Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise A, Question 1

Question:

Use tables of the normal distribution to find the following.

a P($Z < 2.12$)

b P($Z < 1.36$)

c P($Z > 0.84$)

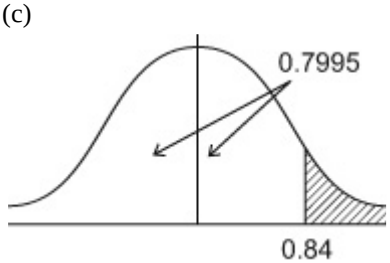
d P($Z < -0.38$)

Solution:

(a) P($z < 2.12$) = $\underline{0.9830}$

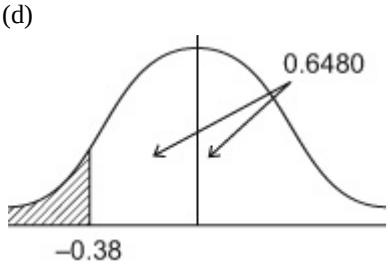
(b) P($z < 1.36$) = $\underline{0.9131}$

(c)



$P(Z > 0.84) = 1 - 0.7995$
$= \underline{0.2005}$

(d)



$P(Z < -0.38)$
$= 1 - 0.6480$
$= \underline{0.352}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise A, Question 2

Question:

Use tables of the normal distribution to find the following.

a P($Z > 1.25$)

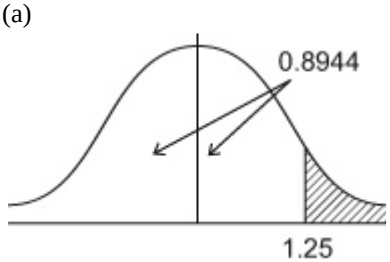
b P($Z > -1.68$)

c P($Z < -1.52$)

d P($Z < 3.15$)

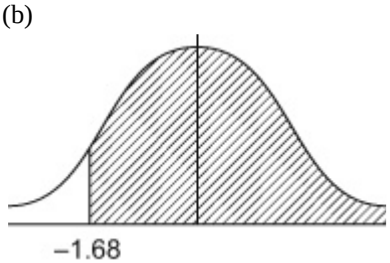
Solution:

(a)



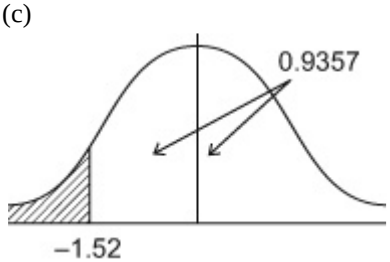
$P(Z > 1.25) = 1 - 0.8944$
$= \underline{0.1056}$

(b)



$P(Z > -1.68) = \underline{0.9535}$

(c)



$P(Z < -1.52) = 1 - 0.9357$
$= \underline{0.0643}$

(d) $P(Z < 3.15) = \underline{0.9992}$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise A, Question 3

Question:

Use tables of the normal distribution to find the following.

a $P(Z > -2.24)$

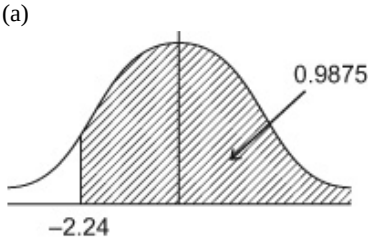
b $P(0 < Z < 1.42)$

c $P(-2.30 < Z < 0)$

d $P(Z < -1.63)$

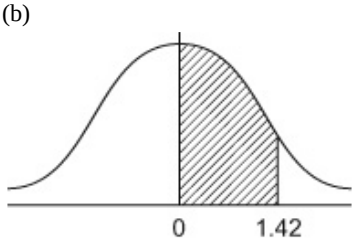
Solution:

(a)



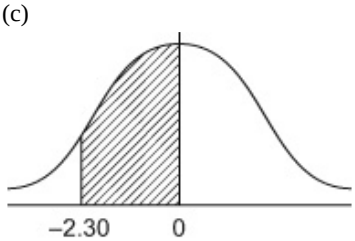
$P(Z > -2.24) = \underline{0.9875}$

(b)



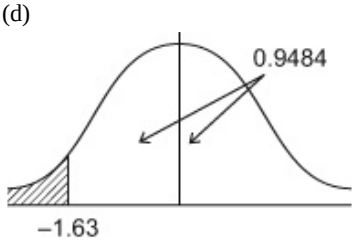
$P(0 < Z < 1.42) = 0.9222 - 0.5$
$= \underline{0.4222}$

(c)



$P(-2.30 < Z < 0) = 0.9893 - 0.5$
$= \underline{0.4893}$

(d)



$P(Z < -1.63) = 1 - 0.9484$
$= \underline{0.0516}$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise A, Question 4

Question:

Use tables of the normal distribution to find the following.

a P(1.25 < Z < 2.16)

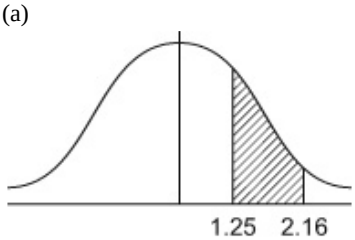
b P(−1.67 < Z < 2.38)

c P(−2.16 < Z < −0.85)

d P(−1.57 < Z < 1.57)

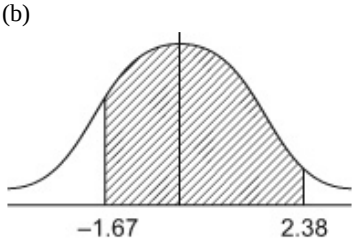
Solution:

(a)



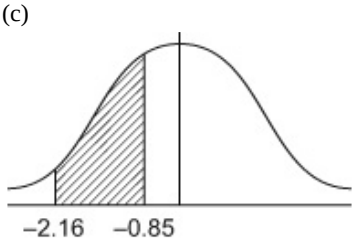
P(1.25 < Z < 2.16)
= P(Z < 2.16) − P(Z < 1.25)
= 0.9846 − 0.8944
= 0.0902

(b)



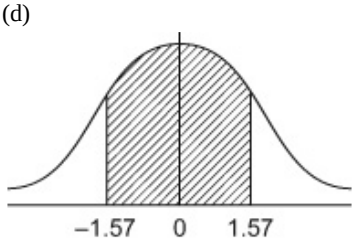
P(−1.67 < Z < 2.38)
= P(Z < 2.38) − [1 − P(Z < 1.67)]
= 0.9913 − [1 − 0.9525]
= 0.9913 − 0.0475
= 0.9438

(c)



P(−2.16 < Z < −0.85)
= P(Z < 2.16) − P(Z < 0.85)
= 0.9846 − 0.8023
= 0.1823

(d)



P(−1.57 < Z < 1.57)
= 2 × P(0 < Z < 1.57)
= 2 × [0.9418 − 0.5]
= 2 × 0.4418
= 0.8836

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise B, Question 1

Question:

Find the value of a in the following.

a $P(Z < a) = 0.9082$

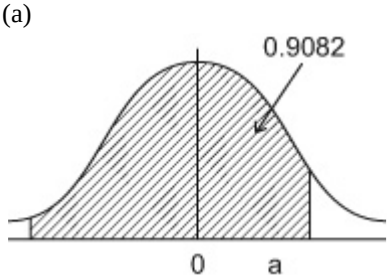
b $P(Z > a) = 0.0314$

c $P(Z < a) = 0.3372$

d $P(Z > a) = 0.6879$

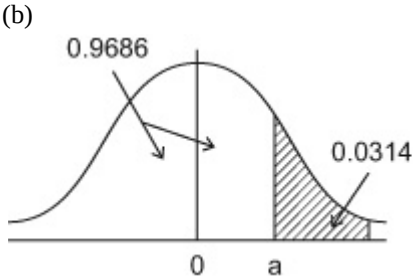
Solution:

(a)



$\underline{a = 1.33}$

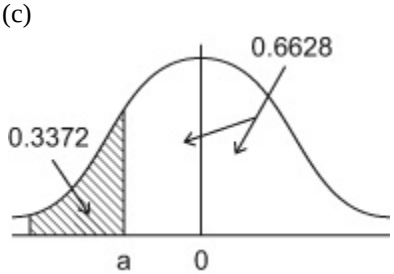
(b)



$1 - 0.0314 = 0.9686$

$\therefore \underline{\underline{a = 1.86}}$

(c)



$1 - 0.3372 = 0.6628$

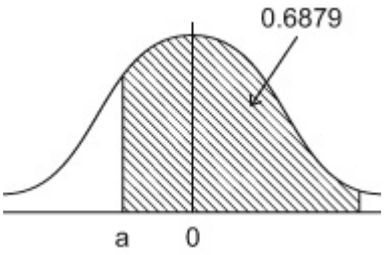
N.B.$0.3372 < 0.5 \therefore a < 0$

$\therefore \underline{\underline{a = -0.42}}$

(d)

