Solutionbank M11
Edexcel AS and A Level Modular Mathematics

Statics of a particle
Exercise A, Question 1

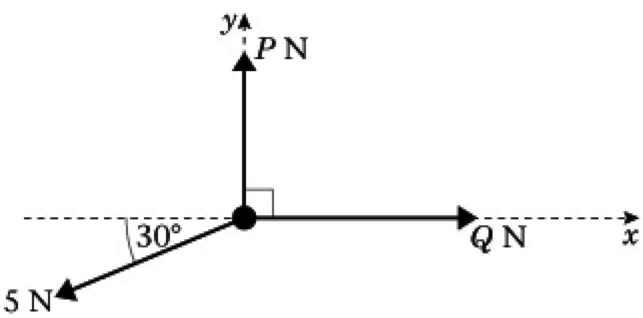
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $Q - 5\cos 30° = 0$
b $P - 5\sin 30° = 0$
c $Q = 5\cos 30° = \frac{5\sqrt{3}}{2} = 4.33 \text{ N } (3 \text{ s.f.})$

$P = 5\sin 30° = 2.5 \text{ N}$

Give exact answers using $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$ or give decimal answers using your calculator.

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Statics of a particle
Exercise A, Question 2

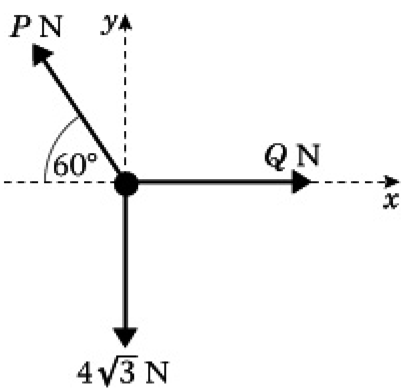
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



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Statics of a particle
Exercise A, Question 3

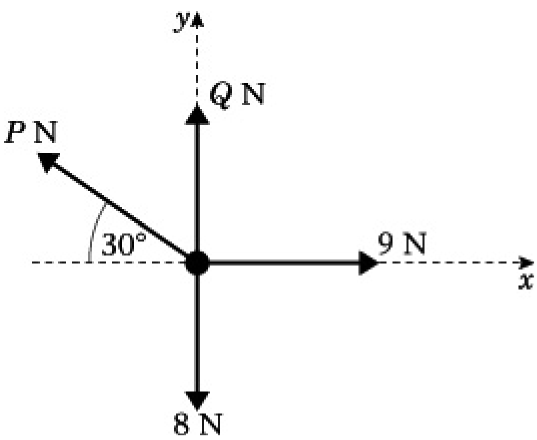
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $9 - P \cos 30° = 0$

b $Q + P \sin 30° - 8 = 0$

c From part **a**,

$$P = \frac{9}{\cos 30°} = 9 \div \frac{\sqrt{3}}{2}$$
$$= 9 \times \frac{2}{\sqrt{3}}$$
$$= \frac{18}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{18\sqrt{3}}{3}$$
$$= 6\sqrt{3}$$
$$= 10.4 \text{ N } (3 \text{ s.f.})$$

Use part **a** to find P, then substitute into **b** to find **a**.

$\cos 30° = \frac{\sqrt{3}}{2}$

Substitute into part **b**

$$Q + 6\sqrt{3} \sin 30° - 8 = 0$$

$$\boxed{\sin 30° = \frac{1}{2}}$$

$$\therefore Q = 8 - 6\sqrt{3} \times \frac{1}{2}$$
$$= 8 - 3\sqrt{3}$$
$$= 2.80 \text{ N} \ \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 4

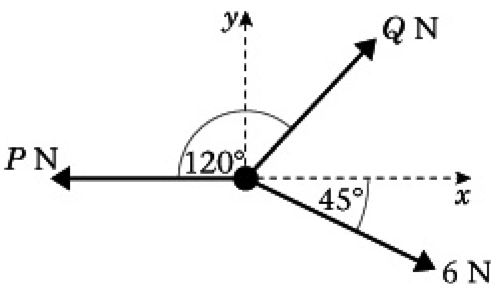
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $Q \cos 60° + 6 \cos 45° - P = 0$
b $Q \sin 60° - 6\sin 45° = 0$
c use part **b** to give

Use angles on **a** straight line to find *Q* makes on angle of 60° with the *x*-axis.

$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 60° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$$Q = \frac{6\sin 45°}{\sin 60°}$$
$$= 6 \times \frac{1}{\sqrt{2}} \div \frac{\sqrt{3}}{2}$$
$$= 6 \times \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$$
$$= \frac{12}{\sqrt{6}}$$
$$= \frac{12\sqrt{6}}{\sqrt{6}\sqrt{6}}$$
$$= 2\sqrt{6} = 4.90 \text{ N (3 s.f.)}$$

Substitute into part **a** to give:

$$2\sqrt{6} \times \frac{1}{2} + 6 \times \frac{1}{\sqrt{2}} - P = 0$$

$$\therefore P = \sqrt{6} + \frac{6}{\sqrt{2}}$$

$$= \sqrt{6} + \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$= \sqrt{6} + 3\sqrt{2}$$

$$= 6.69 \text{ N} \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 5

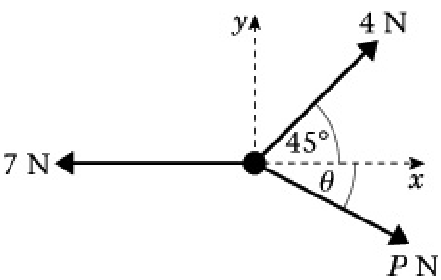
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $4 \cos 45° + P \cos \theta - 7 = 0$

b $4 \sin 45° - P \sin \theta = 0$

c $P \cos \theta = 7 - 4 \cos 45°$ (from **a**) (1)

$P \sin \theta = 4 \sin 45°$ (from **b**) (2)

Divide equation (2) by equation (1)
Then

$$\frac{P \sin \theta}{P \cos \theta} = \frac{4 \sin 45°}{7 - 4 \cos 45°}$$

$$\therefore \tan \theta = \frac{2.828}{4.172}$$

$$= 0.678$$

$$\therefore \theta = 34.1° \quad (3 \text{ s.f.})$$

Use $\frac{P \sin \theta}{P \cos \theta} = \tan \theta$ to eliminate *P* from the equations after resolving.

Substitute this value for *θ* into equation (2)

Then

$$P = \frac{2.828}{\sin 34.1°}$$

$$P = 5.04 \text{ N} \quad (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 6

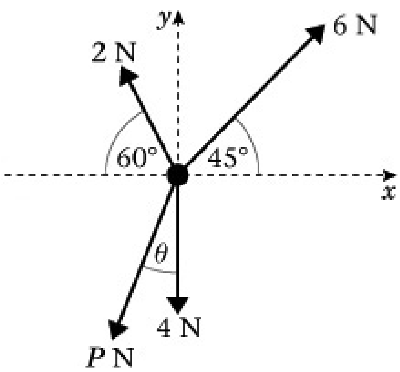
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $6\cos 45° - 2\cos 60° - P\sin\theta = 0$

b $6\sin 45° + 2\sin 60° - P\cos\theta - 4 = 0$

c $P\sin\theta = 6\cos 45° - 2\cos 60°$ (1) [from **a**]

$P\cos\theta = 6\sin 45° + 2\sin 60° - 4$ (2) [from **b**]

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{6\cos 45° - 2\cos 60°}{6\sin 45° + 2\sin 60° - 4}$$

$$\therefore \tan\theta = \frac{3.24264}{1.97469..}$$

$$= 1.642$$

$$\therefore \theta = 58.7° \quad (3 \text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations after resolving.

Substitute θ back into equation (1)

$$P\sin\theta = 6\cos 45° - 2\cos 60°$$

$$\therefore P = \frac{3.24264}{\sin 58.65°}$$

$$P = 3.80 \text{ N} \quad (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 7

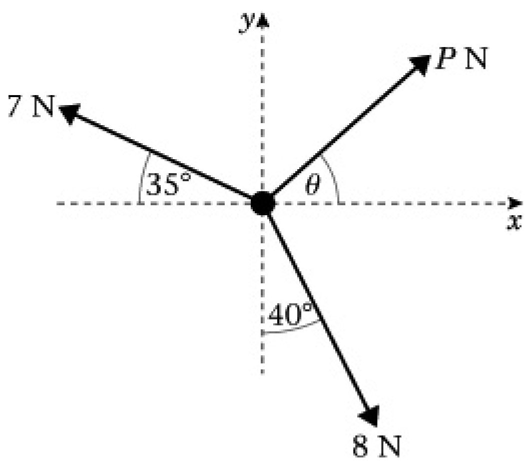
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$

b $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$

c From **a** $P\cos\theta = 7\cos 35° - 8\sin 40° = 0.5918$ (1)

From **b** $P\sin\theta = 8\cos 40° - 7\sin 35° = 2.113$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 40° - 7\sin 35°}{7\cos 35° - 8\sin 40°}$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate *P* from the equations obtained in **a** and **b**.

$$\therefore \tan\theta = \frac{2.113}{0.5918}$$

$$= 3.57$$

$$\therefore \theta = 74.4° \quad (\text{allow } 74.3°) \quad (3 \text{ s.f.})$$

Substitute θ into equation (1)

$\therefore P \cos 74.4° = 0.5918$

$\therefore P = \frac{0.5918}{\cos 74.3569°}$

$= 2.19 \ (3 \text{ s.f.})$

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Statics of a particle
Exercise A, Question 8

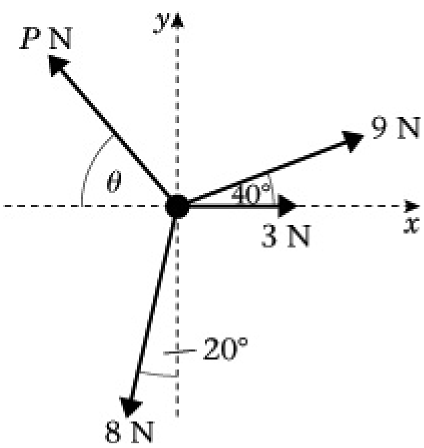
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $9\cos 40° + 3 - P\cos\theta - 8\sin 20° = 0$

b $P\sin\theta + 9\sin 40° - 8\cos 20° = 0$

c From **a**: $P\cos\theta = 9\cos 40° + 3 - 8\sin 20°$ (1)

From **b**: $P\sin\theta = 8\cos 20° - 9\sin 40°$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 20° - 9\sin 40°}{9\cos 40° + 3 - 8\sin 20°}$$

$$\therefore \tan\theta = \frac{1.732}{7.158}$$

$$= 0.242$$

$$\therefore \theta = 13.6° \quad (3 \text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate *P* from the equations obtained in parts **a** and **b**.

Substitute into equation (2)

$$P\cos 13.6° = 9\cos 40° + 3 - 8\sin 20°$$
$$= 7.158$$
$$\therefore P = \frac{7.158}{\cos 13.6°}$$
$$= 7.36 \ (3 \text{ s.f.})$$

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Statics of a particle
Exercise A, Question 9

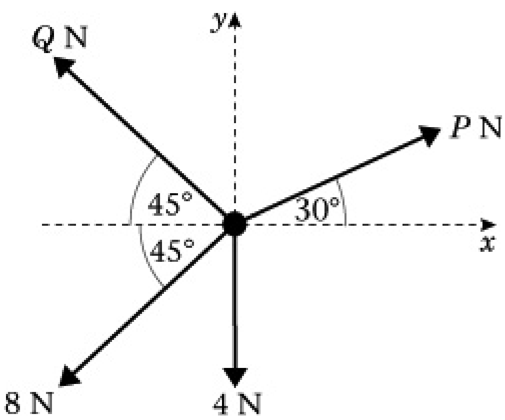
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $P \cos 30° - Q \cos 45° - 8 \cos 45° = 0$

b $P \sin 30° + Q \sin 45° - 8 \sin 45° - 4 = 0$

c From **a** $P \frac{\sqrt{3}}{2} - \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}}$ (1)

From **b** $\frac{P}{2} + \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} + 4$ (2)

You will have two equations in two unknown forces *P* and *Q*, so should use simultaneous equations to solve them. Give your answers to 3 s.f.

$\cos 30° = \frac{\sqrt{3}}{2} \cos 45° = \frac{1}{\sqrt{2}} \sin 45° = \frac{1}{\sqrt{2}}$

These are simultaneous equations.
Add (1) and (2)

$$\frac{P\sqrt{3}}{2} + \frac{P}{2} - \frac{Q}{\sqrt{2}} + \frac{Q}{\sqrt{2}} = \frac{8}{\sqrt{2}} + \frac{8}{\sqrt{2}} + 4$$

$$\therefore P \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) + 0 = \frac{16}{\sqrt{2}} + 4$$

$$\therefore P = \left(\frac{16\sqrt{2}}{\sqrt{2}\sqrt{2}} + 4 \right) \div \left(\frac{\sqrt{3} + 1}{2} \right)$$

$$= \left(8\sqrt{2} + 4 \right) \times \frac{2}{\sqrt{3} + 1}$$

$$= 8 \frac{\left(2\sqrt{2} + 1 \right)}{\sqrt{3} + 1} = 11.21 = 11.2 \ (3 \text{ s.f.})$$

Substitute into equation (2)

$Q = 8 + 4\sqrt{2} - \frac{P}{2}\sqrt{2} = 5.73$ (3 s.f.) .

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Statics of a particle
Exercise A, Question 10

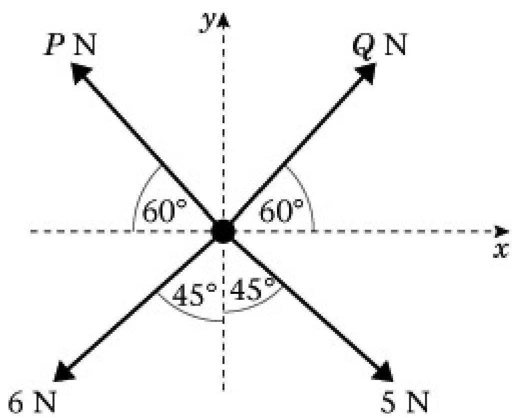
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $Q\cos 60° - P\cos 60° + 5\sin 45° - 6\sin 45° = 0$

b $P\sin 60° + Q\sin 60° - 5\cos 45° - 6\cos 45° = 0$

c From **a** $\frac{Q}{2} - \frac{P}{2} = \frac{6}{\sqrt{2}} - \frac{5}{\sqrt{2}}$

$\therefore Q - P = \sqrt{2}$ (1)

From **b** $\frac{P\sqrt{3}}{2} + \frac{Q\sqrt{3}}{2} = \frac{5}{\sqrt{2}} + \frac{6}{\sqrt{2}}$

$\therefore P\sqrt{3} + Q\sqrt{3} = 11\sqrt{2}$ (2)

Use simultaneous equations to solve the equations in parts **a** and **b**.

