misprints occur independently and at random and at a constant average rate then this could be Poisson.
- b Yes because after 1 hour the pigs are probably dispersed fairly randomly and independently around the field.
- c No because after 1 minute the pigs will probably be clustered around the feeding trough and so will not be randomly and independently scattered.
- d No because the salt needs to diffuse so that it is randomly dissolved at a constant rate throughout the contents of the bucket.
- e Yes, this may be Poisson provided that the runners are not in groups, since they need to pass the post independently and at random.

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Exercise C, Question 4

Question:

The number of accidents per week at a certain road intersection has a Poisson distribution with parameter 2.5. Find the probability that

- a exactly 5 accidents will occur in a particular week,
- b more than 14 accidents will occur in 4 weeks. *E*

Solution:

- a X = number of accidents in a week

$$X \sim P_0(2.5)$$

$$\begin{aligned} P(X=5) &= P(X \leq 5) - P(X \leq 4) \\ &= 0.9580 - 0.8912 && \text{(tables)} \\ &= 0.0668 \end{aligned}$$

- b Y = number of accidents in a 4-week period

$$Y \sim P_0(4 \times 2.5 = 10)$$

$$\begin{aligned} P(Y > 14) &= 1 - P(Y \leq 14) \\ &= 1 - 0.9165 && \text{(tables)} \\ &= 0.0835 \end{aligned}$$

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Exercise C, Question 5

Question:

In a particular district it has been found, over a long period, that the number, X , of cases of measles reported per month has a Poisson distribution with parameter 1.5. Find the probability that in this district

- a in any given month, exactly 2 cases of measles will be reported,
- b in a period of 6 months, fewer than 10 cases of measles will be reported.

Solution:

$$X \sim P_0(1.5)$$

$$\begin{aligned} \text{a } P(X = 2) &= P(X \leq 2) - P(X \leq 1) \\ &= 0.8088 - 0.5578 \\ &= 0.2510 \end{aligned}$$

- b Y = number of reported cases of measles in 6 months

$$Y \sim P_0(9)$$

$$\begin{aligned} P(Y < 10) &= P(Y \leq 9) \\ &= 0.5874 \end{aligned}$$

(tables)

$9 = 6 \times 1.5$

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Exercise C, Question 6

Question:

A biologist is studying the behaviour of sheep in a large field. The field is divided into a number of equally sized squares and the average number of sheep per square is 2.5. The sheep are randomly scattered throughout the field.

- Suggest a suitable model for the number of sheep in a square and give a value for any parameter or parameters required.
- Calculate the probability that a randomly selected square contains more than 3 sheep.

A sheep dog has been sent into the field to round up the sheep.

- Explain why the model may no longer be applicable.

Solution:

- X = number of sheep per square
 $X \sim P_0(2.5)$

- $$\begin{aligned} P(X > 3) &= 1 - P(X \leq 3) \\ &= 1 - 0.7576 && \text{(tables)} \\ &= 0.2424 \end{aligned}$$

- The sheep will no longer be randomly scattered.

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Exercise C, Question 7

Question:

During office hours, telephone calls to a single telephone in an office come in at an average rate of 18 calls per hour. Assuming that a Poisson distribution can be applied, find the probability that in a 5-minute period there will be

- fewer than 2 calls,
- more than 3 calls.
- Find the probability of no calls during a 20-minute coffee break.

Solution:

Let X = number of calls in a 5-minute period

$$\lambda = \frac{18}{12} = 1.5$$

$$\therefore X \sim P_0(1.5)$$

$$\begin{aligned} \text{a } P(X < 2) &= P(X \leq 1) \\ &= 0.5578 \end{aligned} \quad \text{(tables)}$$

$$\begin{aligned} \text{b } P(X > 3) &= 1 - P(X \leq 3) \\ &= 1 - 0.9344 \\ &= 0.0656 \end{aligned} \quad \text{(tables)}$$

c Y = number of calls in 20-minute period

$$\lambda = 4 \times 1.5 = 6$$

$$\therefore Y \sim P_0(6)$$

$$P(Y = 0) = 0.0025 \quad \text{(tables)}$$

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Exercise C, Question 8

Question:

A shop sells large birthday cakes at a rate of 2 every 3 days.

- a** Find the probability of selling no large birthday cakes on a randomly selected day. Fresh cakes are baked every 3 days and any cakes older than 3 days can not be sold.
- b** Find how many large birthday cakes should be baked so that the probability of running out of large birthday cakes to sell is less than 1%.