[N.B.$a < 0$again]

$\underline{\underline{a = -0.49}}$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise B, Question 2

Question:

Find the value of a in the following.

a $P(Z < a) = 0.9938$

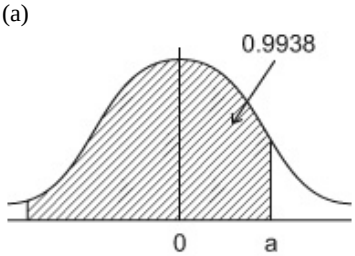
b $P(Z > a) = 0.4129$

c $P(Z > a) = 0.7611$

d $P(Z > a) = 0.2000$

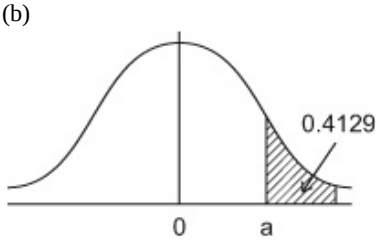
Solution:

(a)



$\underline{\underline{a = 2.50}}$

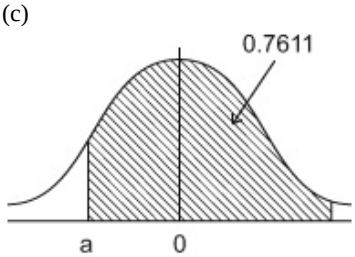
(b)



$1 - 0.4129 = 0.5871$

$\therefore \underline{\underline{a = 0.22}}$

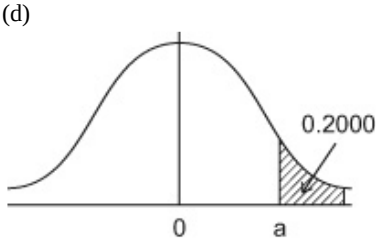
(c)



[N.B. $a < 0 \because P(Z > a) > 0.5$]

$\underline{\underline{a = -0.71}}$

(d)



Using table of percentage points

$\underline{\underline{a = 0.8416}}$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise B, Question 3

Question:

Find the value of a in the following.

a $P(Z > a) = 0.1500$

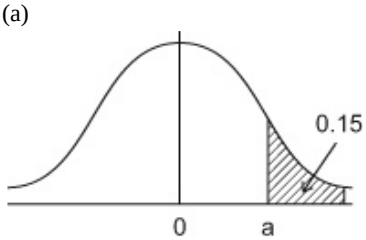
b $P(Z > a) = 0.9500$

c $P(Z > a) = 0.1112$

d $P(Z < a) = 0.9990$

Solution:

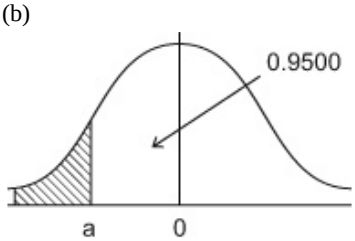
(a)



Use table of percentage points with $p = 0.15$

$\underline{\underline{a = 1.0364}}$

(b)

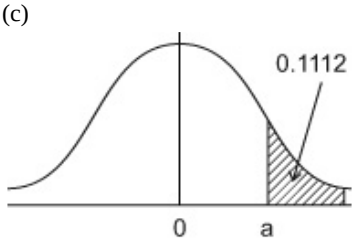


$1 - 0.9500 = 0.05 = p$

Using table of percentage points

$\underline{\underline{a = -1.6449}}$

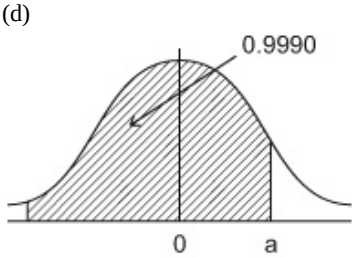
(c)



$1 - 0.1112 = 0.8888$

$\underline{\underline{a = 1.22}}$

(d)



$1 - 0.9990 = 0.0010$

Use table of percentage points with

$p = 0.0010$

$\underline{\underline{a = 3.0902}}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise B, Question 4

Question:

Find the value of a in the following.

a $P(0 < Z < a) = 0.3554$

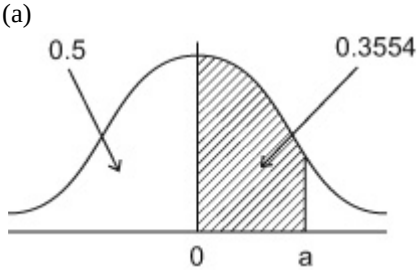
b $P(0 < Z < a) = 0.4946$

c $P(-a < Z < a) = 0.5820$

d $P(-a < Z < a) = 0.8230$

Solution:

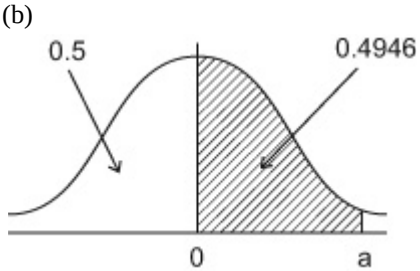
(a)



$P(Z < a) = 0.8554$

$\therefore \underline{\underline{a = 1.06}}$

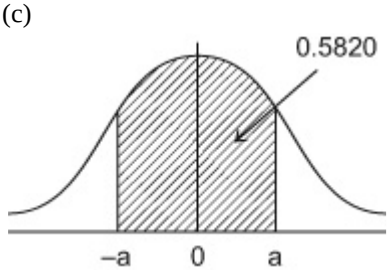
(b)



$P(Z < a) = 0.9946$

$\therefore \underline{\underline{a = 2.55}}$

(c)



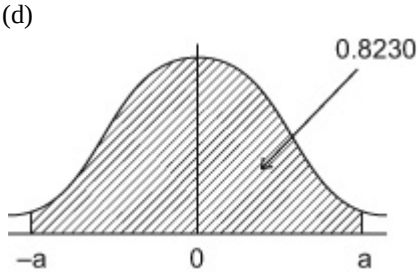
$P(0 < Z < a) = \frac{1}{2} \times 0.5820$

$= 0.2910$

$\therefore P(Z < a) = 0.7910$

$\therefore \underline{\underline{a = 0.81}}$

(d)



$P(0 < Z < a) = \frac{1}{2} \times 0.8230$

$= 0.4115$

$\therefore$ $P(Z < a) = 0.9115$

$\therefore$ $\underline{\underline{a = 1.35}}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise B, Question 5

Question:

Find the value of a in the following.

a $p(0 < Z < a) = 0.10$

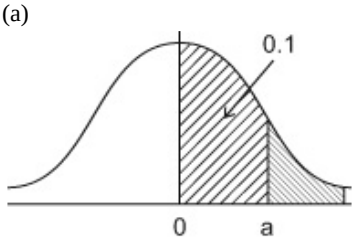
b $p(0 < Z < a) = 0.35$

c $p(-a < Z < a) = 0.80$

d $p(-a < Z < a) = 0.40$

Solution:

(a)

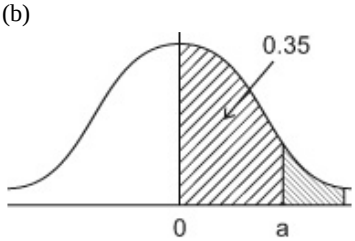


$p = 0.5 - 0.1 = 0.4000$

Use table of percentage points

$\underline{\underline{a = 0.2533}}$

(b)

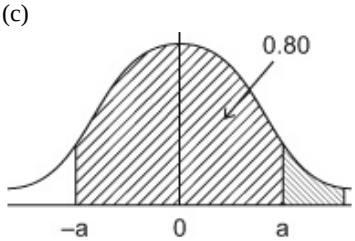


$p = 0.5 - 0.35 = 0.1500$

Use table of percentage points

$\underline{\underline{a = 1.0364}}$

(c)

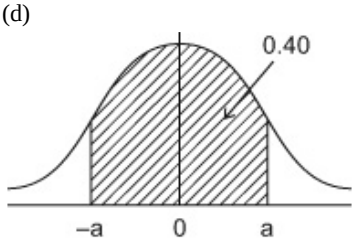


$p = 0.5 - \frac{1}{2} \times 0.8 = 0.1000$

Use table of percentage points

$\underline{\underline{a = 1.2816}}$

(d)



$P(0 < Z < a) = \frac{1}{2} \times 0.4 = 0.20$

$p = 0.5 - 0.2 = 0.3000$

Use table of percentage points

$\underline{\underline{a = 0.5244}}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise C, Question 1

Question:

The random variable $X \sim N(30, 2^2)$.

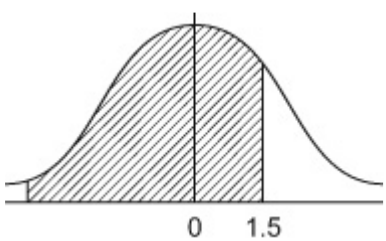
Find **a** $P(X < 33)$,

b $P(X > 26)$.