$\cos 60° = \frac{1}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$
$\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$

Multiply equation (1) by $\sqrt{3}$ and add to equation (2).

$$\therefore 2Q\sqrt{3} = \sqrt{3}\sqrt{2} + 11\sqrt{2}$$

$$\therefore Q = \frac{\sqrt{2}}{2} + \frac{11\sqrt{2}}{2\sqrt{3}}$$

$$= 5.198 \text{ N } (4 \text{ s.f.})$$

Substitute into equation (1)

$\therefore P = Q - \sqrt{2}$

$= 3.784 \text{ N} \; (4 \text{ s.f.})$

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Statics of a particle
Exercise A, Question 11

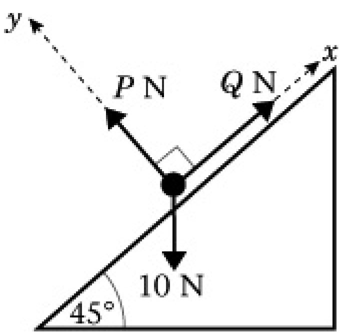
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a R (↗)
$Q - 10\sin 45° = 0$

Resolve along the plane and perpendicular to the plane.

b R (↖)
$P - 10\cos 45° = 0$

c From **b**,

$P = 10\cos 45°$
$= 5\sqrt{2}$
$= 7.07$ N (3 s.f.)

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

From **a**,

$Q = 10\sin 45°$
$= 5\sqrt{2}$
$= 7.07$ N (3 s.f.)

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

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Statics of a particle
Exercise A, Question 12

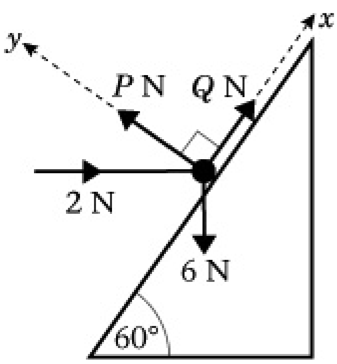
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the x direction,

b resolve in the y direction,

c find the magnitude of any unknown forces (marked P and Q) and the size of any unknown angles (marked θ).



Solution:

a $Q + 2\cos 60° - 6\sin 60° = 0$

b $P - 2\sin 60° - 6\cos 60° = 0$

c From part **b**

$$P = 2\sin 60° + 6\cos 60°$$
$$P = 4.73 \text{ (3 s.f.)}$$

From part **a**

$$Q = 6\sin 60° - 2\cos 60°$$
$$Q = 4.20 \text{ (3 s.f.)}$$

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Statics of a particle
Exercise A, Question 13

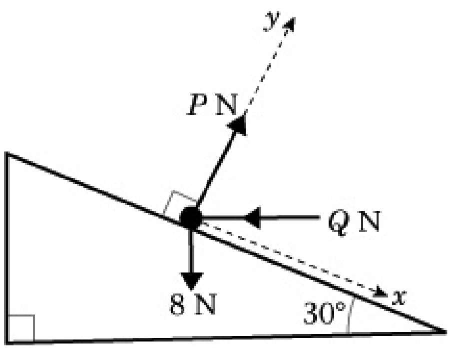
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $8\sin 30° - Q\cos 30° = 0$
b $P - Q\sin 30° - 8\cos 30° = 0.$

Find Q from the equation written down in part **a**, then substitute into the equation obtained in part **b** to find P.

c From **a**

$Q = \frac{8\sin 30°}{\cos 30°}$

$= 8\tan 30°$

$= \frac{8\sqrt{3}}{3}$

$= 4.62$ N (3 s.f.)

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Substitute into **b**

$P = Q\sin 30° + 8\cos 30°$

$= \frac{8\sqrt{3}}{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2}$

$= \frac{4\sqrt{3}}{3} + 4\sqrt{3}$

$= \frac{16\sqrt{3}}{3}$

$= 9.24$ N (3 s.f.)

$\sin 30° = \frac{1}{2}\ \cos 30° = \frac{\sqrt{3}}{2}$

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Statics of a particle
Exercise A, Question 14

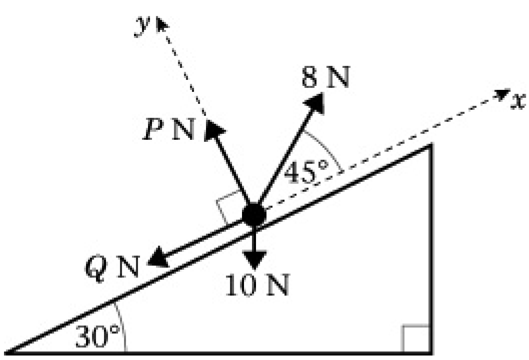
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked θ).



Solution:

a $8\cos 45° - 10\sin 30° - Q = 0$

b $P + 8\sin 45° - 10\cos 30° = 0$

c From part **b**,

$P = 10\cos 30° - 8\sin 45°$
$= 5\sqrt{3} - 4\sqrt{2}$
$= 3.00$ N (3 s.f.)

From part **a**,

$Q = 8\cos 45° - 10\sin 30°$
$= 4\sqrt{2} - 5$
$= 0.657$ N (3 s.f.)

You may give your answers as exact answers using surds as $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$ and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ or you may give answers to 3 significant figures, using a calculator.

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Statics of a particle
Exercise A, Question 15

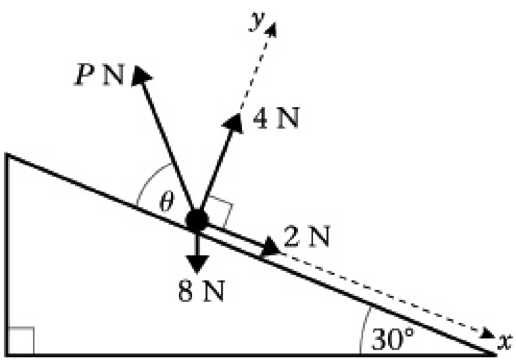
Question:

The diagram shows a particle in equilibrium under the action of three or more forces. Using the information given in the diagram,

a resolve in the *x* direction,

b resolve in the *y* direction,

c find the magnitude of any unknown forces (marked *P* and *Q*) and the size of any unknown angles (marked *θ*).



Solution:

a $2 + 8\sin 30° - P\cos\theta = 0$

b $4 - 8\cos 30° + P\sin\theta = 0$

c From **a** $P\cos\theta = 2 + 8\sin 30°$ (1)

From **b** $P\sin\theta = 8\cos 30° - 4$ (2)

Divide equation (2) by equation (1)

$$\therefore \frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 30° - 4}{2 + 8\sin 30°}$$

$$\text{i.e.} \tan\theta = \frac{4\sqrt{3} - 4}{6}$$

$$= 0.488$$

$$\therefore \theta = 26.0° \ (3 \text{ s.f.})$$

Substitute into equation (1)

$$P\cos 26.0° = 2 + 8\sin 30°$$

$$\therefore P = \frac{6}{\cos 26.0°}$$

$$\therefore P = 6.68 \text{ N} \ (3 \text{ s.f.})$$

Eliminate *P* from your equations by using $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$.

$\cos 30° = \frac{\sqrt{3}}{2} \sin 30° = \frac{1}{2}$

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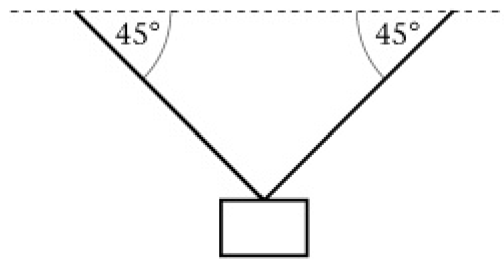
Statics of a particle
Exercise B, Question 1

Question:

A picture of mass 5 kg is suspended by two light inextensible strings, each inclined at 45 ° to the horizontal as shown. By modelling the picture as a particle find the tension in the strings when the system is in equilibrium.



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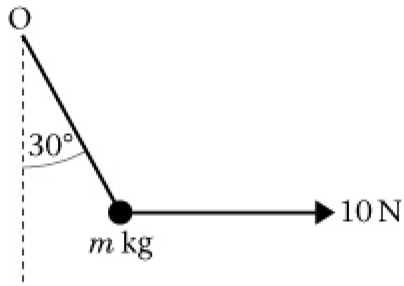
Statics of a particle
Exercise B, Question 2

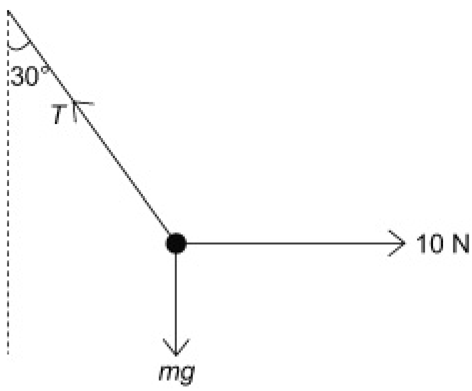
Question:

A particle of mass m kg is suspended by a single light inextensible string. The string is inclined at an angle of $30°$ to the vertical and the other end of the string is attached to a fixed point O. Equilibrium is maintained by a horizontal force of magnitude 10 N which acts on the particle, as shown in the figure.



Find **a** the tension in the string, **b** the value of m.

Solution:



Draw a diagram showing the forces acting on the particle; i.e.: the tension in the string, the weight mg and the force 10 N.

a Let the tension in the string be TN.

The weight is $m g$.

R(←)

$T \sin 30° - 10 = 0$

$$\therefore T = \frac{10}{\sin 30°}$$

$$T = 20 \text{ N}$$

b R(↑)

$T \cos 30° - m g = 0$

As $T = 20$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$mg = 20\cos 30°$$

$$\therefore m = \frac{20\cos 30°}{g}$$

$$= \frac{10\sqrt{3}}{g}$$

$$= 1.8 \ (2 \text{ s.f.})$$

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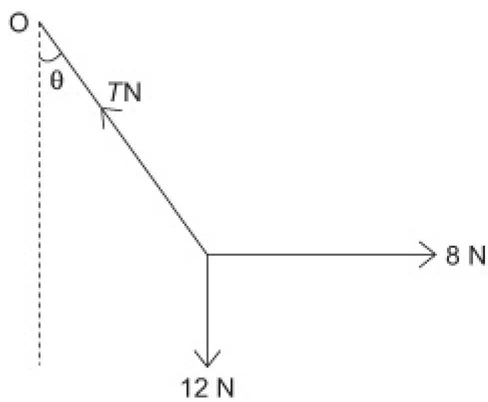
Statics of a particle
Exercise B, Question 3

Question:

A particle of weight 12 N is suspended by a light inextensible string from a fixed point *O*. A horizontal force of 8 N is applied to the particle and the particle remains in equilibrium with the string at an angle θ to the vertical.

Find **a** the angle θ, **b** the tension in the string.

Solution:



Let the tension in the string be *T*N.

>

R(→)

$$8 - T \sin \theta = 0$$
$$\therefore T \sin \theta = 8 \qquad (1)$$

Resolve horizontally and vertically then divide one equation by the other to eliminate the tension *T*.

R(↑)

$$T \cos \theta - 12 = 0$$
$$\therefore T \cos \theta = 12 \qquad (2)$$

Divide equation (1) by equation (2)

$$\frac{T \sin \theta}{T \cos \theta} = \frac{8}{12}$$
$$\therefore \tan \theta = \frac{2}{3}$$
$$\therefore \theta = 33.7° \quad (3 \text{ s.f.})$$

Substitute into equation (1)

$T \sin 33.7° = 8$

$\therefore T = \frac{8}{\sin 33.7°} = 14.4 \text{ (3 s.f.)}$

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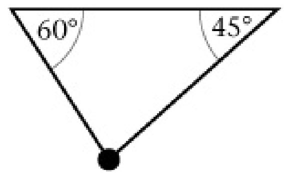
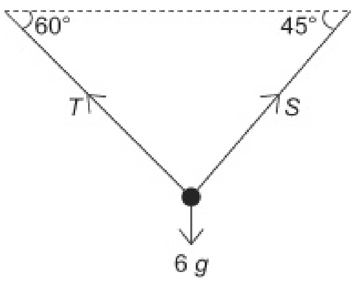
Statics of a particle
Exercise B, Question 4

Question:

A particle of mass 6 kg hangs in equilibrium, suspended by two light inextensible strings, inclined at 60 ° and 45 ° to the horizontal, as shown. Find the tension in each of the strings.



Solution:



Let the tension in the strings be TN and SN as shown in the figure.

R(←)

$T \cos 60° - S \cos 45° = 0$

$\therefore \frac{T}{2} - \frac{S}{\sqrt{2}} = 0$

$\therefore T = S\sqrt{2}$ (1)

Resolve in two directions and obtain simultaneous equations.

R(↑)

$T \sin 60° + S \sin 45° - 6g = 0$ (2)

$\cos 60° = \frac{1}{2} \cos 45° = \frac{1}{\sqrt{2}}$

Substitute $T = S\sqrt{2}$ into equation (2)

$\text{sun } 60° = \frac{\sqrt{3}}{2} \text{sun } 45° = \frac{1}{\sqrt{2}}$

$\therefore S(\sqrt{2} \sin 60° + \sin 45°) = 6g$

$$\begin{aligned}\therefore S &= \frac{6g}{(\sqrt{2} \sin 60° + \sin 45°)} \\ &= \frac{6g\sqrt{2}}{(\sqrt{3}+1)} \\ &= 3g\sqrt{2}(\sqrt{3}-1) \\ &= 30 \ (2 \text{ s.f.})\end{aligned}$$

and $T = 6g(\sqrt{3}-1) = 43 \ (2 \text{ s.f.})$

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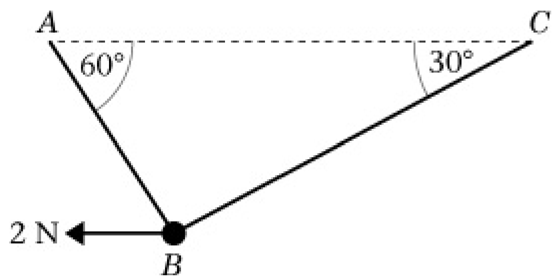
Statics of a particle
Exercise B, Question 5

Question:

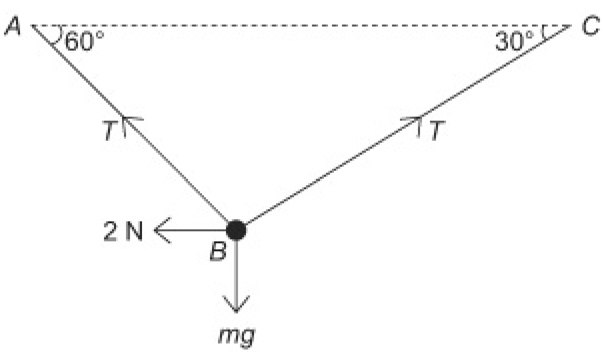
A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 2 N acting parallel to CA. The sections of string make angles of 60 ° and 30 ° with the horizontal.