Solution:

X = number of large cakes sold in a day

$$X \sim P_0\left(\frac{2}{3}\right)$$

a

$$\begin{aligned} P(X=0) &= e^{-\frac{2}{3}} = 0.51341 \\ &= 0.513(3 \text{ s.f.}) \end{aligned}$$

b Y = number of large cakes sold in 3 days

$$Y \sim P_0(2)$$

Let n = number of cakes baked

To run out of cakes you require $Y > n$

Require $P(Y > n) < 0.01$

i.e. $P(Y \leq n) > 0.99$

$$P(Y \leq 5) = 0.9834 < 0.99 \quad (\text{tables})$$

$$P(Y \leq 6) = 0.9955 > 0.99$$

\therefore need $n = 6$

($n \geq 6$ but baking more is likely to create more waste)

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Exercise C, Question 9

Question:

On a typical summer's day a boat company hires rowing boats at a rate of 9 per hour.

- a Find the probability of hiring out at least 6 boats in a randomly selected 30-minute period.

The company has 8 boats to hire and decides to hire them out for 20-minute periods.

- b Show that the probability of running out of boats is less than 1%.
 c Find how many boats the company should have to be 99% sure of meeting all demands if the hire period is extended to 30 minutes.

Solution:

- a Let X = number of boats hired in a 30-minute period

$$\lambda = \frac{1}{2} \times 9 = 4.5$$

$$\therefore X \sim P_0(4.5) \quad (\text{tables})$$

$$\begin{aligned} P(X \geq 6) &= 1 - P(X \leq 5) \\ &= 1 - 0.7029 \\ &= 0.2971 \end{aligned}$$

- b Let Y = number of requests for hire in 20-minute period

$$\lambda = \frac{1}{3} \times 9 = 3$$

$$Y \sim P_0(3)$$

$$\begin{aligned} P(Y > 8) &= 1 - P(Y \leq 8) \\ &= 1 - 0.9962 \quad (\text{tables}) \\ &= 0.0038 < 0.01 \end{aligned}$$

- c Let n = number of boats
 Require $P(X > n) < 0.01$
 or $P(X \leq n) > 0.99$

Use $P_0(4.5)$.

$$P(X \leq 9) = 0.9829 < 0.99 \quad (\text{tables})$$

$$P(X \leq 10) = 0.9933 > 0.99$$

\therefore they would need 10 boats

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Exercise C, Question 10

Question:

Breakdowns on a particular machine occur at random at a rate of 1.5 per week.

- Find the probability that no more than 2 breakdowns occur in a randomly chosen week.
- Find the probability of at least 5 breakdowns in a randomly chosen two-week period.

A maintenance firm offers a contract for repairing breakdowns over a six-week period. The firm will give a full refund if there are more than n breakdowns in a six-week period. The firm want the probability of having to pay a refund to be 5% or less.

- find the smallest value of n .

Solution:

Let X = number of breakdowns in a week

$$X \sim P_0(1.5)$$

$$\text{a } P(X \leq 2) = 0.8088 \quad (\text{tables})$$

b Let Y = no of breakdowns in a 2-week period

$$Y \sim P_0(3)$$

$$\begin{aligned} P(Y \geq 5) &= 1 - P(Y \leq 4) \\ &= 1 - 0.8153 \\ &= 0.1847 \end{aligned} \quad (\text{tables})$$

c Let B = number of breakdowns in a 6-week period

$$B \sim P_0(9)$$

Firm requires $P(B > n) \leq 0.05$

i.e. $P(B \leq n) \geq 0.95$

$$P(B \leq 13) = 0.9261 < 0.95 \quad (\text{tables})$$

$$P(B \leq 14) = 0.9585 > 0.95$$

$$\therefore n = 14$$

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Exercise D, Question 1

Question:

The random variable $X \sim B(80, 0.10)$. Using a suitable approximation, find

a $P(X \geq 1)$,

b $P(X \leq 6)$.

Solution:

$$X \sim B(80, 0.10)$$

$$X \approx P_0(8)$$

a

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &\approx 1 - 0.0003 && \text{(Poisson tables)} \\ &= 0.9997 \end{aligned}$$

b $P(X \leq 6) \approx 0.3134$ (Poisson tables)

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Exercise D, Question 2

Question:

The random variable $X \sim B(120, 0.02)$. Using a suitable approximation, find

- a $P(X = 1)$,
- b $P(X \geq 3)$.