Solution:

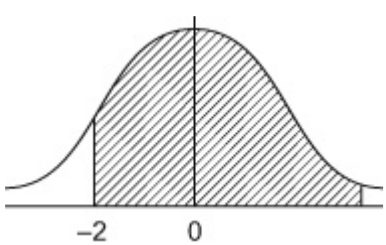
(a)

$$\begin{aligned} P(X < 33) &= P\left(Z < \frac{33 - 30}{2}\right) \\ &= P(Z < 1.5) \\ &= \underline{\underline{0.9332}} \end{aligned}$$



(b)

$$\begin{aligned} P(X > 26) &= P\left(Z > \frac{26 - 30}{2}\right) \\ &= P(Z > -2) \\ &= \underline{\underline{0.9772}} \end{aligned}$$



Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise C, Question 2

Question:

The random variable $X \sim N(40, 9)$.

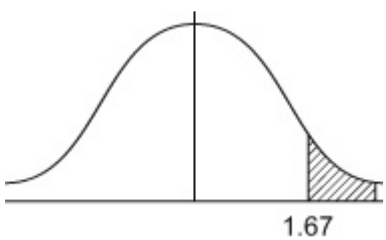
Find **a** $P(X > 45)$,

b $P(X < 38)$.

Solution:

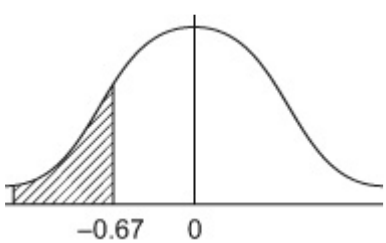
(a)

$$P(X > 45) = P\left(Z > \frac{45 - 40}{\sqrt{9}}\right)$$
$$= P(Z > 1.67)$$
$$= 1 - 0.9525$$
$$= \underline{\underline{0.0475}} \text{ (allow AWRT } \underline{\underline{0.048}}\text{)}$$



(b)

$$P(X < 38) = P\left(Z < \frac{38 - 40}{3}\right)$$
$$= P(Z < -0.67)$$
$$= 1 - 0.7486$$
$$= \underline{\underline{0.2514}} \text{ (allow AWRT } 0.251 \text{ or } 0.252\text{)}$$



Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise C, Question 3

Question:

The random variable $Y \sim N(25, 25)$.

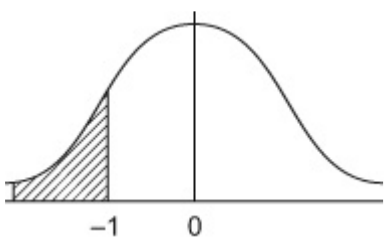
Find **a** $P(Y < 20)$,

b $P(18 < Y < 26)$.

Solution:

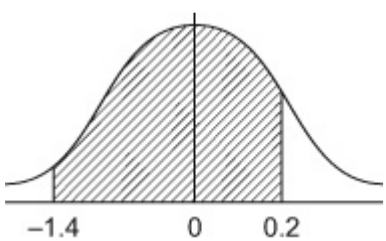
(a)

$$\begin{aligned} P(Y < 20) &= P\left(Z < \frac{20 - 25}{\sqrt{25}}\right) \\ &= P(Z < -1) \\ &= 1 - 0.8413 \\ &= \underline{\underline{0.1587}} \end{aligned}$$



(b)

$$\begin{aligned} P(18 < Y < 26) &= P\left(\frac{18 - 25}{5} < Z < \frac{26 - 25}{5}\right) \\ &= P(-1.4 < Z < 0.2) \\ &= (0.5793 - 0.5) + (0.9192 - 0.5) \\ &= \underline{\underline{0.4985}} \end{aligned}$$



Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise C, Question 4

Question:

The random variable $X \sim N(18, 10)$.

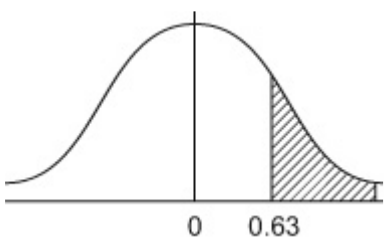
Find **a** $P(X > 20)$,

b $P(X < 15)$.

Solution:

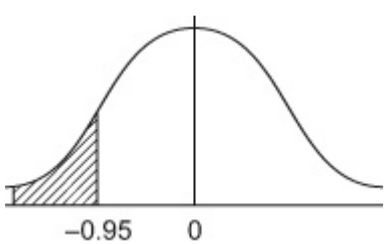
(a)

$$P(X > 20) = P\left(Z > \frac{20 - 18}{\sqrt{10}}\right)$$
$$= P(Z > 0.6324\ldots) \text{ Use } 0.63$$
$$= 1 - 0.7357$$
$$= \underline{\underline{0.2643}} \text{ (Calculator } 0.26354\ldots.)$$
allow AWRT $\underline{\underline{0.264}}$ or $\underline{\underline{0.263}}$



(b)

$$P(X < 15) = P\left(Z < \frac{15 - 18}{\sqrt{10}}\right)$$
$$= P(Z < -0.9486\ldots) \text{ [Use } -0.95]$$
$$= 1 - 0.8289$$
$$= \underline{\underline{0.1711}} \text{ (Calculator: } 0.17139\ldots)$$
allow AWRT $\underline{\underline{0.171}}$



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Normal distribution
Exercise C, Question 5

Question:

The random variable $X \sim N(20, 8)$.

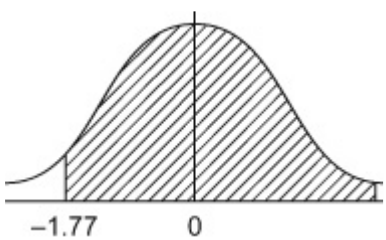
Find **a** $P(X > 15)$,

b the value of a such that $P(X < a) = 0.8051$.

Solution:

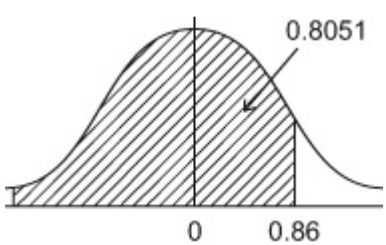
(a)

$$P(X > 15) = P\left(Z > \frac{15 - 20}{\sqrt{8}}\right)$$
$$= P(Z > -1.767\ldots) \text{ Use } -1.77$$
$$= 0.9616 \text{ (Calculator: } 0.96145\ldots)$$
allow AWRT <u>0.961 or 0.962</u>



(b)

$$P(X < a) = 0.8051$$
$$P\left(Z < \frac{a - 20}{\sqrt{8}}\right) = 0.8051$$
$$\therefore \frac{a - 20}{\sqrt{8}} = 0.86$$
$$\therefore a = \underline{\underline{22.43}} \text{ (allow AWRT 22.4)}$$



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Normal distribution
Exercise C, Question 6

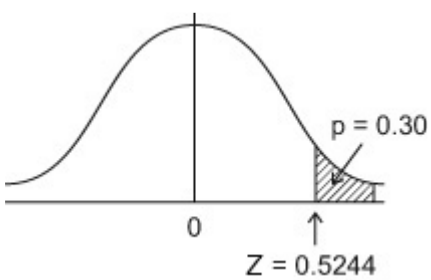
Question:

The random variable $Y \sim N(30, 5^2)$.

Find the value of a such that $P(Y > a) = 0.30$.

Solution:

$$\begin{aligned} P(Y > a) &= 0.30 \\ \frac{a-30}{5} &= 0.5244 \\ \therefore\ a &= 5 \times 0.5244 + 30 \\ a &= 32.622 \text{ or } \underline{\underline{32.6}} \text{ (3sf)} \end{aligned}$$



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Normal distribution
Exercise C, Question 7

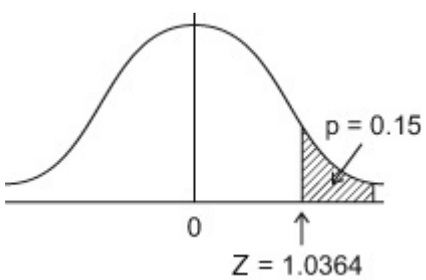
Question:

The random variable $X \sim N(15, 3^2)$.

Find the value of a such that $P(X > a) = 0.15$.

Solution:

$$\begin{aligned} p(X > a) &= 0.15 \\ \frac{a-15}{3} &= 1.0364 \\ \therefore\ a &= 3 \times 1.0364 + 15 \\ a &= 18.1092 \\ a &= \underline{\underline{18.1}} \ \text{(3sf)} \end{aligned}$$



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Normal distribution
Exercise C, Question 8

Question:

The random variable $X \sim N(20, 12)$.

Find the value of a and the value of b such that

a $P(X < a) = 0.40$,

b $P(X > b) = 0.6915$.

c Write down $P(b < X < a)$.