Find **a** the tension in the string,

b the mass of the bead.

Solution:



a

Let the tension in the string be T and the mass of the bead be m.

The tension is the same throughout the string. Resolve horizantally first to find T.

R(→)

$$T\cos 30° - T\cos 60° - 2 = 0$$

$$\therefore T(\cos 30° - \cos 60°) = 2$$

$$\therefore T = \frac{2}{\cos 30° - \cos 60°}$$

$$= \frac{4}{\sqrt{3}-1}$$

$\cos 30° = \frac{\sqrt{3}}{2} \cos 60° = \frac{1}{2}$

$$= \frac{4(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{4(\sqrt{3}+1)}{2}$$

$$= 2(\sqrt{3}+1) = 5.46 \text{ N } (3 \text{ s.f.})$$

b R(↑)

$T\sin 60° + T\sin 30° - mg = 0$

$\therefore mg = T(\sin 60° + \sin 30°)$

$m = \frac{2}{g}(\sqrt{3}+1)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right) = \frac{4+2\sqrt{3}}{g} = 0.76 \text{ kg} \quad (2 \text{ s.f.})$

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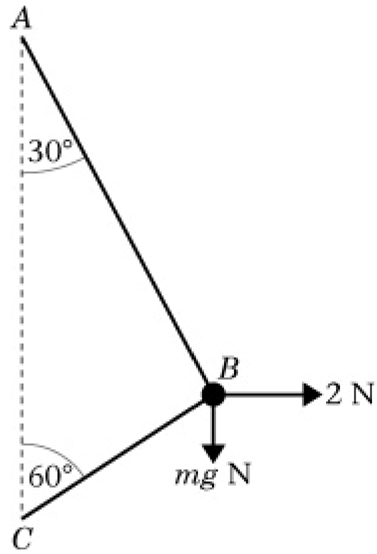
Statics of a particle
Exercise B, Question 6

Question:

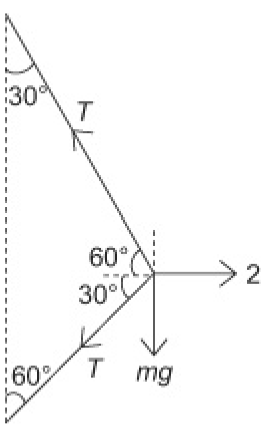
A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force of magnitude 2 N. The sections AB and BC of the string make angles of $30°$ and $60°$ with the vertical respectively.



Find **a** the tension in the string,

b the mass of the bead, giving your answer to the nearest gramme.

Solution:



Let the tension in the string be TN and let the mass of the bead be m kg.

a R(→)

$$2 - T\cos 60° - T\cos 30° = 0$$
$$\therefore T(\cos 60° + \cos 30°) = 2$$

The tension is the same in both sections of the string.
Resolve horizontally first to find T.

$\cos 60° = \frac{1}{2} \cos 30° = \frac{\sqrt{3}}{2}$

$$\therefore T = \frac{2}{\cos 60° + \cos 30°}$$

$$= \frac{4}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= 2(\sqrt{3}-1)$$

$$= 1.46 \text{ (3 s.f.)}$$

b R(↑)

$$T \sin 60° - T \sin 30° - mg = 0$$

$$\boxed{\sin 60° = \frac{\sqrt{3}}{2} \sin 30° = \frac{1}{2}}$$

$$\therefore mg = T(\sin 60° - \sin 30°)$$

$$= 2(\sqrt{3}-1)\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$= (\sqrt{3}-1)^2$$

$$= 4 - 2\sqrt{3}$$

$$m = \frac{(4-2\sqrt{3})}{g} = 0.055 \text{ kg} = 55 \text{ g}$$

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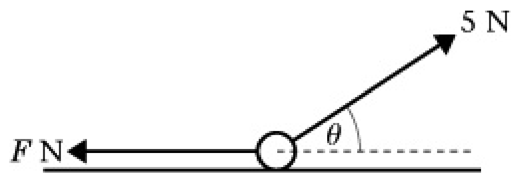
Statics of a particle
Exercise B, Question 7

Question:

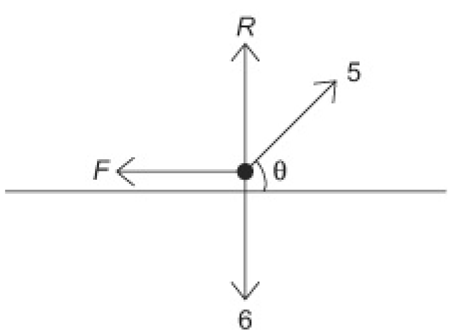
A particle of weight 6 N rests on a smooth horizontal surface. It is acted upon by two external forces as shown in the figure. One of these forces is of magnitude 5 N and acts at an angle θ with the horizontal, where $\tan\theta = \frac{4}{3}$. The other has magnitude F N and acts in a horizontal direction. Find



a the value of F,

b the magnitude of the normal reaction between the particle and the surface.

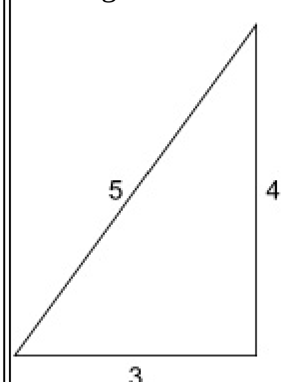
Solution:



Let the normal reaction be RN.

a
R(→)
$5\cos\theta - F = 0$
$\therefore F = 5 \times \frac{3}{5} = 3$ N

The angle θ is defined as satisfying $\tan\theta = \frac{4}{3}$.



From the right-angled triangle shown, $\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$.

b R(↑)

$R + 5 \sin \theta - 6 = 0$

$\therefore R = 6 - 5 \times \frac{4}{5} = 2 \text{ N}$

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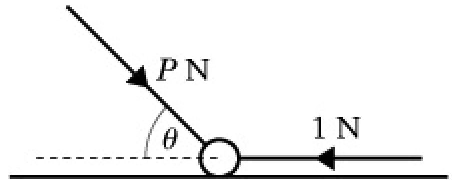
Statics of a particle
Exercise B, Question 8

Question:

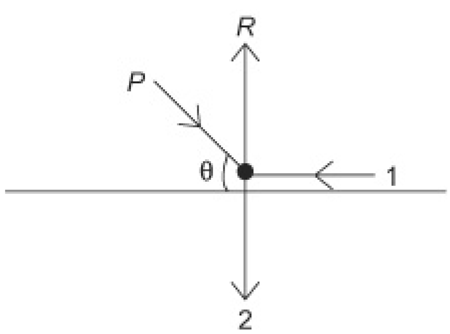
A particle of weight 2 N rests on a smooth horizontal surface and remains in equilibrium under the action of the two external forces shown in the figure. One is a horizontal force of magnitude 1 N and the other is a force *P* N at an angle θ to the horizontal, where $\tan\theta = \frac{12}{5}$. Find



a the magnitude of *P*,

b the normal reaction between the particle and the surface.

Solution:



Let the normal reaction be *R*N.

a

R(→)

$P\cos\theta - 1 = 0$

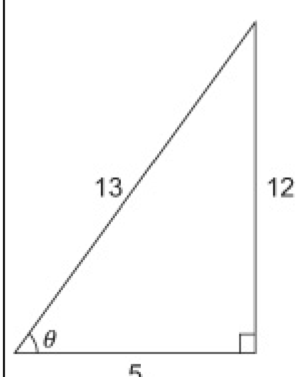
$\therefore P = 1 \div \cos\theta$

$= 1 \div \frac{5}{13}$

$= \frac{13}{5}$

$P = 2.6$

When $\tan\theta = \frac{12}{5}$ we can used the triangle shown to find $\sin\theta = \frac{12}{13}$ and $\cos\theta = \frac{5}{13}$.



b

R(↑)

$R - P \sin \theta - 2 = 0$

$$\therefore R = P \sin \theta + 2$$
$$= 2.6 \times \frac{12}{13} + 2$$
$$= 2.4 + 2$$
$$= 4.4$$

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Statics of a particle
Exercise B, Question 9

Question:

A particle A of mass m kg rests on a smooth horizontal table. The particle is attached by a light inextensible string to another particle B of mass $2m$ kg, which hangs over the edge of the table.

The string passes over a smooth pulley, which is fixed at the edge of the table so that the string is horizontal between A and the pulley and then is vertical between the pulley and B.

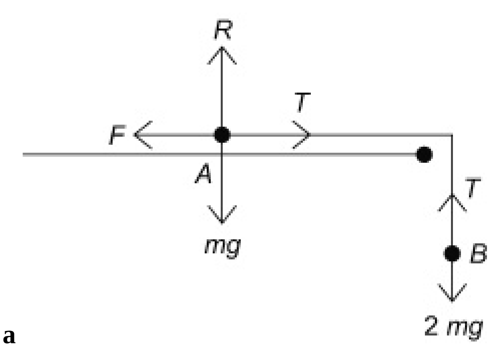
A horizontal force F N applied to A maintains equilibrium. The normal reaction between A and the table is R N.

a Find the magnitudes of F and R in terms of m.

The pulley is now raised to a position above the edge of the table so that the string is inclined at 30 degrees to the horizontal between A and the pulley. Again the string then hangs vertically between the pulley and B. A horizontal force F' N applied to A maintains equilibrium in this new situation. The normal reaction between A and the table is now R' N.

b Find, in terms of m, the magnitudes of F' and R'.

Solution:



a

Consider the mass $2m$ kg.

Consider the mass 2 m kg first, as it has only two forces acting on it. This enables you to find the tension.

R(↑)

$$T - 2m\ g = 0$$
$$\therefore T = 2m\ g.$$

Consider the mass m kg.

R(→)

$$T - F = 0$$
$$\therefore F = T = 2m\ g$$
$$= 19.6m\ \text{(accept } 20m\text{)}$$

R(↑)

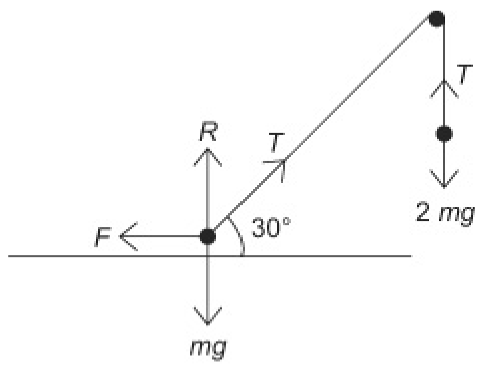
$R - mg = 0$

$\therefore R = mg$

$= 9.8m$

b

Consider the $2m$ kg mass.
Again $T = 2mg$.
Consider the mass m kg.



R(→)

$T \cos 30° - F = 0$

$\therefore F = 2mg \times \frac{\sqrt{3}}{2} = \sqrt{3}mg$

$= 17m$ (2 s.f.)

R(↑)

$R + T \sin 30 - mg = 0$

$\therefore R = mg - T \sin 30$

$= mg - 2mg \times \frac{1}{2}$

$= 0$

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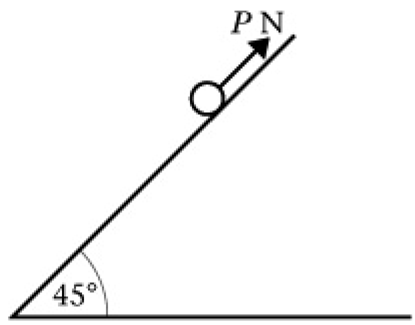
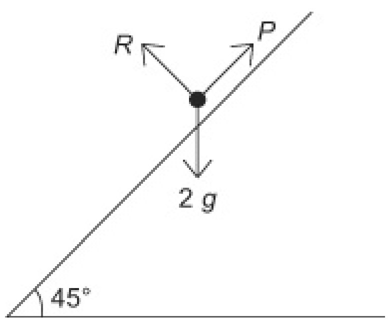
Statics of a particle
Exercise B, Question 10

Question:

A particle of mass 2 kg rests on a smooth inclined plane, which makes an angle of 45 ° with the horizontal. The particle is maintained in equilibrium by a force *P* N acting up the line of greatest slope of the inclined plane, as shown in the figure. Find the value of *P*.



Solution:



Let the normal reaction be *R*N.

R (↗)

Resolve along the plane.

$$P - 2g \sin 45° = 0$$
$$\therefore P = 2g \sin 45°$$
$$= g\sqrt{2}$$
$$= 14 \text{ N } (2 \text{ s.f.})$$

$$\text{sun } 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

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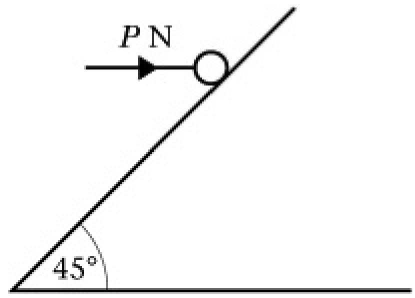
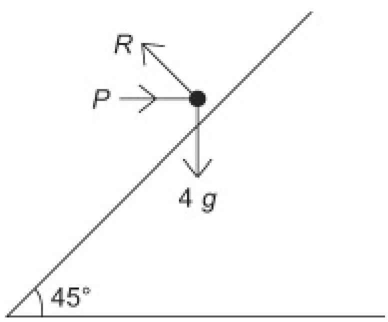
Statics of a particle
Exercise B, Question 11

Question:

A particle of mass 4 kg is held in equilibrium on a smooth plane which is inclined at 45 ° to the horizontal by a horizontal force of magnitude *P* N, as shown in the diagram. Find the value of *P*.



Solution:



Let the normal reaction be *R*N.

R (↗)

$P \cos 45° - 4g \sin 45° = 0$

Resolve along the plane.

$\therefore P = \frac{4g \sin 45°}{\cos 45°}$

$= 4g$

$= 39$ (2 s.f.)