Solution:

$$X \sim B(120, 0.02)$$

$$X \approx \sim P_0(2.4)$$

$$\begin{aligned} \text{a } P(X = 1) &= e^{-2.4}(2.4)^1 \\ &= 0.21772\dots = 0.218 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{b } P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - e^{-2.4} \left[1 + 2.4 + \frac{2.4^2}{2!} \right] \\ &= 1 - 0.56970\dots \\ &= 0.430 \text{ (3 s.f.)} \end{aligned}$$

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Exercise D, Question 3

Question:

The random variable $X \sim B(50, 0.05)$. Find the percentage error in $P(X \leq 4)$ when X is approximated by a Poisson distribution.

Solution:

$$X \sim B(50, 0.05)$$

$$P(X \leq 4) = 0.8964 \quad (\text{Binomial tables})$$

$$X \approx \sim P_0(2.5)$$

$$P(X \leq 4) \approx 0.8912 \quad (\text{Poisson tables})$$

$$\begin{aligned} \text{Percentage error} &= \frac{(0.8964 - 0.8912)}{0.8964} \times 100 \\ &= 0.58\% \end{aligned}$$

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Exercise D, Question 4

Question:

In a certain manufacturing process the proportion of defective articles produced is 2%. In a batch of 300 articles, use a suitable approximation to find the probability that

- a there are fewer than 2 defectives,
- b there are exactly 4 defectives.

Solution:

X = number of defectives in a batch of 300

$$X \sim B(300, 0.02)$$

$$X \approx \sim P_0(6)$$

$$\begin{aligned} \text{a } P(X < 2) &= P(X \leq 1) && \text{(Poisson tables)} \\ &\approx 0.0174 \end{aligned}$$

$$\begin{aligned} \text{b } P(X = 4) &= P(X \leq 4) - P(X \leq 3) \\ &\approx 0.2851 - 0.1512 && \text{(Poisson tables)} \\ &= 0.1339 \end{aligned}$$

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Exercise D, Question 5

Question:

A medical practice screens a random sample of 250 of its patients for a certain condition which is present in 1.5% of the population. Use a suitable approximation to find the probability that they obtain

- a no patients with the condition,
- b at least two patients with the condition.

Solution:

X = number of people with the condition in sample of 250

$$X \sim B(250, 0.015)$$

$$X \approx \sim P_0(3.75)$$

Poisson is suitable approximation.

- a $P(X=0) \approx e^{-3.75} = 0.0235177\dots$ (using Poisson)
So using Poisson approximation probability ≈ 0.0235 (3 s.f)

- b $P(X \geq 2) = 1 - P(X \leq 1)$
 $\approx 1 - e^{-3.75} [1 + 3.75]$
 $= 1 - 4.75 \times e^{-3.75}$
 $= 1 - 0.111709\dots$
 $= 0.8883$

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Exercise D, Question 6

Question:

An experiment involving 2 fair dice is carried out 180 times. The dice are placed in a container, shaken and the number of times a double six is obtained recorded. Use a suitable approximation to find the probability that a double six is obtained

- a once,
- b twice,
- c at least three times.

Solution:

X = number of double sixes in 180 throws

$$X \sim B(180, \frac{1}{36})$$

$$X \approx \sim P_0(5)$$

$$\begin{aligned} \text{a } P(X=1) &\approx \frac{e^{-5} \times 5^1}{1!} = 0.03368 \\ &= 0.0337 \quad (3 \text{ s.f.}) \end{aligned}$$

$$\begin{aligned} \text{b } P(X=2) &\approx \frac{e^{-5} \times 5^2}{2!} = 0.084224\dots \\ &= 0.0842 \quad (3 \text{ s.f.}) \end{aligned}$$

$$\begin{aligned} \text{c } P(X \geq 3) &= 1 - P(X \leq 2) \\ &\approx 1 - 0.1247 \quad (\text{Poisson tables}) \\ &= 0.8753 \\ &= 0.875 \quad (3 \text{ s.f.}) \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 7

Question:

It is claimed that 95% of the population in a certain village are right-handed. A random sample of 80 villagers is tested to see whether or not they are right-handed. Use a Poisson approximation to estimate the probability that the number who are right-handed is

- a 80,
- b 79,
- c at least 78.