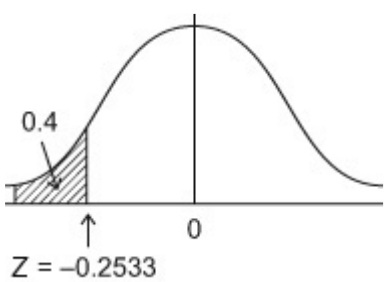
Solution:

(a)

$p(X < a) = 0.40$ Use P= 0.4000

$\frac{a-20}{\sqrt{12}} = -0.2533$

$a = 19.122\ldots \quad \therefore \quad a = \underline{\underline{19.1}}$ (3sf)

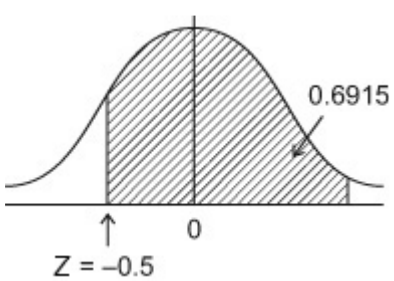


(b)

$P(X > b) = 0.6915$

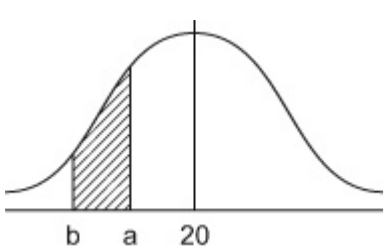
$\frac{b-20}{\sqrt{12}} = -0.5$

$\therefore b = 18.267\ldots \quad \therefore b = \underline{\underline{18.3}}$ (3sf)



(c)

$p(b < X < a)$
$= 0.40 - [1 - 0.6915]$
$= \underline{\underline{0.0915}}$



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Normal distribution
Exercise C, Question 9

Question:

The random variable $Y \sim N(100, 15^2)$.

Find the value of a and the value of b such that

a $P(Y > a) = 0.975$,

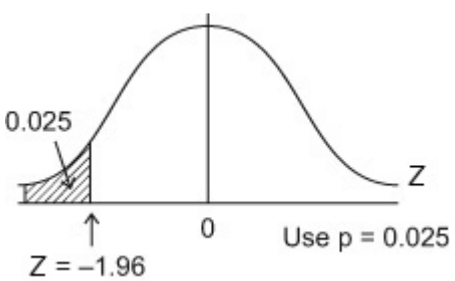
b $P(Y < b) = 0.10$.

c Write down $P(a < Y < b)$.

Solution:

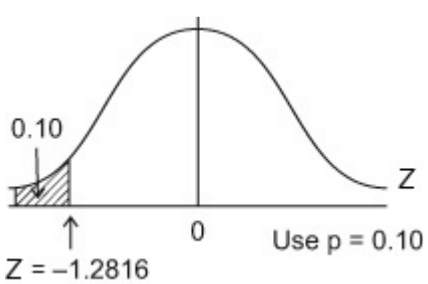
(a)

$$P(Y > a) = 0.975$$
$$\therefore \frac{a - 100}{15} = -1.96$$
$$\therefore a = \underline{\underline{70.6}}$$



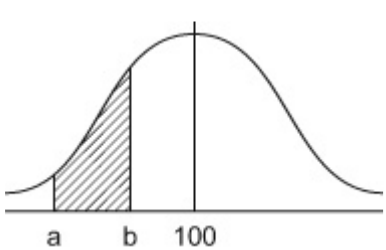
(b)

$$P(Y < b) = 0.10$$
$$\therefore \frac{b - 100}{15} = -1.2816$$
$$\therefore b = 80.776 \text{ or } \underline{\underline{80.8}} \text{ (3sf)}$$



(c)

$$P(a < Y < b)$$
$$= 0.10 - 0.025$$
$$= \underline{\underline{0.075}}$$



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Normal distribution
Exercise C, Question 10

Question:

The random variable $X \sim N(80, 16)$.

Find the value of a and the value of b such that

a $P(X > a) = 0.40$,

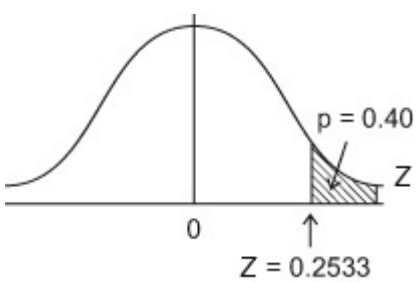
b $P(X < b) = 0.5636$.

c Write down $P(b < X < a)$.

Solution:

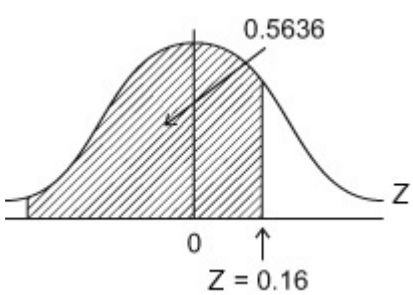
(a)

$$P(X > a) = 0.40$$
$$\therefore \frac{a-80}{\sqrt{16}} = 0.2533$$
$$\therefore a = \underline{\underline{81.0}} \text{ (3sf)}$$



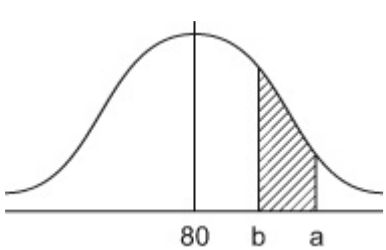
(b)

$$P(X < b) = 0.5636$$
$$\therefore \frac{b-80}{4} = 0.16$$
$$\therefore b = \underline{\underline{80.64}}$$



(c)

$$P(b < X < a)$$
$$= [1 - 0.4] - 0.5636$$
$$= 0.6 - 0.5636$$
$$= \underline{\underline{0.0364}}$$



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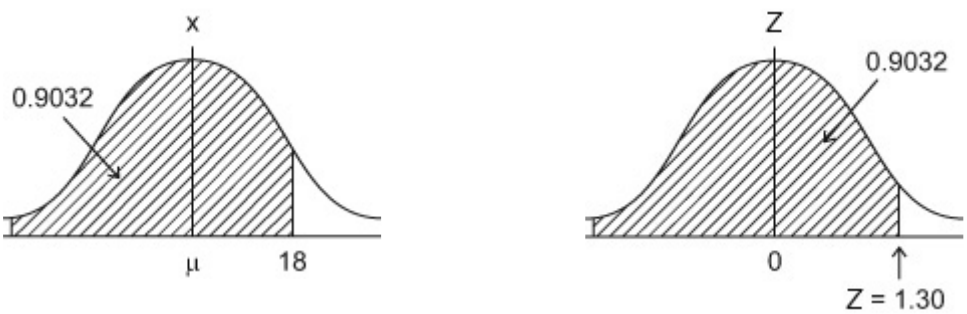
Normal distribution
Exercise D, Question 1

Question:

The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$.

Find the value of μ.

Solution:



$$Z = \frac{X-\mu}{\sigma} \quad \Rightarrow \quad 1.30 = \frac{18-\mu}{5}$$
$$\therefore \quad \mu = 18 - 5 \times 1.30$$
$$\mu = \underline{\underline{11.5}}$$

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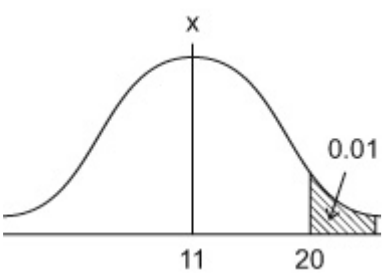
Normal distribution
Exercise D, Question 2

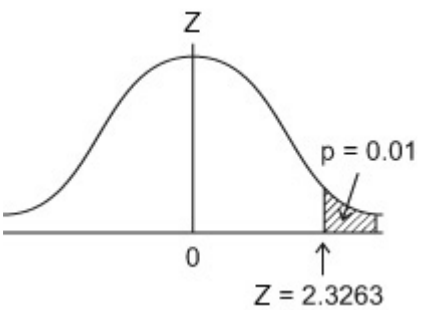
Question:

The random variable $X \sim N(11, \sigma^2)$ and $P(X > 20) = 0.01$.

Find the value of σ.