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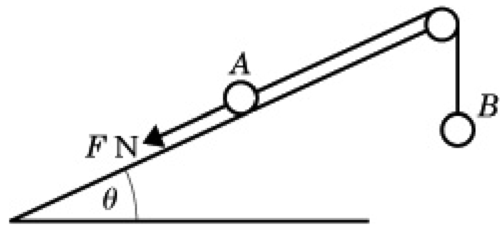
Statics of a particle
Exercise B, Question 12

Question:

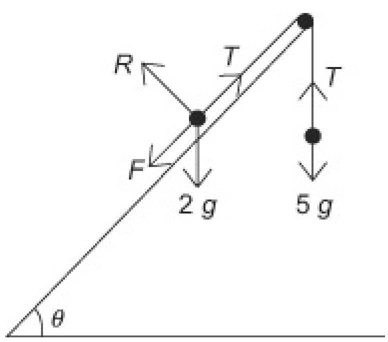
A particle *A* of mass 2 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$.



The particle is attached to one end of a light inextensible string which passes over a smooth pulley, as shown in the figure. The other end of the string is attached to a particle *B* of mass 5 kg. Particle *A* is also acted upon by a force of magnitude *F*N down the plane, along a line of greatest slope.

Find **a** the magnitude of the normal reaction between *A* and the plane, **b** the value *F*.

Solution:



Let the normal reaction between the particle *P* and the plane be *R*N.

Let the tension in the string be *T*N.
Consider first the 5 kg mass.
R(↑)

$T - 5g = 0$

$\therefore T = 5g$

Consider the 2 kg mass.
R (↖)

$R - 2g \cos\theta = 0$

$\therefore R = 2g \times \frac{4}{5} = \frac{8g}{5} = 16 \text{ N} \ (2 \text{ s.f.})$

R (↗)

$T - F - 2g \sin\theta = 0$

$\therefore F = T - 2g \sin\theta$

But as $T = 5g$

Consider the 5 kg mass first to find *T*. Then resolve perpendicular to the plane and parallel to the plane for the forces acting on the 2 kg mass. Use a 3, 4, 5 triangle to find $\sin\theta$ and $\cos\theta$.

$$F = 5g - 2g \times \frac{3}{5}$$

$$= \frac{19g}{5}$$

$$= 37 \text{ N} \ (2 \text{ s.f.})$$

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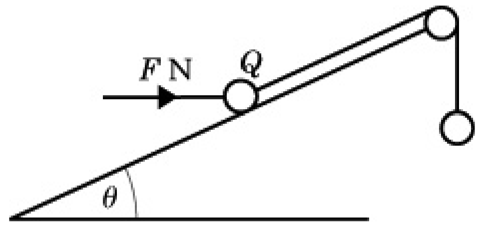
Statics of a particle
Exercise B, Question 13

Question:

A particle Q of mass 5 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan \theta = \frac{3}{4}$.



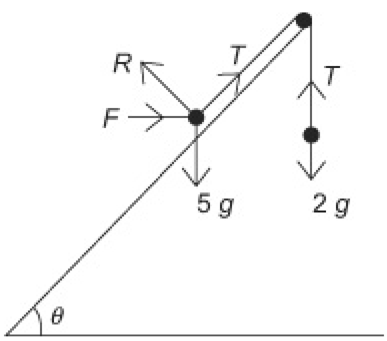
Q is attached to one end of a light inextensible string which passes over a smooth pulley as shown. The other end of the string is attached to a particle of mass 2 kg.

The particle Q is also acted upon by a force of magnitude FN acting horizontally.

Find the magnitude of

a the force FN, **b** the normal reaction between particle Q and the plane.

Solution:



Consider the 2 kg particle.

R(↑)

$$T - 2g = 0$$
$$\therefore T = 2g$$

Consider the 5 kg particle.

R (↗)

$$T + F \cos \theta - 5g \sin \theta = 0$$
$$\therefore F \cos \theta = 5g \sin \theta - T$$

As $T = 2g$, $\cos \theta = \frac{4}{5}$ and $\sin \theta = \frac{3}{5}$

$$F \times \frac{4}{5} = 5g \times \frac{3}{5} - 2g$$

$$\therefore F = g \div \frac{4}{5}$$

$$= \frac{5g}{4}$$

$$= 12 \ (2 \text{ s.f.})$$

R (↖)

$R - F \sin \theta - 5g \cos \theta = 0$

$$\therefore R = F \sin \theta + 5g \cos \theta$$

$$= \frac{5g}{4} \times \frac{3}{5} + 5g \times \frac{4}{5}$$

$$= \frac{19g}{4} = 47 \ (2 \text{ s.f.})$$

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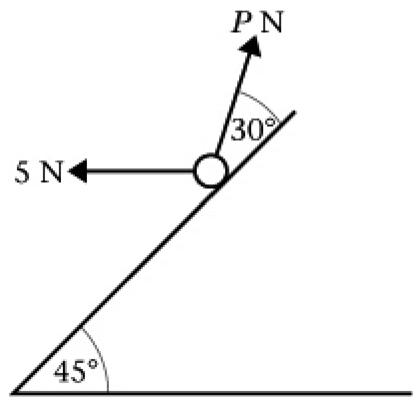
Statics of a particle
Exercise B, Question 14

Question:

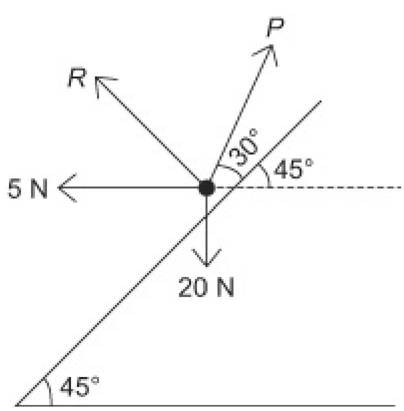
A particle of weight 20 N rests in equilibrium on a smooth inclined plane. It is maintained in equilibrium by the application of two external forces as shown in the diagram. One of the forces is a horizontal force of 5 N, the other is a force *P*N acting at 75 ° to the horizontal.



Find **a** the value of *P*,

b the magnitude of the normal reaction between the particle and the plane.

Solution:



Let the normal reaction be *R*N.

R (↗)

$P \cos 30° - 5 \cos 45° - 20 \sin 45° = 0$

Resolve along the plane to find *P* as it is the only unknown in your equation.

$\cos 45° = \frac{1}{\sqrt{2}} \text{sun } 45° = \frac{1}{\sqrt{2}} \cos 30° = \frac{\sqrt{3}}{2}$

$$\therefore P = \frac{5 \cos 45° + 20 \sin 45°}{\cos 30°}$$

$$= \left(5 \cdot \frac{\sqrt{2}}{2} + 20 \cdot \frac{\sqrt{2}}{2}\right) \div \frac{\sqrt{3}}{2}$$

$$= \frac{25\sqrt{2}}{\sqrt{3}}$$

$$= \frac{25\sqrt{6}}{3}$$

$$= 20.4 \ (3 \text{ s.f.})$$

R (↖)

$R + P \sin 30° + 5 \sin 45° - 20 \cos 45° = 0$

$\therefore R = 20 \cos 45° - 5 \sin 45° - P \sin 30°$

sun $30° = \frac{1}{2}$

As

$P = \frac{25\sqrt{6}}{3}$

$R = \frac{15}{\sqrt{2}} - \frac{25\sqrt{6}}{6} = \frac{45\sqrt{2} - 25\sqrt{6}}{6} = 0.400$ (3 s.f.)

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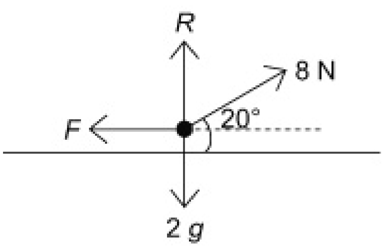
Statics of a particle
Exercise C, Question 1

Question:

A book of mass 2 kg rests on a rough horizontal table. When a force of magnitude 8 N acts on the book, at an angle of 20 ° to the horizontal in an upward direction, the book is on the point of slipping.

Calculate, to three significant figures, the value of the coefficient of friction between the book and the table.

Solution:



Let the normal reaction be RN, the friction force be FN and the coefficient of friction be μ.

R(→)

$8 \cos 20° - F = 0$

$\therefore F = 8 \cos 20°$

Resolve horizontally to find F, vertically to find R and use $F = \mu R$ to find μ.

R(↑)

$R + 8 \sin 20° - 2g = 0$

$\therefore R = 2g - 8 \sin 20°$

As the book is on the point of slipping the friction is limiting and

$F = \mu R$

$$\therefore \mu = \frac{F}{R} = \frac{8\cos 20°}{2g - 8\sin 20°} = \frac{7.518}{16.86} = 0.446 \text{ (3 s.f.)}$$

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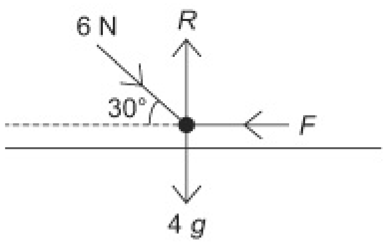
Statics of a particle
Exercise C, Question 2

Question:

A block of mass 4 kg rests on a rough horizontal table. When a force of 6 N acts on the block, at an angle of 30 ° to the horizontal in a downward direction, the block is on the point of slipping. Find the value of the coefficient of friction between the block and the table.

Solution:



Let the normal reaction be RN and the Friction force be FN. Let the coefficient of friction on be μ.

R(→)

$6 \cos 30° - F = 0$

$\therefore F = 6 \cos 30° = 3\sqrt{3} = 5.20$ (3 s.f.)

$R - 6\sin 30° - 4g = 0$

$\therefore R = 6\sin 30° + 4g$

$= 3 + 4 \times 9.8$

$= 42.2$

Resolve horizontally to find F, vertically to find R and use $\mu = \frac{F}{R}$.

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

R(↑)

As the block is on the point of slipping

$F = \mu R$

$\therefore \mu = \frac{F}{R}$

$= 0.123$ (3 s.f.) or 0.12 (2 s.f.)

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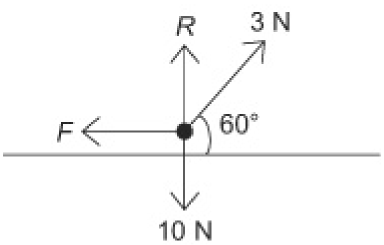
Statics of a particle
Exercise C, Question 3

Question:

A block of weight 10 N is at rest on a rough horizontal surface. A force of magnitude 3 N is applied to the block at an angle of 60 ° above the horizontal in an upward direction. The coefficient of friction between the block and the surface is 0.3.

a Calculate the force of friction, **b** determine whether the friction is limiting.

Solution:



Let the normal reaction force be R and the friction force be F.

a R(→)

$3 \cos 60° - F = 0$

$\therefore F = 3 \cos 60°$

$F = 1.5$ and so friction is 1.5 N

b R(↑)

$R + 3\sin 60° - 10 = 0$

$\therefore R = 10 - 3\sin 60°$

$= 10 - \frac{3\sqrt{3}}{2}$

$= 7.40$ (3 s.f.)

$\therefore \mu R = 0.3 \times 7.40$

$= 2.22$ (3 s.f.)

Find the friction force necessary to maintain equilibrium. Find the normal reaction force. Check whether $F < \mu R$ – non limiting equilibrium or $F = \mu R$ – limiting equilibrium.

$\sin 60° = \frac{\sqrt{3}}{2}$

As $F < \mu R$ the friction force is 1.5 N and is not limiting.

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Statics of a particle
Exercise C, Question 4

Question:

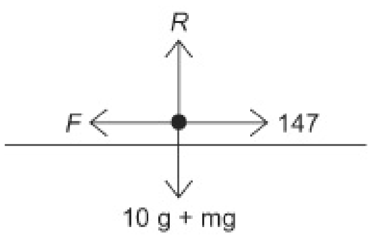
A packing crate of mass 10 kg rests on rough ground. It is filled with books which are evenly distributed through the crate. The coefficient of friction between the crate and the ground is 0.3.

a Find the mass of the books if the crate is in limiting equilibrium under the effect of a horizontal force of magnitude 147 N.

b State what modelling assumptions you have made.

Solution:

a



Let the normal reaction be RN and the friction force be FN.

Let the mass of the books be m kg.

Find F and R by resolving and use $F = \mu R$ for limiting friction.

R(→)

$$147 - F = 0$$
$$\therefore F = 147$$

R(↑)

$$R - 10g - mg = 0$$
$$\therefore R = 10g + mg$$

As the equilibrium is limiting, $F = \mu R$

$$\therefore 147 = 0.3(10g + mg)$$
$$\therefore 147 = 3g + 0.3mg$$
$$\therefore m = \frac{147 - 3g}{0.3g}$$
$$= 40$$

b The assumption is that the crate and books may be modelled as a particle.

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Statics of a particle
Exercise C, Question 5

Question:

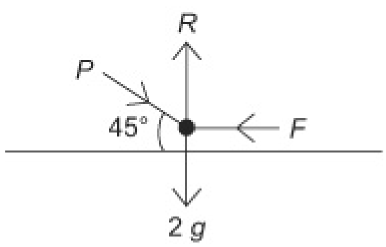
A block of mass 2 kg rests on a rough horizontal plane. A force P acts on the block at an angle of 45 ° to the horizontal. The equilibrium is limiting, with $\mu = 0.3$.

Find the magnitude of P if

a P acts in a downward direction, **b** P acts in an upward direction.

Solution:

a



Let R be the normal reaction and F be the force of friction.

R(→)

$$P\cos 45° - F = 0$$
$$\therefore F = P\cos 45°$$

Resolve horizontally and vertically to find F and R, then use the condition for limiting friction.

R(↑)

$$R - P\sin 45° - 2g = 0$$
$$\therefore R = P\sin 45° + 2g$$

As equilibrium is limiting, $F = \mu R$

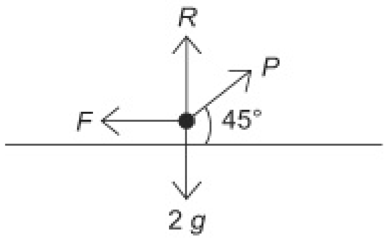
$$\therefore P\cos 45° = 0.3(P\sin 45° + 2g)$$
$$\therefore P\cos 45° - 0.3P\sin 45° = 0.6g$$
$$\therefore P(\cos 45° - 0.3\sin 45°) = 0.6g$$

$$P = \frac{0.6g}{\cos 45° - 0.3\sin 45°}$$
$$= \frac{6g\sqrt{2}}{7}$$
$$= 11.9 \text{ N } (3 \text{ s.f.}) \text{ or } 12 \text{ N } (2 \text{ s.f.})$$

$$\cos 45° = \sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

b

Let R be the normal reaction and F be the force of friction.