Solution:

X = number of villagers who are left-handed

$$X \sim B(80, 0.05)$$

(Need P small to use Poisson approximation)

$$X \approx P_0(4)$$

$$\begin{aligned} \text{a } P(80 \text{ are right-handed}) &= P(X = 0) \approx e^{-4} && \text{(Poisson Tables)} \\ &= 0.0183 \end{aligned}$$

$$\begin{aligned} \text{b } P(79 \text{ are right-handed}) &= P(X = 1) \\ &= P(X \leq 1) - P(X = 0) \\ &\approx 0.0916 - 0.0183 && \text{(Poisson Tables)} \\ &= 0.0733 \end{aligned}$$

$$\begin{aligned} \text{c } P(\text{at least } 78 \text{ are right-handed}) &= P(X \leq 2) \\ &\approx 0.2381 && \text{(Poisson tables)} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 8

Question:

In a computer simulation 500 dots were fired at a target and the probability of a dot hitting the target was 0.98. Find the probability that

- a all the dots hit the target,
- b at least 495 hit the target.

Solution:

X = number of hits out of 500

$X \sim B(500, 0.98)$

Y = number of misses out of 500 ($\because p$ is small and n large)

$Y \sim B(500, 0.02)$

$Y \approx P_0(10)$

a

$$\begin{aligned} P(X=500) &= P(Y=0) \\ &\approx e^{-10} = 0.000045399 \\ &\approx 0.0000454 \text{ (3 s.f.)} \end{aligned}$$

b

$$\begin{aligned} P(X \geq 495) &= P(Y \leq 5) \\ &\approx 0.0671 \quad (\text{Poisson tables}) \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 9

Question:

- a State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

Independently for each call into the telephone exchange of a large organisation, there is a probability of 0.002 that the call will be connected to a wrong extension.

- b Find, to 3 significant figures, the probability that, on a given day, exactly one of the first 5 incoming calls will be wrongly connected.
- c Use a Poisson approximation to find, to 3 decimal places, the probability that, on a day when there are 1000 incoming calls, at least 3 of them are wrongly connected during that day. *E*

Solution:

- a For large n
and small p
 $B(n, p) \approx P_0(np)$

- b X = number of wrongly connected calls in sample of 5
 $X \sim B(5, 0.002)$
 $P(X=1) = 5 \times (0.002)^1 (0.998)^4$
 $= 0.00992$

- c Y = number of wrongly connected calls in sample of 1000
 $Y \sim B(1000, 0.002)$
So $Y \approx P_0(2)$

$$\begin{aligned} P(Y \geq 3) &= 1 - P(Y \leq 2) \\ &\approx 1 - 0.6767 && \text{(Poisson Tables)} \\ &= 0.3233 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

Question:

- a State conditions under which the Poisson distribution is a suitable model to use in statistical work.

Flaws in a certain brand of tape occur at random and at a rate of 0.75 per 100 metres. Assuming a Poisson distribution for the number of flaws in a 400 metre roll of tape,

- b find the probability that there will be at least one flaw.
 c Show that the probability that there will be at most 2 flaws is 0.423 (to 3 decimal places).

In a batch of 5 rolls, each of length 400 metres,

- d find the probability that at least 2 rolls will contain fewer than 3 flaws.

Solution:

- a If the outcomes occur:

- 1 singly
- 2 at a constant rate
- 3 independently and at random then a Poisson distribution can be suitable.

- b Let F = number of flaws in 400 m of tape

$$F \sim P_0(3)$$

$$\begin{aligned} P(F \geq 1) &= 1 - P(F = 0) && \text{(tables)} \\ &= 1 - 0.0498 \\ &= 0.9502 \end{aligned}$$

- c $P(F \leq 2) = 0.4232$ (tables)
 $= 0.423$ (3 d.p.)

- d Let R = number of rolls (out of 5) with fewer than 3 flaws.

$$P(\text{fewer than 3 flaws}) = P(F < 3) = P(F \leq 2) = 0.423.$$

$$R \sim B(5, 0.423)$$

$$\begin{aligned} P(R \geq 2) &= 1 - P(R \leq 1) \\ &= 1 - [5 \times (0.423)^1 \times (0.577)^4 + (0.577)^5] \\ &= 1 - 0.29838\dots \\ &= 0.702 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 2

Question:

An archer fires arrows at a target and for each arrow, independently of all others, the probability that it hits the bull's eye is $\frac{1}{8}$.

- a Given that the archer fires 5 arrows, find the probability that fewer than 2 arrows hit the bull's eye.

The archer fires 5 arrows, collects them and then fires all 5 again.

- b Find the probability that on both occasions fewer than 2 hit the bull's eye.