Solution:





$$Z = \frac{X-\mu}{\sigma} \Rightarrow \quad 2.3263 = \frac{20-11}{\sigma}$$

$$\therefore \quad \sigma = \frac{9}{2.3263}$$

$$= 3.8688\ldots.$$

$$\therefore \quad \sigma = \underline{\underline{3.87}} \quad (3sf)$$

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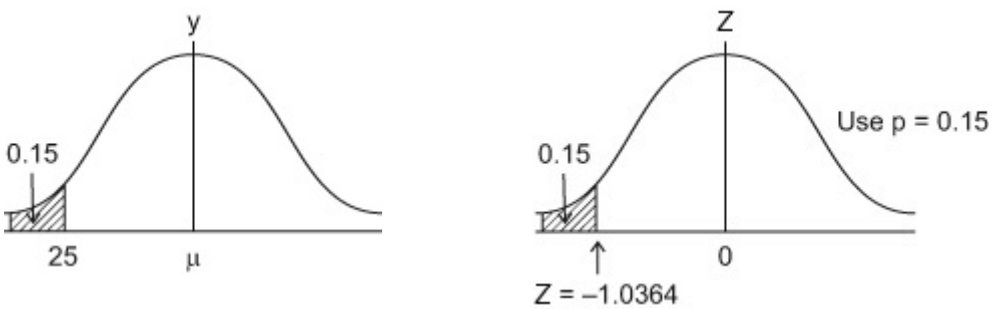
Normal distribution
Exercise D, Question 3

Question:

The random variable $Y \sim N(\mu, 40)$ and $P(Y < 25) = 0.15$.

Find the value of μ.

Solution:



$$Z = \frac{X-\mu}{\sigma} \quad \Rightarrow \quad -1.0364 = \frac{25-\mu}{\sqrt{40}}$$

$$\mu = 31.554\ldots.$$

$$= \underline{\underline{31.6}} \quad \text{(3sf)}$$

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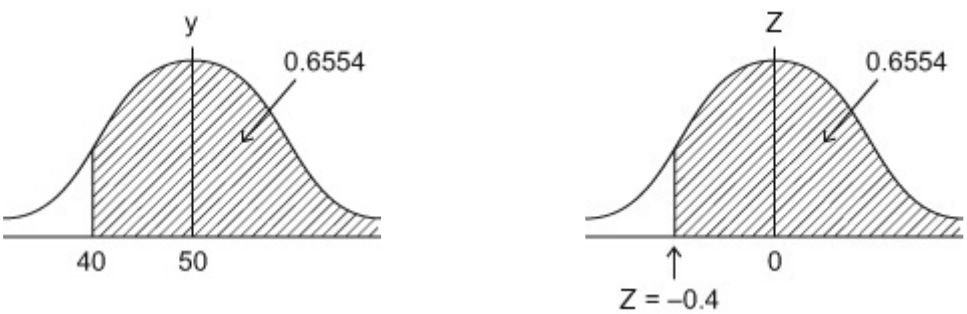
Normal distribution
Exercise D, Question 4

Question:

The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$.

Find the value of σ.

Solution:



$$Z = \frac{X-\mu}{\sigma} \quad \Rightarrow \quad -0.4 = \frac{40-50}{\sigma}$$

$$\therefore \quad \sigma = \frac{10}{0.4}$$

$$\sigma = \underline{\underline{25}}$$

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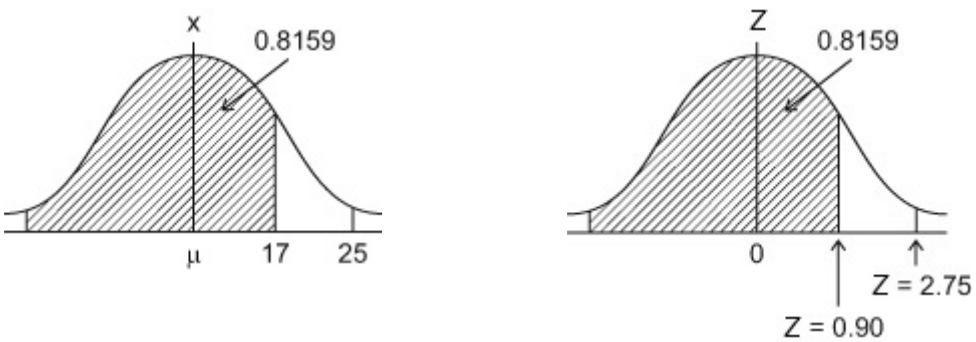
Normal distribution
Exercise D, Question 5

Question:

The random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and the value of σ.

Solution:



$Z = \frac{X-\mu}{\sigma} \Rightarrow 0.90\ \sigma = 17 - \mu$

$2.75\sigma = 25 - \mu$

Subtract $1.85\ \sigma = 8$

$\therefore \quad \sigma = \frac{8}{1.85} = 4.3243$

$\mu = 17 - 0.90 \times \sigma.$

$= 13.1081$

$\therefore \ \underline{\underline{\mu = 13.1}} \quad \sigma = \underline{\underline{4.32}}$ (3sf)

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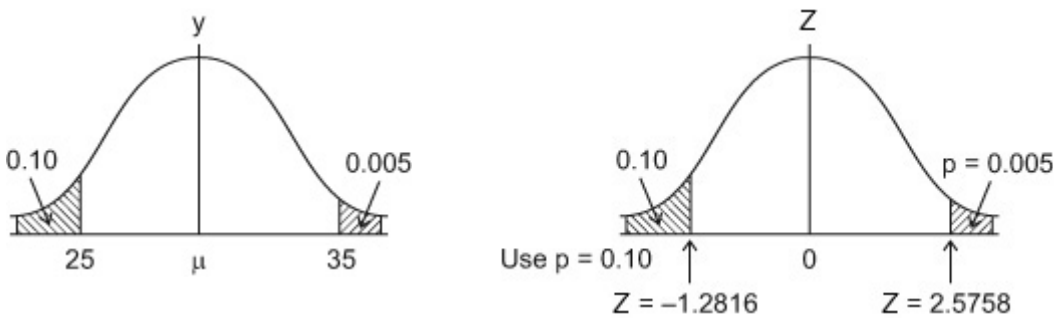
Normal distribution
Exercise D, Question 6

Question:

The random variable $Y \sim N(\mu, \sigma^2)$.

Given that $P(Y < 25) = 0.10$ and $P(Y > 35) = 0.005$, find the value of μ and the value of σ.

Solution:



$Z = \frac{X-\mu}{\sigma} \Rightarrow 2.5758\sigma = 35 - \mu$

$\underline{-1.2816\sigma = 25 - \mu.}$

Subtract $3.8574\sigma = 10$

$\therefore \sigma = 2.59241 \ldots ..$

$\therefore \mu = 35 - 2.5758 \times 2.59241 \ldots .$

$= 28.322 \ldots$

$\therefore \mu = \underline{28.3}$, $\sigma = \underline{2.59}$ (3sf)

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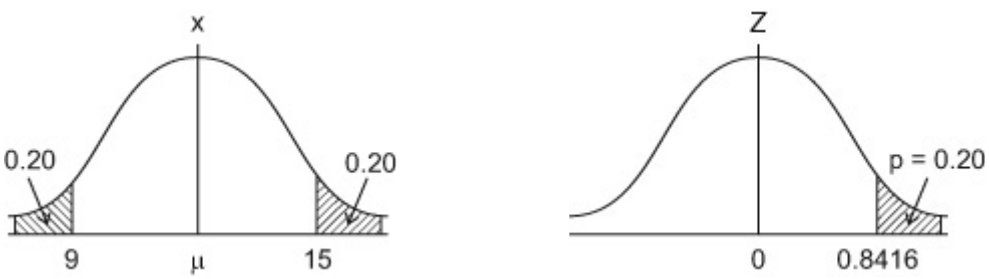
Normal distribution
Exercise D, Question 7

Question:

The random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X > 15) = 0.20$ and $P(X < 9) = 0.20$, find the value of μ and the value of σ.

Solution:



By symmetry $\mu = \frac{1}{2}(9 + 15) = \underline{\underline{12}}$

$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad 0.8416 = \frac{15 - \mu}{\sigma} = \frac{3}{\sigma}$$

$$\therefore \quad \sigma = \frac{3}{0.8416}$$

$$\sigma = \underline{\underline{3.56}}$$

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Normal distribution
Exercise D, Question 8

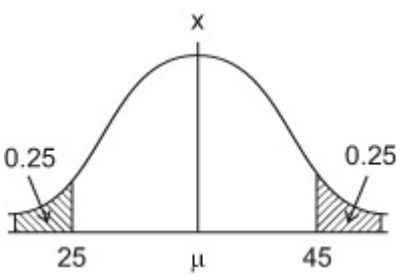
Question:

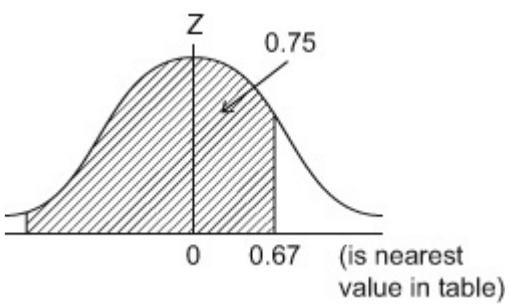
The random variable $X \sim N(\mu, \sigma^2)$.

The lower quartile of X is 25 and the upper quartile of X is 45.

Find the value of μ and the value of σ.