R(→)

$$P\cos 45° - F = 0$$
$$\therefore F = P\cos 45°$$

R(↑)

$$R + P \sin 45° - 2g = 0$$
$$\therefore R = 2g - P \sin 45°$$

As equilibrium is limiting, $F = \mu R$.

$$\therefore P\cos 45° = 0.3(2g - P \sin 45°)$$
$$\therefore P\cos 45° + 0.3P \sin 45° = 0.6g$$
$$\therefore P(\cos 45° + 0.3\sin 45°) = 0.6g$$

$$\therefore P = \frac{6g\sqrt{2}}{13}$$
$$= 6.40 \text{ N } (3 \text{ s.f.}) \text{ or } 6.4 \text{ N } (2 \text{ s.f.})$$

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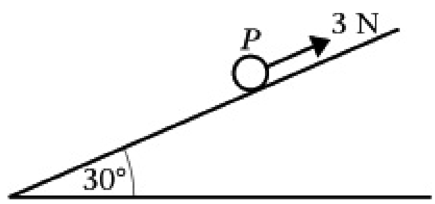
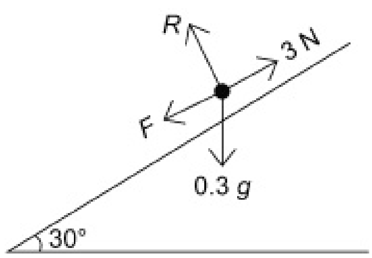
Statics of a particle
Exercise C, Question 6

Question:

A particle P of mass 0.3 kg is on a rough plane which is inclined at an angle 30° to the horizontal. The particle is held at rest on the plane by a force of magnitude 3 N acting up plane by a force of magnitude 3 N acting up the plane, in a direction parallel to a line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between P and the plane.



Solution:



Let R be the normal reaction and F be the force of friction.

R (↗)

$3 - F - 0.3g\sin 30° = 0$

$\therefore F = 3 - 0.3g\sin 30°$

$= 1.53$ N

Note that the force of friction acts down the plane. Resolve parallel and perpendicular to the plane to obtain F and R and use $F = \mu R$.

R (↖)

$R - 0.3g\cos 30° = 0$

$\therefore R = 0.3g\cos 30°$

$= 2.546$ N

As the particle is on the point of slipping

$F = \mu R$

$\therefore 1.53 = \mu \times 2.546$

$\therefore \mu = \frac{1.53}{2.546}$

$= 0.601$ (3 s.f.) (accept 0.6)

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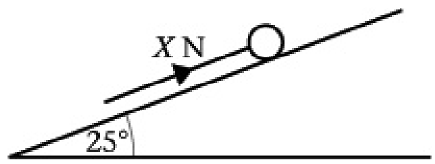
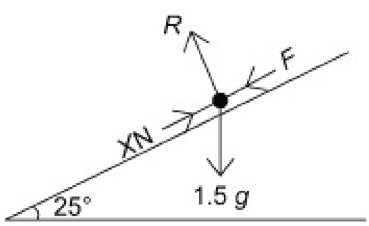
Statics of a particle
Exercise C, Question 7

Question:

A particle of mass 1.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude XN acting up a line of greatest slope of the plane. The plane is inclined at 25 ° to the horizontal. The particle is in limiting equilibrium and on the point of moving up the plane. The coefficient of friction between the particle and the plane is 0.25.
Calculate **a** the normal reaction of the plane on P, **b** the value of X.



Solution:



Let R be the normal reaction and F be the force of friction.

a R (↖)

$R - 1.5 \ g\cos \ 25° = 0$

$\therefore R = 1.5 \ g\cos \ 25° = 13.3$ N (3 s.f.) or 13 N (2 s.f.)

The force of friction acts down the plane, and as the friction is limiting $F = \mu R$.

b R (↗)

$X - F - 1.5g\sin \ 25° = 0$

$\therefore X = F + 1.5 \ g\sin \ 25°$

But the friction is limiting

$$\begin{aligned}\therefore F &= \mu R \\ &= 0.25 \times 13.3227 \\ &= 3.3306...\\ \therefore X &= 3.33 + 1.5g\sin \ 25° \\ &= 9.54 \text{ N (3 s.f.) or } 9.5 \text{ N (2 s.f.)}\end{aligned}$$

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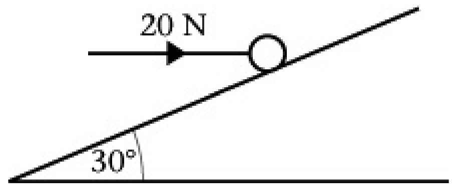
Statics of a particle
Exercise C, Question 8

Question:

A horizontal force of magnitude 20 N acts on a block of mass 1.5 kg, which is in equilibrium resting on a rough plane inclined at 30 ° to the horizontal. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane.

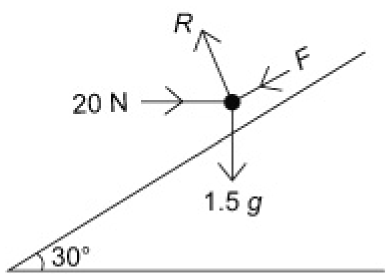


a Find the magnitude and direction of the frictional force acting on the block.

b Find the normal reaction between the block and the plane.

c What can you deduce about the coefficient of friction between the block and the plane?

Solution:



Let the normal reaction be R and the friction force be F acting down the plane.

a R (↗)

$20\cos 30° - F - 1.5g\sin 30° = 0$

$\therefore F = 20\cos 30° - 1.5g\sin 30°$

$= 9.97$ (3 s.f.)

Draw a diagram showing all the forces acting with friction down the plane.
Resolve along the plane. If $F > 0$ then you have chosen the correct direction. If $F < 0$ then friction acts up the plane.

The friction force has magnitude 9.97 N or 10 N (2 s.f.) and acts down the plane.

b R (↖)

$R - 20\sin 30° - 1.5g\cos 30° = 0$

$\therefore R = 20\sin 30° + 1.5g\cos 30°$

$= 22.7$ (3 s.f.)

The normal reaction has magnitude 22.7 N or 23 N (2 s.f.).

c For equilibrium $F \le \mu R$

$\therefore \mu \geq \frac{F}{R}$

i.e. : $\mu \geq 0.439$ or $\mu \geq 0.44$ (2 s.f.)

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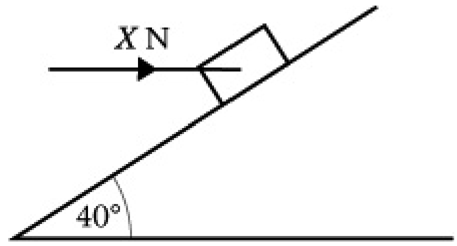
Statics of a particle
Exercise C, Question 9

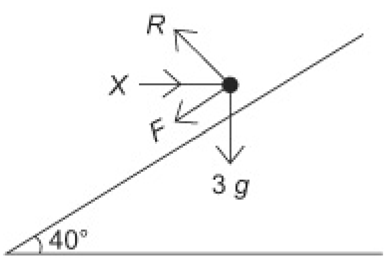
Question:

A box of mass 3 kg lies on a rough plane inclined at 40° to the horizontal. The box is held in equilibrium by means of a horizontal force of magnitude *X*N. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane. The coefficient of friction between the box and the plane is 0.3 and the box is in limiting equilibrium and is about to move up the plane.



a Find the normal reaction between the box and the plane. **b** Find *X*.

Solution:



Let the normal reaction be *R* and the friction force be *F* acting down the plane.

R (↖)

$R - X\sin 40° - 3g\cos 40° = 0$

$\therefore R = X\sin 40° + 3g\cos 40°$ *

You may decide to find *X* first and then use your answer to find *R*.
The simplest way to find *R* first is to resolve vertically as there is no *X* term in the equation obtained.

R (↗)

$X\cos 40° - F - 3g\sin 40° = 0$

$\therefore F = X\cos 40° - 3g\sin 40°$

As the friction is limiting, $F = \mu R$

$\therefore X\cos 40° - 3g\sin 40° = 0.3(X\sin 40° + 3g\cos 40°)$

$\therefore X\cos 40° - 0.3X\sin 40° = 0.9g\cos 40° + 3g\sin 40°$

$\therefore X(\cos 40° - 0.3\sin 40°) = 0.9g\cos 40° + 3g\sin 40°$

$$\therefore X = \frac{0.9g\cos 40° + 3g\sin 40°}{\cos 40° - 0.3\sin 40°}$$

$$= \frac{25.65}{0.5732}$$

$\therefore X = 44.8$ (accept 44.7) or $x = 45$ (2 s.f.)

Substitute into equation $*$ to give

$R = 51.3$ or $R = 51$ (2 s.f.)

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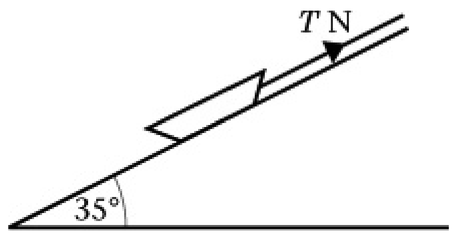
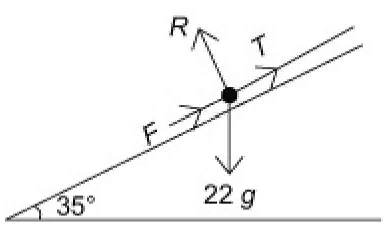
Statics of a particle
Exercise C, Question 10

Question:

A small child, sitting on a sledge, rests in equilibrium on an inclined slope. The sledge is held by a rope which lies along the slope and is under tension. The sledge is on the point of slipping down the plane. Modelling the child and sledge as a particle and the rope as a light inextensible string, calculate the tension in the rope, given that the mass of the child and sledge is 22 kg, the coefficient of friction is 0.125 and that the slope is a plane inclined at 35 ° with the horizontal. The direction of the rope is along a line of greatest slope of the plane.



Solution:



Let the normal reaction be R and the friction be F acting up the plane.

The friction acts up the plane, as the sledge is on the point of slipping down the plane.

R (↗)

$T + F - 22g\sin 35° = 0$ *

R (↖)

$$R - 22g\cos 35° = 0$$
$$\therefore R = 22g\cos 35°$$
$$R = 176.6$$

Normal reaction is 180 N (2 s.f.).

As the friction is limiting, $F = \mu R$

$$\therefore F = 0.125 \times 176.6$$
$$= 22.1 \ (3 \text{ s.f.})$$

Friction is 22 N (2 s.f.)

Substitute into equation ∗ to give

$$
\begin{aligned}
T &= 22g\sin\ 35 - 22.0.76... \\
&= 101.6 \\
&= 102\ \ (\ 3 \text{ s.f. })
\end{aligned}
$$

Tension is 100 N (2 s.f.)

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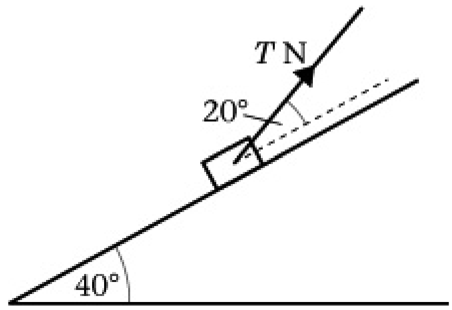
Statics of a particle
Exercise C, Question 11

Question:

A box of mass 0.5 kg is placed on a plane which is inclined at an angle of 40 ° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{5}$. The box is kept in equilibrium by a light inextensible string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20 ° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is *T*N. Find *T*

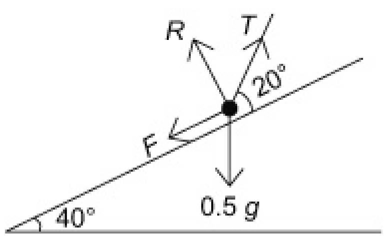


a if the box is about to move up the plane,

b if the box is about to move down the plane.

Solution:

a



Let the normal reaction be *R* and the friction force be *F*.

In part **a** *F* acts down the plane and in part **b** *F* ' acts up the plane.

R (↖)

$$R + T\sin\ 20° - 0.5g\cos\ 40° = 0$$
$$\therefore R = 0.5g\cos\ 40° - T\sin\ 20°$$

R (↗)

$$T\ \cos\ 20° - F - 0.5g\sin\ 40° = 0$$
$$\therefore F = T\ \cos\ 20° - 0.5g\sin\ 40°$$

As the friction is limiting $F = \mu R$

$$\therefore T\cos 20° - 0.5g\sin 40° = \frac{1}{5}(0.5g\cos 40° - T\sin 20°)$$

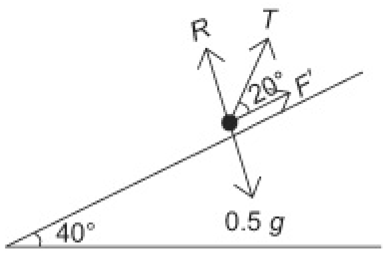
$$\therefore T\cos 20° + \frac{1}{5}T\sin 20° = 0.1g\cos 40° + 0.5g\sin 40°$$

$$\therefore T\left(\cos 20° + \frac{1}{5}\sin 20°\right) = 3.900$$

$$\therefore T = \frac{3.900}{\cos 20° + 0.2\sin 20°} = 3.87 \text{ N}$$

Tension is 3.9 N (2 s.f.)

b



Let the normal reaction be R and the friction force be F'.

The change from **a** is the direction of the friction force.

$\therefore R = 0.5g\cos 40° - T\sin 20°$ as before
and $F' = 0.5g\sin 40° - T\cos 20°$ (i.e. $F' = -F$)

As $F' = \mu R$

$$0.5g\sin 40° - T\cos 20° = \frac{1}{5}(0.5g\cos 40° - T\sin 20°)$$

$$\therefore 0.5g\sin 40° - 0.1g\cos 40° = T\cos 20° - \frac{1}{5}T\sin 20°$$

$$\therefore 2.399 = T(\cos 20° - 0.2\sin 20°)$$

$$\therefore T = \frac{2.399}{(\cos 20° - 0.2\sin 20°)}$$
$$= 2.75$$
$$= 2.8\text{N} \ (2 \text{ s.f.})$$

Tension is 2.8 N (2 s.f.).