The archer now fires 60 arrows at the target. Using a suitable approximation find

- c the probability that fewer than 10 hit the bull's eye,
 d the greatest value of m such that the probability that the archer hits the bull's eye with at least m arrows is greater than 0.5.

Solution:

X = number of arrows that hit bull's eye

$$X \sim B(5, \frac{1}{8})$$

$$\begin{aligned} \text{a } P(X < 2) &= P(X \leq 1) \\ &= 5 \times \left(\frac{1}{8}\right) \times \left(\frac{7}{8}\right)^4 + \left(\frac{7}{8}\right)^5 \\ &= 0.87927\dots = 0.879 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{b } [P(X < 2)]^2 &= (0.87927)^2 \\ &= 0.773 \text{ (3 s.f.)} \end{aligned}$$

Y = number of arrows that hit bull's eye out of 60

$$\text{c } Y \sim B(60, \frac{1}{8})$$

$$Y \sim P_0(7.5)$$

$$\begin{aligned} P(Y < 10) &= P(Y \leq 9) && \text{(Poisson tables)} \\ &\approx 0.7764 \end{aligned}$$

$$\text{d } \text{Require } P(Y \geq m) > 0.5$$

$$\text{i.e. } P(Y \leq m-1) < 0.5$$

$$P(Y \leq 6) = 0.3782 < 0.5$$

$$P(Y \leq 7) = 0.5246 > 0.5$$

$$\therefore m-1 = 6$$

$$\text{so } m = 7$$

(Using Poisson tables)

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 3

Question:

In Joe's roadside café $\frac{2}{5}$ of the customers buy a cup of tea.

- a** Find the probability that at least 4 of the next 10 customers will buy a cup of tea.
Joe has calculated that, on a typical morning, customers arrive in the café at a rate of 0.5 per minute.
- b** Find the probability that at least 10 customers arrive in the next 15 minutes.
- c** Find the probability that exactly 10 customers arrive in the next 20 minutes.
- d** Find the probability that in the next 20 minutes exactly 10 customers arrive and at least 4 of them buy a cup of tea.

Solution:

X = number of customers out of 10 who buy a cup of tea
 $X \sim B(10, 0.4)$

$$\begin{aligned} \mathbf{a} \quad P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - 0.3823 && \text{(binomial tables)} \\ &= 0.6177 \end{aligned}$$

C = number of customers who arrive
 in next 15 minutes
 $C \sim P_0(7.5)$

$$\lambda = 7.5 = 0.5 \times 15$$

$$\begin{aligned} \mathbf{b} \quad P(C \geq 10) &= 1 - P(C \leq 9) \\ &= 1 - 0.7764 \\ &= 0.2236 \end{aligned}$$

T = number of customers who arrive in next 20 minutes
 $T \sim P_0(10)$

$$\lambda = 20 \times 0.5 = 10$$

$$\begin{aligned} \mathbf{c} \quad P(T = 10) &= P(T \leq 10) - P(T \leq 9) \\ &= 0.5830 - 0.4579 \\ &= 0.1251 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad P(T = 10) \times P(X \geq 4) &= 0.1251 \times 0.6177 \\ &= 0.07727 \dots \\ &= 0.0773 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

Question:

The number, X , of breakdowns per week of the lifts in a large block of flats has a Poisson distribution with mean 0.25. Find, to 3 decimal places, the probability that in a particular week

- there will be at least one breakdown,
- there will be at most 2 breakdowns.
- Show that the probability that during a 12-week period there will be no lift breakdowns is 0.050 (to 3 decimal places).

The residents in the flats have a maintenance contract with *Liftserve*. The contract is for a set of 20, 12-week periods. For every 12-week period with no breakdowns the residents pay *Liftserve* £500. If there is at least 1 breakdown in a 12-week period then *Liftserve* will mend the lift free of charge and the residents pay nothing for that period of 12 weeks.

- Find the probability that over the course of the contract the residents pay no more than £1000.