Solution:





By Symmetry $\mu = \frac{1}{2}(25 + 45) = \underline{\underline{35}}$

$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad 0.67 = \frac{45 - 35}{\sigma}$$

$$\therefore \quad \sigma = \frac{10}{0.67}$$

$$\sigma = 14.92 \quad (\text{or } 14.826\ldots \text{ on Calculator})$$

$\therefore$ accept $\sigma = \underline{\underline{14.9 \text{ or } 14.8}}$

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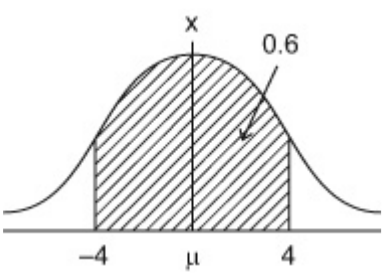
Normal distribution
Exercise D, Question 9

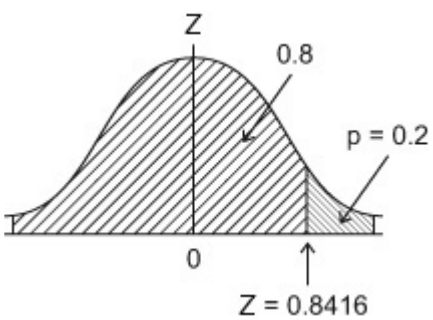
Question:

The random variable $X \sim N(0, \sigma^2)$.

Given that $P(-4 < X < 4) = 0.6$, find the value of σ.

Solution:





By Symmetry $\mu = \underline{0}$

$$Z = \frac{X-\mu}{\sigma} \quad \Rightarrow \quad 0.8416 = \frac{4}{\sigma}$$

$$\therefore \quad \sigma = \frac{4}{0.8416}$$

$$\sigma = \underline{\underline{4.75}} \quad 3\text{sf}$$

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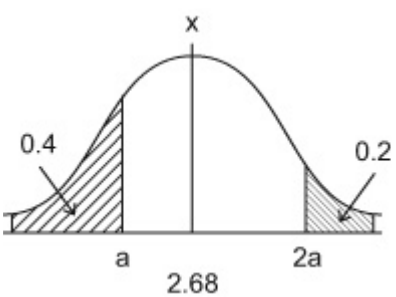
Normal distribution
Exercise D, Question 10

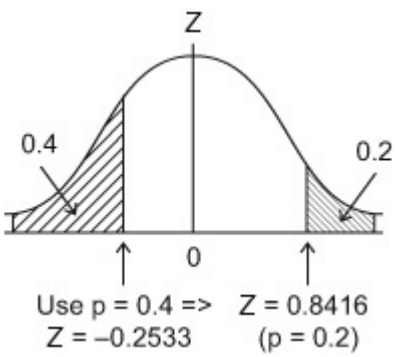
Question:

The random variable $X \sim N(2.68, \sigma^2)$.

Given that $P(X > 2a) = 0.2$ and $P(X < a) = 0.4$, find the value of σ and the value of a.

Solution:





$Z = \frac{X-\mu}{\sigma} \Rightarrow \quad 0.8416\sigma = 2a - 2.68$

$\underline{-0.2533\sigma = a - 2.68} \times 2$

$-0.5066\sigma = 2a - 5.36$

$\underline{0.8416\sigma = 2a - 2.68}$

Subtract : $1.3482\sigma = 2.68$

$\sigma = 1.9878\ldots$

$\sigma = \underline{1.99}$ 3sf

$a = 2.68 - 0.2533\sigma$

$\therefore \quad a = 2.176\ldots$

$a = \underline{\underline{2.18}}$ 3sf

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Normal distribution
Exercise E, Question 1

Question:

The heights of a large group of men are normally distributed with a mean of 178 cm and a standard deviation of 4 cm.

A man is selected at random from this group.

a Find the probability that he is taller than 185 cm.

A manufacturer of door frames wants to ensure that fewer than 0.005 men have to stoop to pass through the frame.

b On the basis of this group, find the minimum height of a door frame.

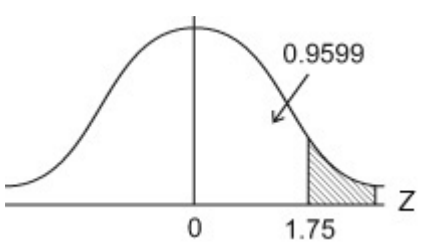
Solution:

$M \sim N(178, 4^2)$

(a)

$$P(M > 185) = P\left(Z > \frac{185 - 178}{4}\right)$$

$$= P(Z > 1.75)$$
$$= 1 - 0.9599$$
$$= \underline{\underline{0.0401}}$$

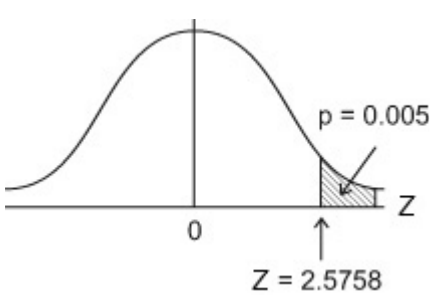


(b)

$$P(M > h) = 0.005$$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow \quad 2.5758 = \frac{h - 178}{4}$$

$$\therefore \quad h = 188.3032$$
$$\therefore \quad h = \underline{\underline{188\text{cm}}}$$



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Normal distribution
Exercise E, Question 2

Question:

The weights of steel sheets produced by a factory are known to be normally distributed with mean 32.5 kg and standard deviation 2.2 kg.

a Find the percentage of sheets that weigh less than 30 kg.

Bob requires sheets that weigh between 31.6 kg and 34.8 kg.

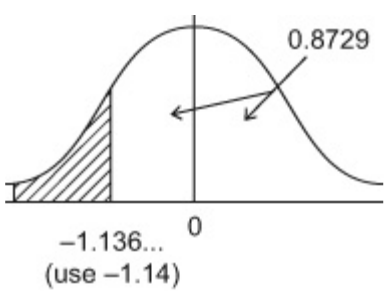
b Find the percentage of sheets produced that satisfy Bob's requirements.

Solution:

$W \sim N(32.5, 2.2^2)$

(a)

$$P(W < 30) = P\left(Z < \frac{30 - 32.5}{2.2}\right)$$
$$= P(Z < -1.14)$$
$$= 1 - 0.8729$$
$$= 0.1271 \quad \underline{12.7\% \text{ or } 12.8\%}$$

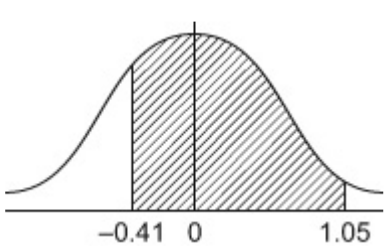


(Calculator gives 0.1279.. so allow <u>AWRT (0.127 - 0.128)</u>)

(b)

$$P(31.6 < W < 34.8)$$

$$= P\left(\frac{31.6 - 32.5}{2.2} < Z < \frac{34.8 - 32.5}{2.2}\right)$$
$$= P(-0.41 < Z < 1.05)$$
$$= 0.8531 - (1 - 0.6591)$$
$$= 0.5122$$



(Calculator gives 0.510856 … So allow AWRT <u>0.511 or 0.512</u>)

So 51.1% or 51.2% of sheets satisfy Bob's requirements

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Normal distribution
Exercise E, Question 3

Question:

The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

a Find the probability that a battery will last for more than 60 hours.

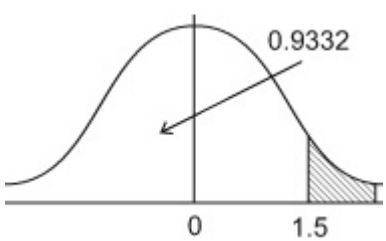
b Find the probability that the battery lasts less than 35 hours.

Solution:

$T \sim N(48, 8^2)$

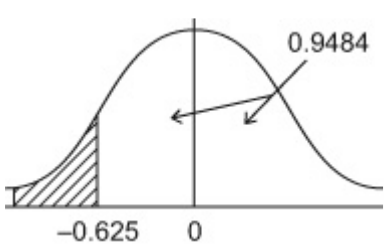
(a)

$$\begin{aligned} P(T > 60) &= P\left(Z > \frac{60-48}{8}\right) \\ &= P(Z > 1.5) \\ &= 1 - 0.9332 \\ &= \underline{\underline{0.0668}} \end{aligned}$$



(b)

$$\begin{aligned} P(T < 35) &= P\left(Z < \frac{35-48}{8}\right) \\ &= P(Z < -1.63) \\ &= 1 - 0.9484 \\ &= 0.0516 \end{aligned}$$



(Calculator gives 0.05208… so allow AWRT <u>0.052</u>)

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Normal distribution
Exercise E, Question 4

Question:

The random variable $X \sim N(24, \sigma^2)$.

Given that $P(X > 30) = 0.05$, find

a the value of σ,

b $P(X < 20)$,

c the value of d so that $P(X > d) = 0.01$.