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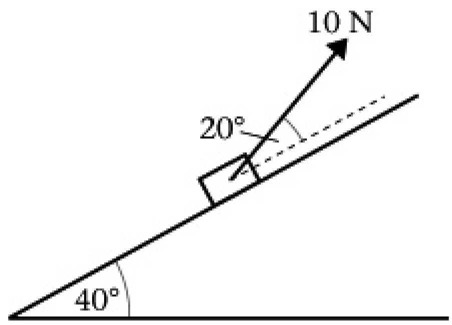
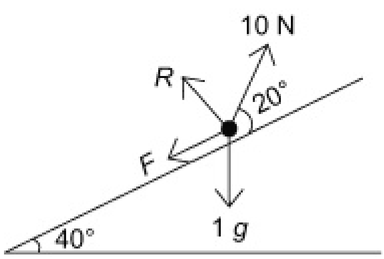
Statics of a particle
Exercise C, Question 12

Question:

A box of mass 1 kg is placed on a plane, which is inclined at an angle of 40 ° to the horizontal. The box is kept in equilibrium by a light inextensible string, which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20 ° with the plane, as shown in the diagram. The box is in limiting equilibrium and may be modelled as a particle. The tension in the string is 10 N and the coefficient of friction between the box and the plane is μ. Find μ if the box is about to move up the plane.



Solution:



R (↖)

$R + 10\sin 20° - g\cos 40° = 0$

$$\therefore R = g\cos 40° - 10\sin 20°$$
$$= 4.087$$

R (↗)

$$10 \cos 20° - F - g\sin 40° = 0$$

$$\therefore F = 10 \cos 20° - g\sin 40°$$
$$= 3.0976...$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{3.0976}{4.087}$$

$$= 0.758 \ (3 \text{ s.f.})$$

$$\text{So } \mu = 0.76 \ (2 \text{ s.f.})$$

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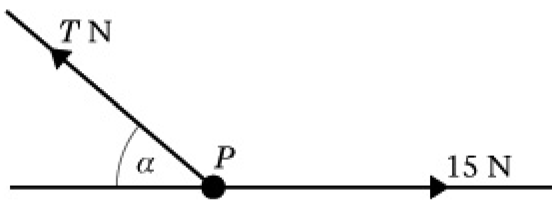
Statics of a particle
Exercise D, Question 1

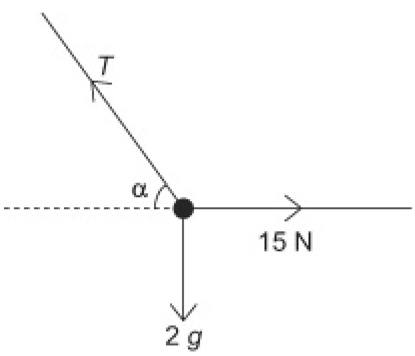
Question:



A particle P of mass 2 kg is held in equilibrium under gravity by two light inextensible strings. One string is horizontal and the other is inclined at an angle α to the horizontal, as shown in the diagram. The tension in the horizontal string is 15 N. The tension in the other string is T newtons.

a Find the size of the angle α. **b** Find the value of T.

Solution:



R(→)

$$15 - T \cos \alpha = 0$$
$$\therefore T \cos \alpha = 15 \qquad (1)$$

R(↑)

$$T \sin \alpha - 2g = 0$$
$$\therefore T \sin \alpha = 2g \qquad (2)$$

Divide equation (2) by equation (1)

$$\frac{T \sin \alpha}{T \cos \alpha} = \frac{2g}{15}$$
$$\therefore \tan \alpha = \frac{2g}{15}$$
$$= 1.307$$
$$\therefore \alpha = 52.6 °$$

Substitute into equation (1)

$T\cos 52.6° = 15$

$\therefore T = \frac{15}{\cos 52.6°} = 24.7$

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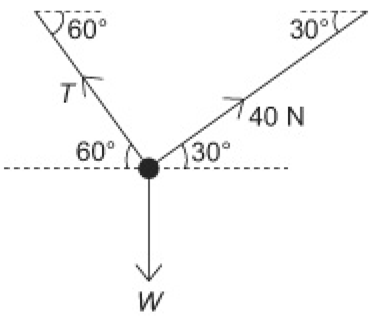
Statics of a particle
Exercise D, Question 2

Question:

A particle is suspended by two light inextensible strings and hangs in equilibrium. One string is inclined at 30° to the horizontal and the tension in that string is of magnitude 40 N. the second string is inclined at 60° to the horizontal. Calculate in N

a the weight of the particle, **b** the magnitude of the tension in the second string.

Solution:



Let the weight of the particle be WN and the tension in the second string be TN.

R(→)

$40 \cos 30° - T \cos 60° = 0$

$$\therefore T = \frac{40 \cos 30°}{\cos 60°} = 40\sqrt{3} \; (= 69.3 \text{ N})$$

R(↑)

$T \sin 60° + 40 \sin 30° - W = 0$

$\therefore W = T \sin 60° + 40 \sin 30°$

Substitute $T = 40\sqrt{3}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$

$$\text{Then } W = 60 + 20 = 80$$

$\therefore$ the weight of the particle is 80 N and the tension in the second string is 69.3 N

Alternative method

R (↗)

$40 - W\cos 60° = 0$

$\therefore W\cos 60° = 40$

$\therefore W = \dfrac{40}{\cos 60°}$

$= 80.$

b R (↖)

$T - W\sin 60° = 0$

$\therefore T = W\sin 60°$

$= 80\dfrac{\sqrt{3}}{2}$

$= 40\sqrt{3}$

$= 69.3$ (3 s.f.)

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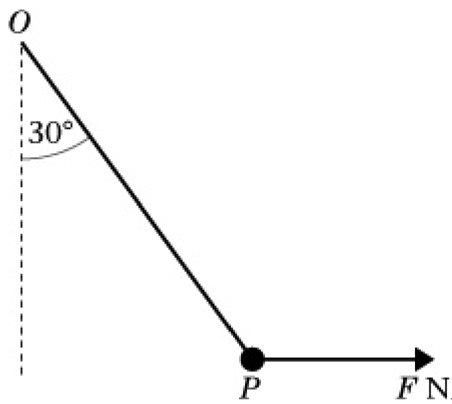
Statics of a particle
Exercise D, Question 3

Question:

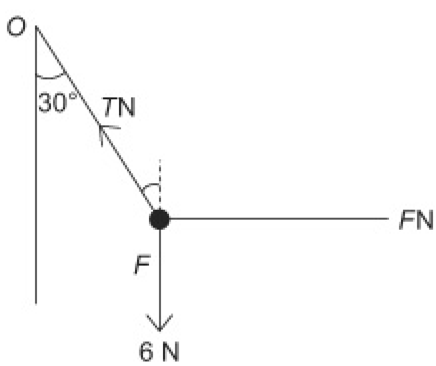


A particle P of weight 6 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude F newtons is applied to P. The particle P is in equilibrium under gravity with the string making an angle of 30 ° with the vertical, as shown in the diagram.

Find, to three significant figures,

a the tension in the string, **b** the value of F.

Solution:



Let the tension in the string $= T$ N.

a R(↑)

$T \cos 30° - 6 = 0$

$$\therefore T = \frac{6}{\cos 30°}$$

= The 6.93 tension is 6.93 N (3 s.f.)

b R(→)

$F - T \sin 30° = 0$

$$\therefore F = T \sin 30°$$
$$= T \times \frac{1}{2}$$
$$= 3.46. \ (3 \text{ s.f.})$$

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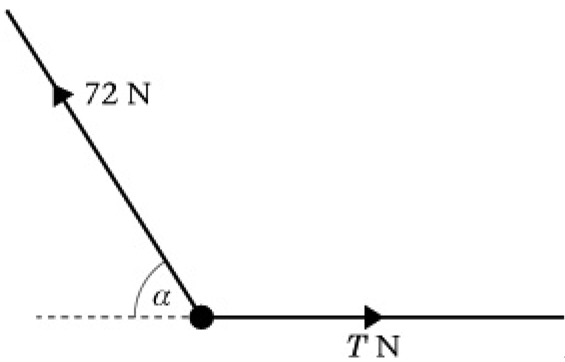
Statics of a particle
Exercise D, Question 4

Question:



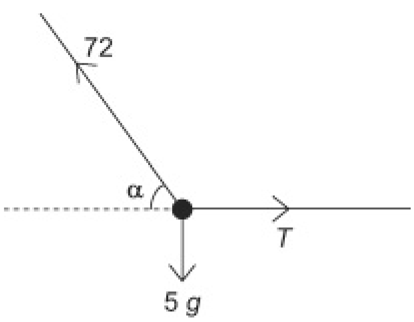
A body of mass 5 kg is held in equilibrium under gravity by two inextensible light ropes. One rope is horizontal, the other is at an angle α to the horizontal, as shown in the diagram.

The tension in the rope inclined at α to the horizontal is 72 N. Find

a the angle α, giving your answer to the nearest degree,

b the value of T to the nearest whole number.

Solution:



a R(↑)

$72 \sin \alpha - 5g = 0$

$$\therefore \sin \alpha = \frac{5g}{72}$$
$$= 0.681$$
$$\therefore \alpha = 42.9 \ (3 \text{ s.f.})$$
$$= 43° \ (\text{nearest degree})$$

b R(→)

$T - 72 \cos \alpha = 0$

$$\therefore T = 72 \cos \alpha$$
$$= 52.8$$

$\therefore$ Tension is 53 N to the nearest Newton

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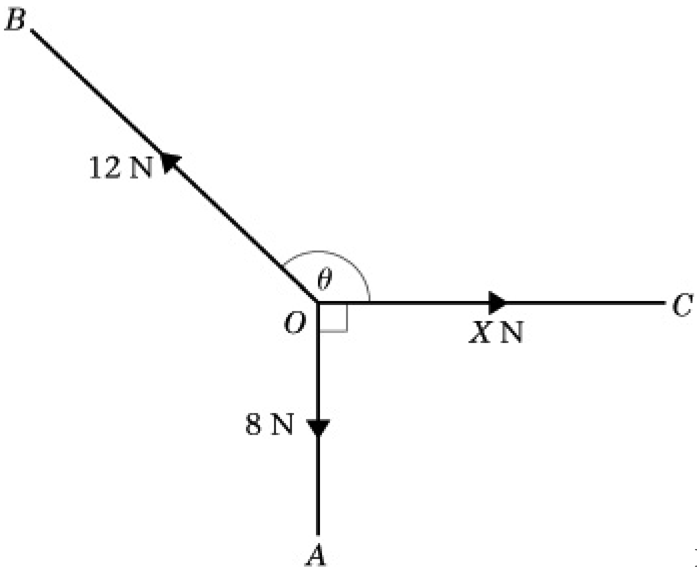
Statics of a particle
Exercise D, Question 5

Question:

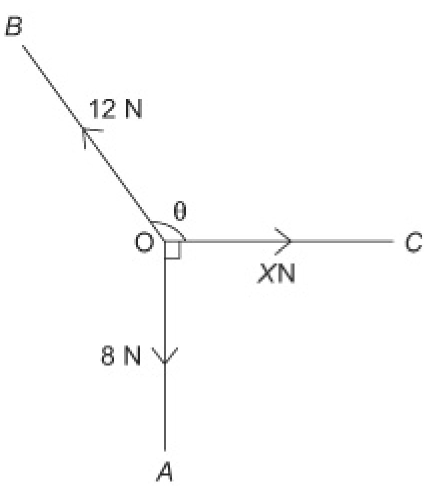


In the diagram, $\angle AOC = 90°$ and $\angle BOC = \theta°$. A particle at O is in equilibrium under the action of three coplanar forces. The three forces have magnitudes 8 N, 12 N and XN and act along OA, OB and OC respectively. Calculate

a the value, to one decimal place, of θ,

b the value, to two decimal places, of X.

Solution:



a

R(↑)

$12 \cos (\theta - 90°) - 8 = 0$

$$\therefore \cos(\theta - 90°) = \frac{8}{12}$$

$$\therefore \theta - 90° = 48.2°$$

$$\therefore \theta = 138.2° \text{ (1 d.p.)}$$

b R(→)

$X - 12\sin(\theta - 90°) = 0$

$$\therefore X = 12\sin 48.2°$$
$$= 8.95 \text{ (2 d.p.)}$$

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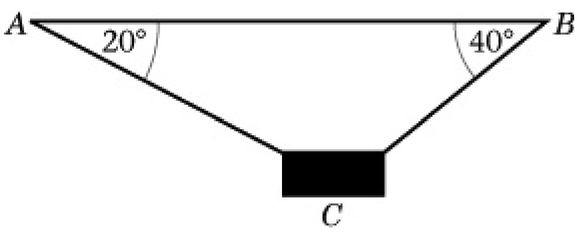
Statics of a particle
Exercise D, Question 6

Question:

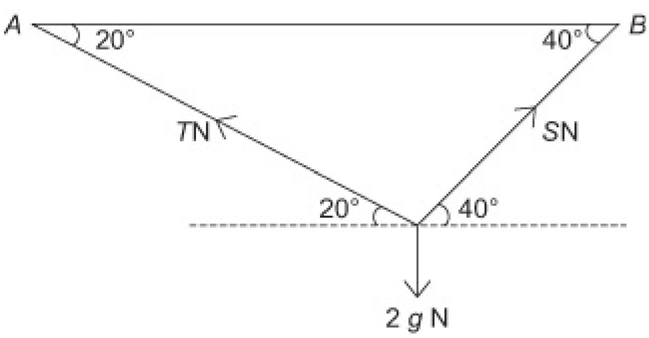
The two ends of a string are attached to two points *A* and *B* of a horizontal beam. A package of mass 2 kg is attached to the string at the point *C*. When the package hangs in equilibrium $\angle BAC = 20^\circ$ and $\angle ABC = 40^\circ$, as shown below.



By modelling the package as a particle and the string as light and inextensible, find, to three significant figures,

a the tension in *AC*, **b** the tension in *BC*.

Solution:



Let the tension in *AC* be *T*N and the tension in *BC* be *S*N.