Solution:

$$X \sim P_0(0.25)$$

$$\begin{aligned} \text{a } P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - e^{-0.25} \\ &= 0.221199\dots = 0.221 \text{ (3 d.p.)} \end{aligned}$$

$$\begin{aligned} \text{b } P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= e^{-0.25} \left[1 + 0.25 + \frac{0.25^2}{2!} \right] \\ &= 0.99783\dots = 0.998 \text{ (3 d.p.)} \end{aligned}$$

- T = number of breakdowns in 12 weeks

$$T \sim P_0(3)$$

$$\begin{aligned} \therefore P(T = 0) &= e^{-3} = 0.04978\dots \\ &= 0.050 \text{ (3 d.p.)} \end{aligned}$$

$\lambda = 3 = 12 \times 0.25$

- Y = number of 12-week periods with no breakdowns out of 20

$$Y \sim B(20, 0.050)$$

Residents pay \leq £1000 if $Y \leq 2$

$$\begin{aligned} P(Y \leq 2) &= 0.9245 \\ &= 0.925 \text{ (3 d.p.)} \end{aligned} \quad \text{(binomial tables)}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 5

Question:

Accidents occur in a school playground at the rate of 3 per year.

- Suggest a suitable model for the number of accidents in the playground next month.
- Using this model calculate the probability of 1 or more accidents in the playground next month.

Solution:

- a X = number of accidents in a month

$$\lambda = 3 \times \frac{1}{12} = 0.25$$

$$X \sim P_0(0.25)$$

- b $P(X \geq 1) = 1 - P(X = 0)$
 $= 1 - e^{-0.25}$
 $= 0.221$ (3 s.f.)

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 1

Question:

During working hours an office switchboard receives telephone calls at random and at a rate of one call every 40 seconds.

- a** Find, to 3 decimal places, the probability that during a given one-minute period
- i** no call is received,
 - ii** at least 2 calls are received.
- b** Find, to 3 decimal places, the probability that no call is received between 10:30 a.m. and 10:31 a.m. and that at least two calls are received between 10:31 a.m. and 10:32 a.m. *E*

Solution:

X = number of calls received during one minute

$$X \sim P_0(1.5) \text{ since } \frac{1}{\left(\frac{2}{3}\right)} 1.5$$

$$\begin{aligned} \mathbf{i} \quad P(X=0) &= 0.2231 && \text{(tables)} \\ &= 0.223 \text{ (3 d.p.)} \end{aligned}$$

$$\begin{aligned} \mathbf{ii} \quad P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - 0.5578 && \text{(tables)} \\ &= 0.4422 \\ &= 0.442 \text{ (3 d.p.)} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad P(X=0) \times P(X \geq 2) &= 0.2231 \times 0.4422 \\ &= 0.09865\dots \\ &= 0.0987 \text{ (3 s.f.)} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 2

Question:

State conditions under which the Poisson distribution is a suitable model to use in statistical work.

The number of typing errors per 1000 words made by a typist has a Poisson distribution with mean 2.5.

- a Find, to 3 decimal places, the probability that in an essay of 4000 words there will be at least 12 typing errors.

The typist types 3 essays, each of length 4000 words.

- b Find the probability that each contains at least 12 typing errors. *E*

Solution:

Items occur in continuous space or time:

- 1 singly
- 2 at a constant rate
- 3 independently of one another and at random.

- a X = number of errors in 1,000 words

$$X \sim P_0(10)$$

$\lambda = 4 \times 2.5$

$$\begin{aligned} P(X \geq 12) &= 1 - P(X \leq 11) \\ &= 1 - 0.6968 \\ &= 0.3032 \end{aligned}$$

- b Y = number of 4000 word essays with at least 12 errors

$$Y \sim B(3, 0.3032)$$

$$\begin{aligned} P(Y = 3) &= (0.3032)^3 \\ &= 0.027873\dots \\ &= 0.0279 \text{ (3 s.f.)} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 3

Question:

- a State conditions under which the binomial distribution $B(n, p)$ may be approximated by a Poisson distribution and write down the mean of this Poisson distribution.

Samples of blood were taken from 250 children in a region of India. Of these children, 4 had blood type $A2B$.

- b Write down an estimate of p , the proportion of children in this region having blood type $A2B$.

Consider a group of n children from this region and let X be the number having blood type $A2B$. Assuming that X is distributed $B(n, p)$ and that p has the value estimated above, calculate, to 3 decimal places, the probability that the number of children with blood type $A2B$ in a group of 6 children from this region will be

- i zero,
ii more than 1.
- c Use a Poisson approximation to calculate, to 4 decimal places, the probability that, in a group of 800 children from this region, there will be fewer than 3 children of blood type $A2B$.