Solution:

$X \sim N(24, \sigma^2)$

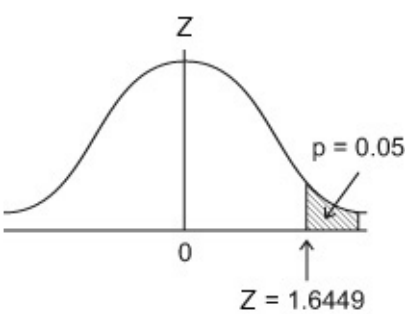
(a)

$P(X > 30) = 0.05$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow \quad 1.6449 = \frac{30-24}{\sigma}$$

$$\therefore \quad \sigma = \frac{6}{1.6449} = 3.6476\ldots$$

$$\therefore \quad \sigma = \underline{3.65 (3\ sf)}$$



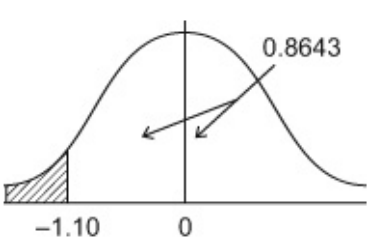
(b)

$$P(X < 20) = P\left(Z < \frac{20-24}{\sigma}\right)$$

$$= P(Z < -1.10)$$

$$= 1 - 0.8643$$

$$= \underline{\underline{0.1357}}$$



(Calculator gives 0.1364… so allow AWRT $\underline{0.136}$)

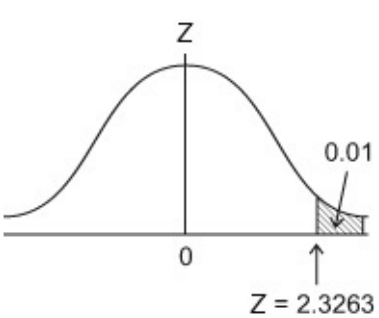
(c)

$P(X > d) = 0.01$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow 2.3263 = \frac{d-24}{\sigma}$$

$$\therefore \quad d = 32.485\ldots.$$

$$d = \underline{\underline{32.5}} \quad (3sf)$$



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Normal distribution
Exercise E, Question 5

Question:

A machine dispenses liquid into plastic cups in such a way that the volume of liquid dispensed is normally distributed with a mean of 120 ml. The cups have a capacity of 140 ml and the probability that the machine dispenses too much liquid so that the cup overflows is 0.01.

a Find the standard deviation of the volume of liquid dispensed.

b Find the probability that the machine dispenses less than 110 ml.

Ten percent of customers complain that the machine has not dispensed enough liquid.

c Find the largest volume of liquid that will lead to a complaint.

Solution:

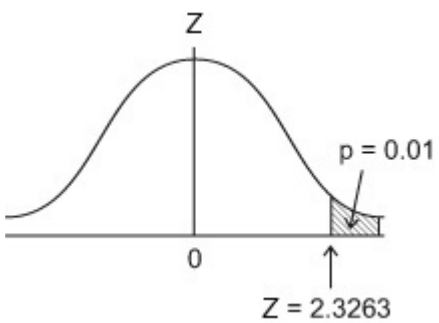
$L \sim N(120, \sigma^2)$

(a)

$$P(L > 140) = 0.01$$

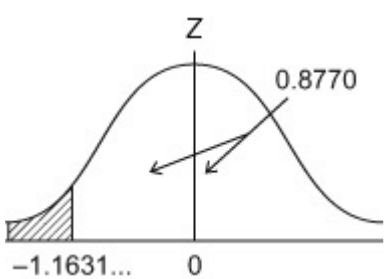
$$Z = \frac{X-\mu}{\sigma} \Rightarrow 2.3263 = \frac{140-120}{\sigma}$$

$$\therefore \quad \sigma = 8.5973\ldots = \underline{\underline{8.60 \text{ ml}}}$$



(b)

$$P(L < 110) = P\left(Z < \frac{110-120}{\sigma}\right)$$
$$= P(Z < -1.16)$$
$$= 1 - 0.8770$$
$$= 0.123$$



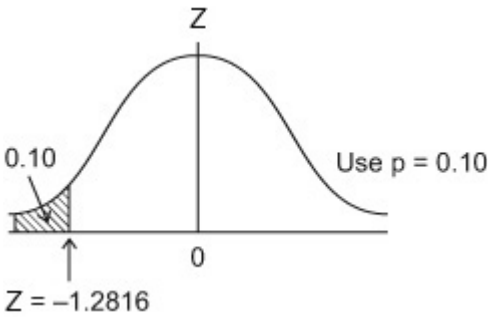
(Calculator gives 0.12238… so allow AWRT <u>0.122 or 0.123</u>)

(c)

$$P(L<c) = 0.10$$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow -1.2816 = \frac{c-120}{\sigma}$$

$$\therefore \quad c = 108.98\ldots$$
$$= \underline{\underline{109 \text{ ml}}} \quad (3sf)$$



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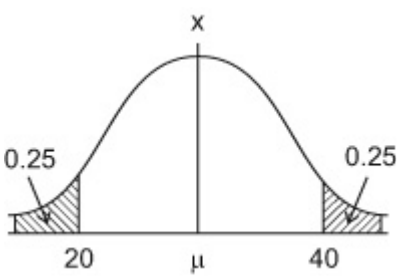
Normal distribution
Exercise E, Question 6

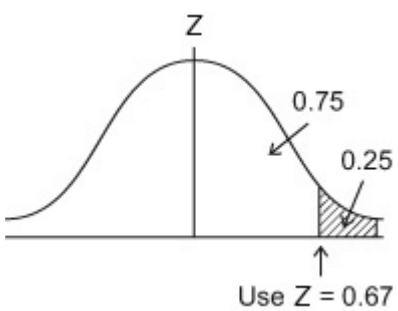
Question:

The random variable $X \sim N(\mu, \sigma^2)$. The lower quartile of X is 20 and the upper quartile is 40. Find μ and σ.

Solution:

$X \sim N(\mu, \sigma^2)$





By symmetry $\mu = \frac{1}{2}(20+40) = \underline{\underline{30}}$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow \frac{40-30}{\sigma} = 0.67$$

$$\therefore \quad \sigma = 14.925\ldots..$$

(Calculator gives 14.82… so allow AWRT 14.8 or 14.9)

Solutionbank S1
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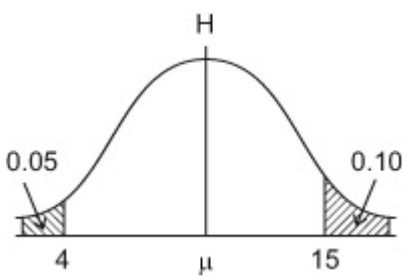
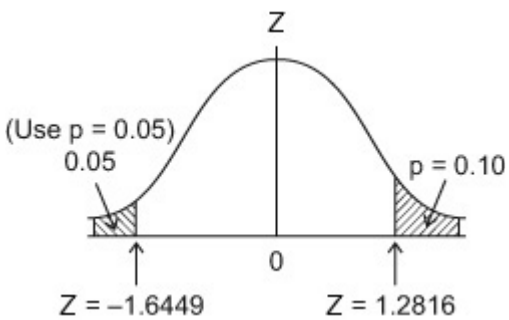
Normal distribution
Exercise E, Question 7

Question:

The heights of seedlings are normally distributed. Given that 10% of the seedlings are taller than 15 cm and 5% are shorter than 4 cm, find the mean and standard deviation of the heights.

Solution:





$Z\sigma = X - \mu \Rightarrow$ $\quad -1.6449\ \sigma = 4 - \mu$

$\quad 1.2816\ \sigma = 15 - \mu$

Subtract $\quad 2.9265\ \sigma = 11$

$\therefore \quad \sigma = 3.7587\ldots = \underline{3.76}$ cm (3sf)

$\mu = 15 - 1.2816\ \sigma$

$\therefore \quad \mu = 10.2$ cm (3sf)

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Normal distribution
Exercise E, Question 8

Question:

A psychologist gives a student two different tests. The first test has a mean of 80 and a standard deviation of 10 and the student scored 85.

a Find the probability of scoring 85 or more on the first test.

The second test has a mean of 100 and a standard deviation of 15. The student scored 105 on the second test.

b Find the probability of a score of 105 or more on the second test.

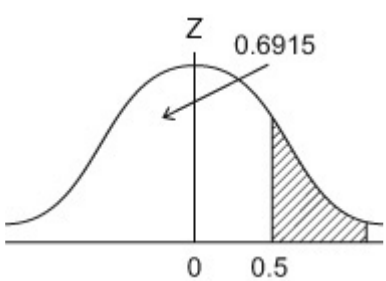
c State, giving a reason, which of the student's two test scores was better.