R(→)

$S\cos 40^\circ - T\cos 20^\circ = 0 \qquad (1)$

R(↑)

$S\sin 40^\circ + T\sin 20^\circ - 2g = 0 \qquad (2)$

Solve the simultaneous equations (1) and (2)

$(1)\times\sin 40^\circ - (2)\times\cos 40^\circ$

$-T\sin 40^\circ\cos 20^\circ - T\sin 20^\circ\cos 40^\circ + 2g\cos 40^\circ = 0$

$\therefore T(\sin 40^\circ\cos 20^\circ + \cos 40^\circ\sin 20^\circ) = 2g\cos 40^\circ$

$$\therefore T = \frac{2g \cos 40°}{\sin 40° \cos 20° + \cos 40° \sin 20°}$$

$= 17.3$ [NB $\sin 40° \cos 20° + \cos 40° \sin 20° = \sin 60°$]

Substitute the value of T into equation (1)

$$\text{Then } S = \frac{T \cos 20°}{\cos 40°}$$

$= 21.3$

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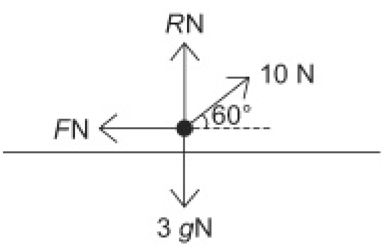
Statics of a particle
Exercise D, Question 7

Question:

A block of mass 3 kg rests on a rough, horizontal table. When a force of magnitude 10 N acts on the block at an angle of 60 ° to the horizontal in an upwards direction, the block is on the point of slipping. Calculate, to two significant figures, the value of the coefficient of friction between the block and the table.

Solution:



Let the normal reaction be R, N the friction force be FN and the coefficient of friction be μ.

R(→)

$$\begin{aligned} 10 \cos 60° - F &= 0 \\ \therefore F &= 10 \cos 60° \\ F &= 5 \end{aligned}$$

R(↑)

$R + 10 \sin 60° - 3g = 0$

$$\begin{aligned} \therefore R &= 3g - 10 \sin 60° \\ &= 3g - 5\sqrt{3} \\ &= 20.7 \end{aligned}$$

As the block is an the point of slipping, the friction is limiting and $F = \mu R$.

$$\begin{aligned} \therefore \mu &= \frac{F}{R} \\ &= \frac{5}{20.7} \\ &= 0.241\ 0.24 \ (2 \text{ s.f.}) \end{aligned}$$

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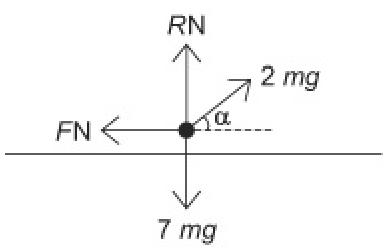
Statics of a particle
Exercise D, Question 8

Question:

A particle P of mass $7m$ is placed on a rough horizontal table, the coefficient of friction between P and the table being μ. A force of magnitude $2mg$, acting upwards at an acute angle α to the horizontal, is applied to P and equilibrium is on the point of being broken by the particle sliding on the table. Given that $\tan\alpha = \frac{5}{12}$, find the value of μ.

Solution:



Let the normal reaction be RN and the friction force be FN.

R(→)

$2mg\cos\alpha - F = 0$

$\therefore F = 2mg\cos\alpha$

R(↑)

$R + 2mg\sin\alpha - 7mg = 0$

$\therefore R = 7mg - 2mg\sin\alpha$

As the friction is limiting, $F = \mu R$.

$$\therefore \mu = \frac{F}{R} = \frac{2mg\cos\alpha}{7mg - 2mg\sin\alpha}$$

As $\tan\alpha = \frac{5}{12}$, $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$

$$\therefore \mu = \frac{2mg \times \frac{12}{13}}{7mg - 2mg \times \frac{5}{13}}$$

$$= \frac{24}{81}$$

$$= \frac{8}{27}$$

$$= 0.296 \ (3 \text{ s.f.})$$

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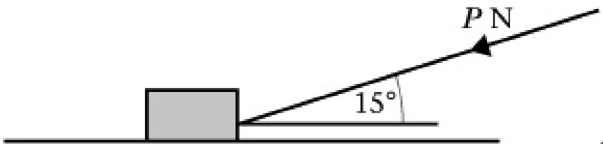
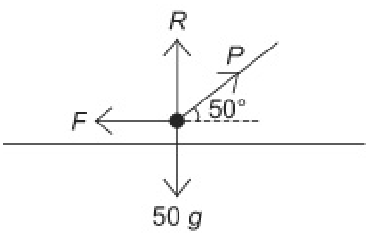
Statics of a particle
Exercise D, Question 9

Question:



A box of mass 50 kg rests on rough horizontal ground. The coefficient of friction between the box and the ground is 0.6. A force of magnitude *P* newtons is applied to the box at an angle of 15 ° to the horizontal, as shown in the diagram, and the box is now in limiting equilibrium. By modelling the box as a particle find, to three significant figures, the value of *P*.

Solution:



Let the normal reaction be *R*N and the frictional force be *F*N.

R(→)

$$F - P\cos 15^\circ = 0$$
$$\therefore F = P\cos 15^\circ$$

R(↑)

$R - P\sin 15^\circ - 50g = 0$

$\therefore R = P\sin 15^\circ + 50g$

As the friction is limiting, $F = \mu R$

$\therefore P\cos 15^\circ = 0.6(P\sin 15^\circ + 50g)$

$\therefore P\cos 15^\circ - 0.6P\sin 15^\circ = 30g$

$\therefore P(\cos 15^\circ - 0.6\sin 15^\circ) = 30g$

$$\therefore P = \frac{30g}{\cos 15^\circ - 0.6\sin 15^\circ}$$
$$\therefore P = 363$$

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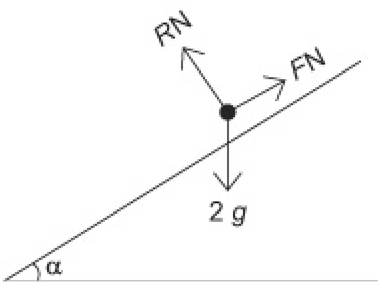
Statics of a particle
Exercise D, Question 10

Question:

A book of mass 2 kg rests on a rough plane inclined at an angle α° to the horizontal. Given that the coefficient of friction between the book and the plane is 0.2, and that the book is on the point of slipping down the plane, find, to the nearest degree, the value of α.

Solution:



R (↗)

$F - 2g \sin \alpha = 0$

$\therefore F = 2g \sin \alpha$

R (↖)

$R - 2g \cos \alpha = 0$

$\therefore R = 2g \cos \alpha$

As the friction is limiting, $F = \mu R$

$\therefore 2g \sin \alpha = 0.2 \times 2g \cos \alpha$

Divide both sides by $2g \cos \alpha$

$\therefore \tan \alpha = 0.2$

$\therefore \alpha = 11.3^\circ$

$\alpha = 11^\circ$ (to the nearest degree)

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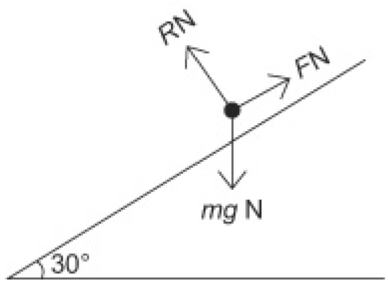
Statics of a particle
Exercise D, Question 11

Question:

a A book is placed on a desk lid which is slowly tilted. Given that the book begins to slide when the inclination of the lid to the horizontal is 30 ° , find the coefficient of friction between the book and the desk lid.

b State an assumption you have made about the book when forming the mathematical model you used to solve part **a**.

Solution:



Let the normal reaction be RN, the friction be FN and the mass be m kg.

a

R (↗)

$F - mg \sin 30° = 0$

R (↖)

$R - mg \cos 30° = 0$

As the friction is limiting $F = \mu R$

$\therefore mg \sin 30° = \mu mg \cos 30°$

Divide both sides by $mg \cos 30°$.

$$\frac{mg \sin 30°}{mg \cos 30°} = \mu$$

$$\therefore \mu = \tan 30° = \frac{\sqrt{3}}{3} = 0.577$$

b The book was modelled as a particle.

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Statics of a particle
Exercise D, Question 12

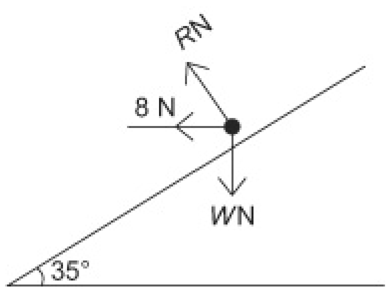
Question:

A particle is placed on a smooth plane inclined at 35 ° to the horizontal. The particle is kept in equilibrium by a horizontal force, of magnitude 8 N, acting in the vertical plane containing the line of greatest slope of the inclined plane through the particle. Calculate, in N to one decimal place,

a the weight of the particle,

b the magnitude of the force exerted by the plane on the particle.

Solution:



Let the normal reaction be RN and let the weight of the particle be WN.

a R (↗)

$8 \cos 35° - W \sin 35° = 0$

$$\therefore W \sin 35° = 8 \cos 35°$$
$$\therefore W = 8 \frac{\cos 35°}{\sin 35°}$$
$$W = 11.4$$

b R (↖)

$R - 8 \sin 35° - W \cos 35° = 0$

$$\therefore R = 8 \sin 35° + W \cos 35°$$
$$R = 13.9$$

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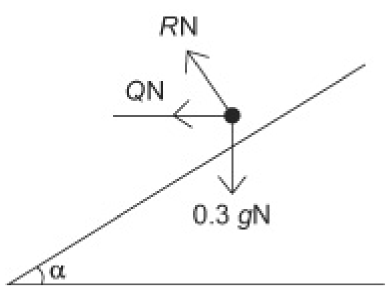
Statics of a particle
Exercise D, Question 13

Question:

A particle of mass 0.3 kg lies on a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is held in equilibrium by a horizontal force of magnitude Q newtons. The line of action of this force is in the same vertical plane as a line of greatest slope of the inclined plane. Calculate the value of Q, to one decimal place.

Solution:



Let the normal reaction force be RN.

R (↗)

$$Q\cos\alpha - 0.3g\sin\alpha = 0$$
$$\therefore Q\cos\alpha = 0.3g\sin\alpha$$
$$\therefore Q = \frac{0.3g\sin\alpha}{\cos\alpha}$$
$$= 0.3g\tan\alpha$$

But $\tan\alpha = \frac{3}{4}$

$$\therefore Q = 0.3g \times \frac{3}{4}$$
$$= 2.2 \ (1 \text{ d.p.})$$

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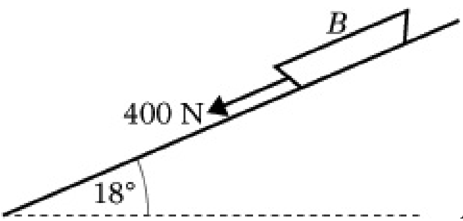
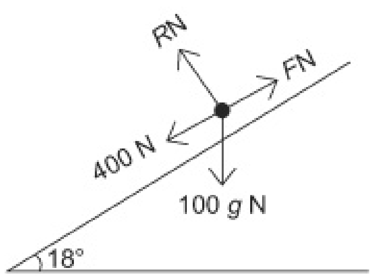
Statics of a particle
Exercise D, Question 14

Question:



A small boat B of mass 100 kg is standing on a ramp which is inclined at 18° to the horizontal. A force of magnitude 400 N is applied to B and acts down the ramp as shown in the diagram. The boat is in limiting equilibrium on the point of sliding down the ramp. Find the coefficient of friction between B and the ramp, giving your answer to two decimal places.

Solution:



Let the normal reaction be RN and the friction be FN.

R (↗)

$$F - 400 - 100g \sin 18^\circ = 0$$
$$\therefore F = 400 + 100g \sin 18^\circ$$
$$= 702.8$$

R (↖)

$$R - 100g \cos 18^\circ = 0$$
$$\therefore R = 100g \cos 18^\circ$$
$$= 932.0$$

As the Friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$
$$= 0.754$$
$$= 0.75 \text{ (2 d.p.) .}$$

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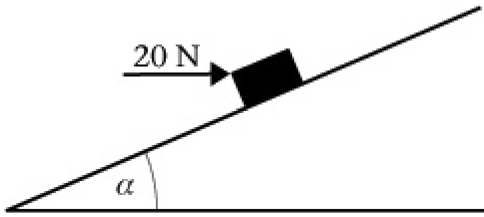
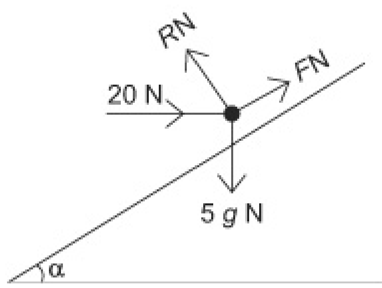
Statics of a particle
Exercise D, Question 15

Question:



A parcel of mass 5 kg lies on a rough plane inclined at an angle of α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The parcel is held in equilibrium by the action of a horizontal force of magnitude 20 N, as shown in the diagram. The force acts in a vertical plane through a line of greatest slope of the plane. The parcel is on the point of sliding down the plane. Find the coefficient of friction between the parcel and the plane.

Solution:



Let the normal reaction be RN and the friction be FN.

R (↗)

$$F - 5g \sin \alpha + 20 \cos \alpha = 0$$
$$\therefore F = 5g \sin \alpha - 20 \cos \alpha$$

As $\tan \alpha = \frac{3}{4}$, $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$

$\therefore F = 3g - 16$

R (↖)

$$R - 20 \sin \alpha - 5g \cos \alpha = 0$$
$$\therefore R = 20 \sin \alpha + 5g \cos \alpha$$
$$= 12 + 4g$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{3g - 16}{12 + 4g}$$

$$= \frac{13.4}{51.2}$$

$$= 0.262 \text{ (3 s.f.)}$$

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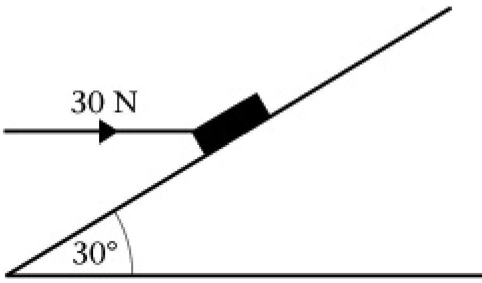
Statics of a particle
Exercise D, Question 16

Question:



A small parcel of mass 3 kg is held in equilibrium on a rough plane by the action of a horizontal force of magnitude 30 N acting in a vertical plane through a line of greatest slope. The plane is inclined at an angle of 30 ° to the horizontal, as shown in the diagram.

The parcel is modelled as a particle. The parcel is on the point of moving up the slope.

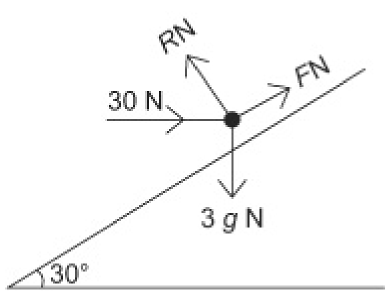
a Draw a diagram showing all the forces acting on the parcel.

b Find the normal reaction on the parcel.

c Find the coefficient of friction between the parcel and the plane.