Solution:

- a $B(n, p)$ can be approximated to $P_0(np)$

If n is large and p is small

Then mean = np

b $\hat{p} = \frac{4}{250} = 0.016$

$X \sim B(6, 0.016)$

i

$$\begin{aligned} P(X=0) &= (0.984)^6 = 0.907759\dots \\ &= 0.908 \text{ (3 d.p.)} \end{aligned}$$

ii

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - [6 \times 0.016 \times (0.984)^5 + (0.984)^6] \\ &= 0.003679\dots \\ &= 0.00370 \text{ (3 s.f.)} \end{aligned}$$

- c Y = number of children out of 800 with the blood group $A2B$

$Y \sim B(800, 0.016)$

$Y \approx P_0(12.8)$

$P(Y < 3) = P(Y \leq 2)$

$$\begin{aligned} &= e^{-12.8} \left[1 + 12.8 + \frac{12.8^2}{2!} \right] \\ &= 0.00026426\dots \\ &= 0.000264 \text{ (3 s.f.)} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 4

Question:

Which of the following variables is best modelled by a Poisson distribution and which is best modelled by a binomial distribution?

- a The number of hits by an arrow on a target, when 20 arrows are fired.
- b The number of earth tremors that take place in a village over a given period of time.
- c The number of particles emitted per minute by a radioactive isotope.
- d The number of heads you get when tossing 2 coins 100 times.
- e The number of accidents in a city in a year.
- f The number of flying bomb hits in specified areas of London during World War 2.

Solution:

- a Binomial a fixed number of arrows ($n = 20$)
need to assume that $p =$ probability of an arrow hitting is constant
- b Poisson no fixed number of trials
need to assume earth tremors occur at random with a constant rate
- c Poisson no fixed number of particles
need to assume particles emitted at a constant average rate
- d Binomial $n = 200$ the number of tosses of coins $p = \frac{1}{2}$
- e Poisson no fixed number
need to assume accidents occur at a constant rate
- f Poisson no fixed number
need to assume flying bomb hits occur at a constant rate

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 5

Question:

Loaves of bread on a production line pass a monitoring point at a constant rate of 300 loaves per hour.

- Find how many loaves you would expect to pass the monitoring point in 2 minutes.
- Find the probability that no loaves pass the monitoring point in a given 1-minute period.

Solution:

X = number of loaves passing in 2 minutes

a $X = \frac{300}{60} \times 2 = 10 = E(X)$

b Y = number of loaves passing in 1 minute

$$Y \sim P_0(5)$$

$$P(Y = 0) = e^{-5} = 0.0067 \quad (\text{tables})$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 6

Question:

Accidents occur at a certain road junction at a rate of 3 per year.

- a Suggest a suitable model for the number of accidents at this road junction in the next month.
- b Show that, under this model, the probability of 2 or more accidents at this road junction in the next month is 0.0265 to 4 decimal places.

The local residents have applied for a crossing to be installed.

The planning committee agree to monitor the situation for the next 12 months.

If there is at least one month with 2 or more accidents in it they will install a crossing.

- c Find the probability that the crossing is installed. *E*

Solution:

$$\text{a } \lambda = \frac{3}{12} = 0.25$$

X = number of accidents in a month

$$X \sim P_0(0.25)$$

$$\begin{aligned} \text{b } P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 1) + P(X = 0)] \\ &= 1 - e^{-0.25} [0.25 + 1] \\ &= 1 - 0.97350\dots \\ &= 0.026499\dots \\ &= 0.0265 \end{aligned}$$

Y = number of months with 2 or more accidents

$$Y \sim B(12, 0.0265)$$

$$\begin{aligned} \text{c } P(Y \geq 1) &= 1 - P(Y = 0) \\ &= 1 - (0.9735)^{12} \\ &= 1 - 0.724488\dots \\ &= 0.275511\dots \\ &= 0.276 \text{ (3 s.f.)} \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 7

Question:

Breakdowns occur on a particular machine at a rate of 2.5 per month. Assuming that the number of breakdowns can be modelled by a Poisson distribution, find the probability that

- a exactly 3 occur in a particular month,
- b more than 10 occur in a three-month period,
- c exactly 3 occur in each of 2 successive months. *E*

Solution:

X = number of breakdowns per month
 $X \sim P_0(2.5)$

$$\begin{aligned} \text{a } P(X = 3) &= P(X \leq 3) - P(X \leq 2) && \text{(tables)} \\ &= 0.7576 - 0.5438 \\ &= 0.2138 \end{aligned}$$

Y = number of breakdowns in 3 months
 $Y \sim P_0(7.5)$

$$\begin{aligned} \text{b } P(Y > 10) &= 1 - P(Y \leq 10) && \text{(tables)} \\ &= 1 - 0.8622 \\ &= 0.1378 \end{aligned}$$

$$\begin{aligned} \text{c } P(X = 3) \times P(X = 3) & \\ &= (0.2138)^2 \\ &= 0.04571 \\ &= 0.0457 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 8

Question:

A geography student is studying the distribution of telephone boxes in a large rural area where there is an average of 300 boxes per 500 km^2 . A map of part of the area is divided into 50 squares, each of area 1 km^2 and the student wishes to model the number of telephone boxes per square.

a Suggest a suitable model the student could use and specify any parameters required.