Solution:

$T \sim N(80, 10^2)$

(a)

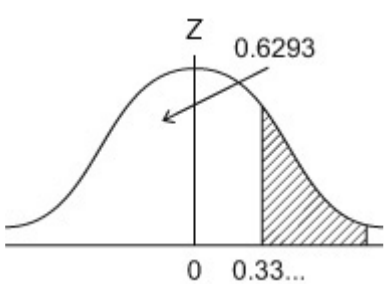
$$P(T > 85) = P\left(Z > \frac{85-80}{10}\right)$$
$$= P(Z > 0.5)$$
$$= 1 - 0.6915$$
$$= \underline{\underline{0.3085}}$$



(b)

$S \sim N(100, 15^2)$

$$P(S > 105) = P\left(Z > \frac{105-100}{15}\right)$$
$$= P(Z > 0.33)$$
$$= 1 - 0.6293$$
$$= 0.3707$$



(Calculator gives 0.36944… so allow 0.369, 0.370 or 0.371)

(c) 1st score is best since a lower proportion of scores will beat it. (or Z value of 1st test in higher so this is the better result)

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Normal distribution
Exercise E, Question 9

Question:

Jam is sold in jars and the mean weight of the contents is 108 grams. Only 3% of jars have contents weighing less than 100 grams. Assuming that the weight of jam in a jar is normally distributed find

a the standard deviation of the weight of jam in a jar,

b the proportion of jars where the contents weigh more than 115 grams.

Solution:

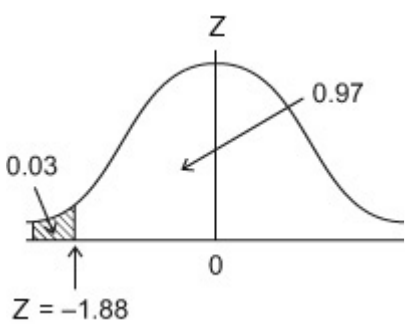
J~ N(108, σ^2)

(a)

$$\text{P}(J < 100) = 0.03$$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow -1.88 = \frac{100-108}{\sigma}$$

$$\therefore \quad \sigma = 4.255\ldots..$$



(Calculator gives 4.2535… so allow AWRT 4.25–4.26)

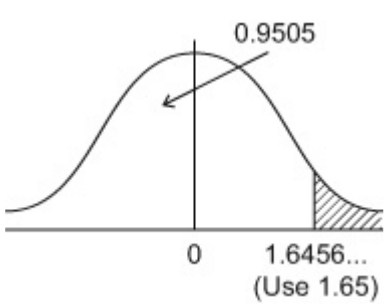
(b)

$$\text{P}(J > 115) = \text{P}\left(Z > \frac{115-108}{\sigma}\right)$$

$$= \text{P}(Z > 1.65)$$

$$= 1 - 0.9505$$

$$= 0.0495$$



(Calculator gives: 0.0499… so allow AWRT 0.050)

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Normal distribution
Exercise E, Question 10

Question:

The waiting time at a doctor's surgery is assumed to be normally distributed with standard deviation of 3.8 minutes. Given that the probability of waiting more than 15 minutes is 0.0446, find

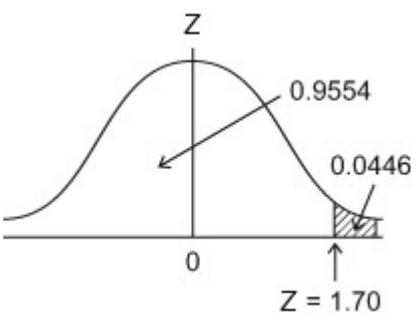
a the mean waiting time,

b the probability of waiting fewer than 5 minutes.

Solution:

$T \sim N(\mu, 3.8^2)$

$P(T > 15) = 0.0446$



(a) $Z = \frac{X-\mu}{\sigma} \Rightarrow 1.70 = \frac{15-\mu}{3.8}$

$\therefore \mu = 15 - 3.8 \times 1.70$

$\mu = \underline{8.54}$ (3sf) minutes

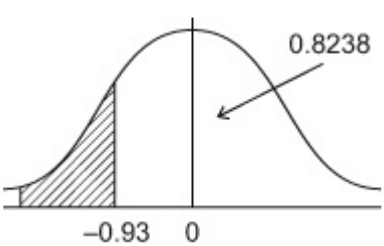
(b)

$P(T < 5) = P\left(Z < \frac{5-8.54}{3.8}\right)$

$= P(Z < -0.93\ldots)$

$= 1 - 0.8238$

$= \underline{\underline{0.1762}}$



(Calculater gives 0.17577… so allow AWRT $\underline{0.176}$)

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Normal distribution
Exercise E, Question 11

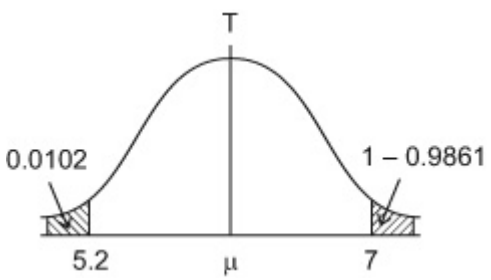
Question:

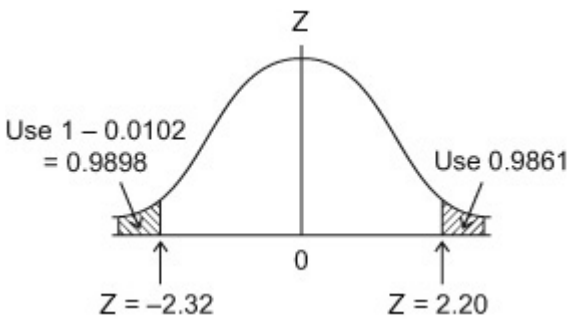
The thickness of some plastic shelving produced by a factory is normally distributed. As part of the production process the shelving is tested with two gauges. The first gauge is 7 mm thick and 98.61% of the shelving passes through this gauge. The second gauge is 5.2 mm thick and only 1.02% of the shelves pass through this gauge.

Find the mean and standard deviation of the thickness of the shelving.

Solution:

$T \sim N(\mu, \sigma^2)$





$Z\sigma = X - \mu \Rightarrow 2.20\ \sigma = 7 - \mu$
$-2.32\ \sigma = 5.2 - \mu$

Subtract $4.52\ \sigma = 1.8$
$\sigma = 0.3982\ldots.$

$\mu = 7 - 2.20\ \sigma \Rightarrow \mu = 6.1238\ldots.$
$\therefore \mu = 6.12$ mm, $\sigma = 0.398$ mm

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Normal distribution
Exercise E, Question 12

Question:

The random variable $X \sim N(14, 9)$. Find

a $P(X \geq 11)$,

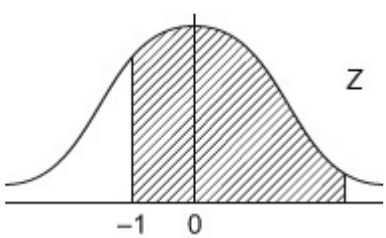
b $P(9 < X < 11)$.

Solution:

$X \sim N(14, 3^2)$

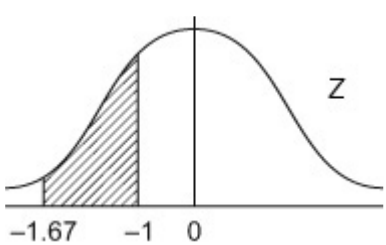
(a)

$$P(X \geq 11) = P\left(Z \geq \frac{11-14}{3}\right)$$
$$= P(Z \geq -1)$$
$$= \underline{\underline{0.8413}}$$



(b)

$$P(9 < X < 11) = P\left(\frac{9-14}{3} < Z < \frac{11-14}{3}\right)$$
$$= P(-1.67 < Z < -1)$$
$$= 0.9525 - 0.8413$$
$$= 0.1112$$



(Calculater gives: 0.11086 … so allow AWRT $\underline{0.111}$)

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Normal distribution
Exercise E, Question 13

Question:

The random variable $X \sim N(20, 5^2)$. Find

a $P(X \leq 16)$,

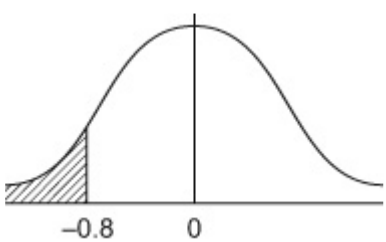
b the value of d such that $P(X < d) = 0.95$.

Solution:

$X \sim N(20, 5^2)$

(a)

$$\begin{aligned} P(X \leq 16) &= P\left(Z < \frac{16-20}{5}\right) \\ &= P(Z < -0.8) \\ &= 1 - 0.7881 \\ &= \underline{\underline{0.2119}} \end{aligned}$$



(b)

$$P(X < d) = 0.95$$

$$Z = \frac{X-\mu}{\sigma} \Rightarrow 1.6449 = \frac{d-20}{5}$$

$$\therefore d = 20 + 5 \times 1.6449$$

$$d = 28.2$$