One of the squares is picked at random.

b Find the probability that this square does not contain any telephone boxes.

c Find the probability that this square contains at least 3 telephone boxes.

The student suggests using this model on another map of a large city and surrounding villages.

d Comment, giving your reason briefly, on the suitability of the model in this situation. **E**

Solution:

X = number of telephone boxes per square

a $X \sim P_0(0.6)$

$\lambda = \frac{300}{500} = 0.6$

b $P(X=0) = e^{-0.6} = 0.5488\dots$
 $= 0.549$ (3 s.f.)

c $P(X \geq 3) = 1 - P(X \leq 2)$
 $= 1 - e^{-0.6} \left[1 + 0.6 + \frac{0.6^2}{2} \right]$
 $= 1 - 0.97688$
 $= 0.02312$
 $= 0.0231$ (3 s.f.)

d Not suitable

The rate of telephone boxes will be different in cities and they are more likely to occur in clusters.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 9

Question:

All the letters in a particular office are typed either by Pat, a trainee typist, or by Lyn, who is a fully-trained typist. The probability that a letter typed by Pat will contain one or more errors is 0.3.

- a Find the probability that a random sample of 4 letters typed by Pat will include exactly one letter free from error.

The probability that a letter typed by Lyn will contain one or more errors is 0.05.

- b Use tables, or otherwise, to find, to 3 decimal places, the probability that in a random sample of 20 letters typed by Lyn, not more than 2 letters will contain one or more errors.

On any one day, 6% of the letters typed in the office are typed by Pat. One letter is chosen at random from those typed on that day.

- c Show that the probability that it will contain one or more errors is 0.065.
Given that each of 2 letters chosen at random from the day's typing contains one or more errors,
- d find, to 4 decimal places, the probability that one was typed by Pat and the other by Lyn. E

Solution:

X = number of letters out of 4 with at least one error

$$X \sim B(4, 0.3)$$

$$\begin{aligned} \text{a } P(X=3) &= 4 \times 0.3^3 \times 0.7^1 \\ &= 0.0756 \end{aligned}$$

L = number of letters out of 20 containing one or more errors

$$L \sim B(20, 0.05)$$

$$\begin{aligned} \text{b } P(L \leq 2) &= 0.9245 \quad (\text{tables}) \\ &= 0.925 \text{ (3 d.p.)} \end{aligned}$$

$$\begin{array}{rcc} \text{c} & & \text{Pat} \quad \text{Lyn} \\ P(\text{letter has errors}) &= & 0.06 \times 0.3 + 0.94 \times 0.05 \\ &= & 0.065 \end{array}$$

$$\begin{array}{rcc} \text{d} & & \text{Pat} \quad \text{Lyn} \\ \text{Probability} &= & \frac{2 \times (0.06 \times 0.3) \times (0.94 \times 0.05)}{(0.065)^2} \\ &= & 0.4004733\dots \\ &= & 0.4005 \text{ (4 d.p.)} \end{array}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 10

Question:

The number of breakdowns per day of the lifts in a large block of flats is modelled by a Poisson distribution with mean 0.2.

- Find, to 3 decimal places, the probability that on a particular day there will be at least one breakdown.
- Find the probability that there are fewer than 2 days in a 30-day month with at least one breakdown.

Solution:

X = number of breakdowns per day

$$X \sim P_0(0.2)$$

$$\begin{aligned} \text{a } P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - e^{-0.2} \\ &= 1 - 0.8187\dots \\ &= 0.181(3 \text{ d.p.}) \end{aligned}$$

Y = number of days in 30 day months with at least one breakdown

$$Y \sim B(30, 0.181)$$

$$\begin{aligned} \text{b } P(Y < 2) &= P(Y \leq 1) \\ &= (0.819)^{30} + 30(0.819)^{29} \times (0.181) \\ &= 0.0191 \end{aligned}$$