Solution:

a



Let the normal reaction be RN and the friction be FN.

b R (↗)

$30 \cos 30° - F - 3g \sin 30° = 0$

$$\therefore F = 30 \cos 30° - 3g \sin 30°$$
$$= 11.28$$

R (↖)

$$R - 30 \sin 30° - 3g \cos 30° = 0$$
$$\therefore R = 30 \sin 30° + 3g \cos 30°$$
$$= 40.46$$

c As the friction is limiting, $F = \mu R$.

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{11.28}{40.46}$$

$$= 0.2788$$

$$= 0.279 \text{ (3 s.f.) .}$$

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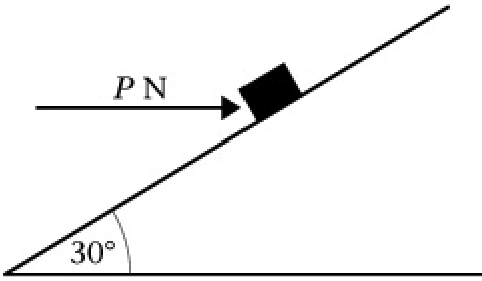
Statics of a particle
Exercise D, Question 17

Question:



A box of mass 6 kg lies on a rough plane inclined at an angle of 30 ° to the horizontal. The box is held in equilibrium by means of a horizontal force of magnitude *P* newtons, as shown in the diagram.

The line of action of the force is in the same vertical plane as a line of greatest slope of the plane. The coefficient of friction between the box and the plane is 0.4. The box is modelled as a particle. Given that the box is in limiting equilibrium and on the point of moving up the plane, find,

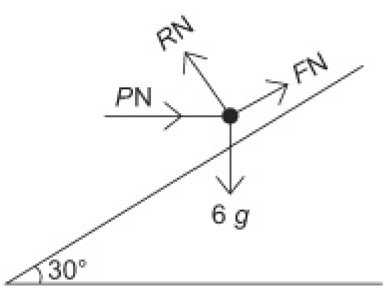
a the normal reaction exerted on the box by the plane,

b the value of *P*.

The horizontal force is removed.

c Show that the box will now start to move down the plane.

Solution:



Let the normal reaction be *R*N and the friction be *F*N.

a R(↑)

$R \cos 30° - F \sin 30° - 6g = 0$ (1)

As the friction is limiting, $F = \mu R$

$\therefore F = 0.4R$

Substitute into equation (1)

$\therefore R\cos 30° - 0.4R\sin 30° = 6g$

$\therefore R(\cos 30° - 0.4\sin 30°) = 6g$

$$\therefore R = \frac{6g}{\cos 30° - 0.4\sin 30°}$$

$R = 88.3$

b R(→)

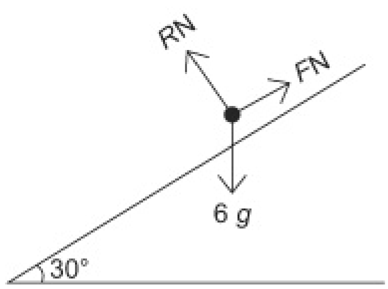
$P - R\sin 30° - F\cos 30° = 0$

$\therefore P = R\sin 30° + 0.4R\cos 30°$ (as $F = 0.4R$)

$= 88.3(\sin 30° + 0.4\cos 30°)$

$= 74.7$

c



Draw a new sketch with the new normal reaction *R*N and friction force *F*N.

R (↖)

$R - 6g\cos 30° = 0$

$\therefore R = 6g\cos 30°$

$= 50.9$

$\therefore \mu R = 20.4$

R (↗)

Resultant force down the plane $= 6g\sin 30° - F$

$= 29.4 - F$

As maximum value *F* can take is 20.4, there is a resultant force of 9 N down the plane and the box will move.

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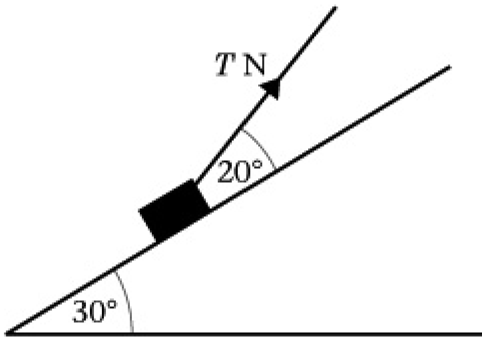
Statics of a particle
Exercise D, Question 18

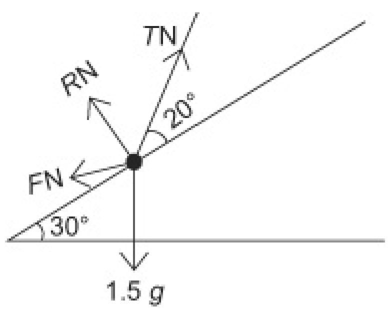
Question:



A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30 ° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20 ° with the plane, as shown in the diagram.

The box is in limiting equilibrium and is about to move up the plane. The tension in the string is *T* newtons. The box is modelled as a particle. Find the value of *T*.

Solution:



Let the normal reaction be *R*N and the friction force be *F*N.

R (↗)

$T \cos 20° - F - 1.5g \sin 30° = 0$

$\therefore F = T \cos 20° - 1.5g \sin 30°$

R (↖)

$R + T \sin 20° - 1.5g \cos 30° = 0$

$\therefore R = 1.5g \cos 30° - T \sin 20°$

As the box is in limiting equilibrium, $F = \mu R$

$\therefore T \cos 20° - 1.5g \sin 30° = \frac{1}{3}(1.5g \cos 30° - T \sin 20°)$

$\therefore T \cos 20° + \frac{1}{3}T \sin 20° = 0.5g \cos 30° + 1.5g \sin 30°$

$\therefore T(\cos 20° + \frac{1}{3} \sin 20°) = 0.5g \cos 30° + 1.5g \sin 30°$

$\therefore T = \frac{11.59}{1.054}$

$\therefore T = 11.0$

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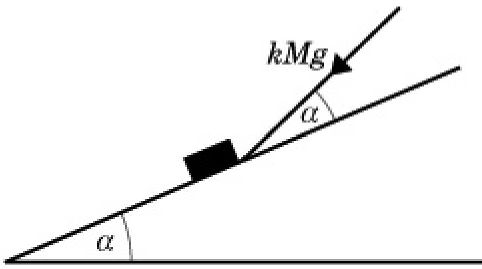
Statics of a particle
Exercise D, Question 19

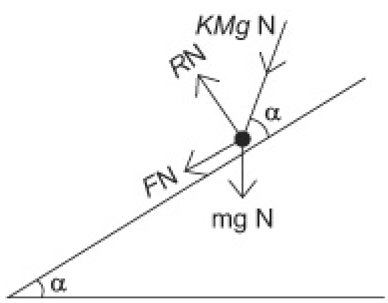
Question:



A rough slope is inclined at an angle α to the horizontal, where $\alpha < 45^\circ$. A small parcel of mass M is at rest on the slope, and the coefficient of friction between the parcel and the slope is μ. A force of magnitude kMg, where k is a constant, is applied to the parcel in a direction making an angle α with a line of greatest slope, as shown in the diagram.

The line of action of the force is in the same vertical plane as the line of greatest slope. Given that the parcel is on the point of moving down the slope, show that: $k = \dfrac{\mu \cos\alpha - \sin\alpha}{\cos\alpha - \mu \sin\alpha}$

Solution:



Let R be the normal reaction and F be the force of friction.

R (↗)

$$F - kMg\cos\alpha - Mg\sin\alpha = 0$$
$$\therefore F = Mg(k\cos\alpha + \sin\alpha)$$

R (↖)

$$R - kMg\sin\alpha - Mg\cos\alpha = 0$$
$$\therefore R = Mg(k\sin\alpha + \cos\alpha)$$

As the friction is limiting; $F = \mu R$.

$\therefore Mg(k\cos\alpha + \sin\alpha) = \mu Mg(k\sin\alpha + \cos\alpha)$

$\therefore k\cos\alpha - \mu k\sin\alpha = \mu\cos\alpha - \sin\alpha$

$\therefore k(\cos\alpha - \mu\sin\alpha) = \mu\cos\alpha - \sin\alpha$

$\therefore k = \dfrac{\mu\cos\alpha - \sin\alpha}{\cos\alpha - \mu\sin\alpha}$

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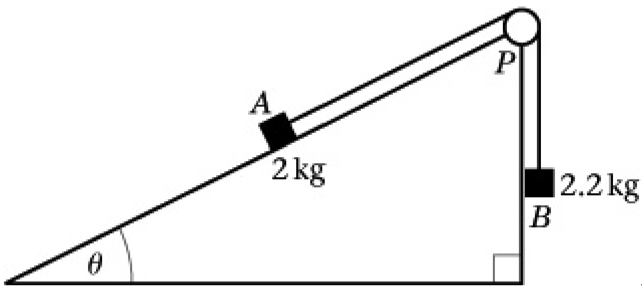
Statics of a particle
Exercise D, Question 20

Question:



A parcel *A* of mass 2 kg rests on a rough slope inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$. A string is attached to *A* and passes over a small smooth pulley fixed at *P*. The other end of the string is attached to a weight *B* of mass 2.2 kg, which hangs freely, as shown in the diagram.

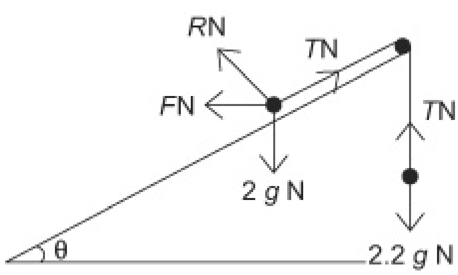
The parcel *A* is in limiting equilibrium and about to slide up the slope. By modelling *A* and *B* as particles and the string as light and inextensible, find

a the normal contact force acting on *A*,

b the coefficient of friction between *A* and the slope.

Solution:

a



Let the normal reaction be *R*N and the friction be *F*N acting down the plane. Let the tension in the string be *T*N.

Consider the 2 kg mass.

R (↗)

$$R - 2g \cos \theta = 0$$
$$\therefore R = 2g \cos \theta$$

As $\tan \theta = \frac{3}{4}$, $\sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$

$$\therefore R = 2g \times \frac{4}{5}$$

$$= \frac{8g}{5}$$

$$R = 15.68$$

$$= 15.7 \text{ (3 s.f.)}$$

b Consider the 2.2 kg mass.

R(↑)

$$T - 2.2g = 0$$

$$\therefore T = 2.2g$$

Consider the 2 kg mass.

R (↗)

$$T - F - 2g \sin \theta = 0$$

$$\therefore F = T - 2g \sin \theta$$

But $T = 2.2g$ and $\sin \theta = \frac{3}{5}$

$$\therefore F = 2.2g - 2g \times \frac{3}{5}$$

$$\text{i.e. : } F = g$$

$$\therefore F = 9.8$$

As the Friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{9.8}{15.68} \text{ (or } g \div \frac{8g}{5} \text{)}$$

$$= 0.625 \text{ or } \frac{5}{8}$$

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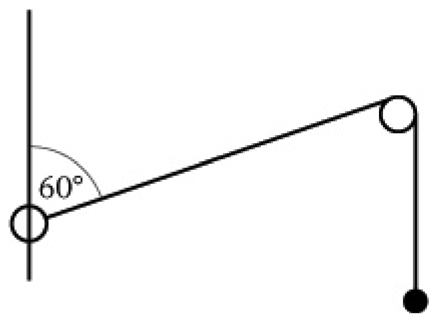
Statics of a particle
Exercise D, Question 21

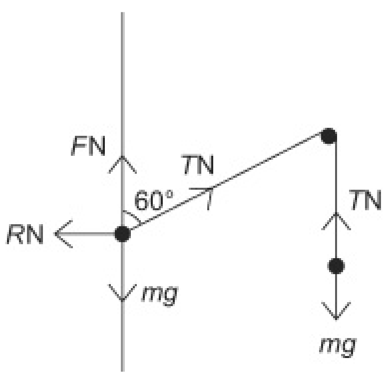
Question:



A light inextensible string passes over a smooth peg, and is attached at one end to a particle of mass *m* kg and at the other end to a ring also of mass *m* kg. The ring is threaded on a rough vertical wire as shown in the diagram. The system is in limiting equilibrium with the part of the string between the ring and the peg making an angle of 60 ° with the vertical wire.

Calculate the coefficient of friction between the ring and the wire, giving your answer to three significant figures.

Solution:



Let the normal reaction, the friction and the tension be *R*N, *F*N and *T*N respectively.

Consider the particle of mass *m*.

R(↑)

$$T - mg = 0$$
$$\therefore T = mg$$

Consider the ring.

R(→)

$$T \sin 60 - R = 0$$
$$\therefore R = T \sin 60°$$
$$= mg \sin 60°$$
$$= mg \frac{\sqrt{3}}{2}$$

R(↑)

$$\begin{aligned} F + T\cos 60° - mg &= 0 \\ \therefore F &= mg - T\cos 60° \\ &= mg - \tfrac{1}{2}mg \\ &= \tfrac{1}{2}mg \end{aligned}$$

As the friction is limiting $F = \mu R$

$$\begin{aligned} \therefore \mu &= \frac{F}{R} \\ &= \frac{1}{2}mg \div mg\frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{3}} \\ &= 0.577 \ (3 \text{ s.f.}) \end{aligned}$$

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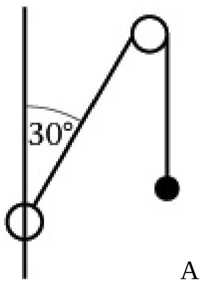
Statics of a particle
Exercise D, Question 22

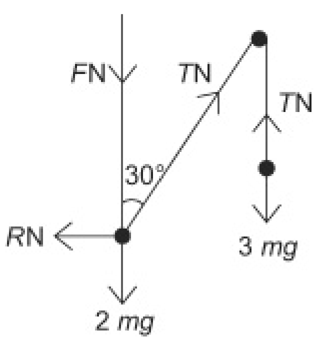
Question:



A light inextensible string passes over a smooth peg, and is attached at one end to a particle of mass $3m$ kg and at the other end to a ring of mass $2m$ kg. The ring is threaded on a rough vertical wire as shown in the diagram. The system is in limiting equilibrium with the part of the string between the ring and the peg making an angle of 30 ° with the vertical wire.

Calculate the coefficient of friction between the ring and the wire, giving your answer to three significant figures.

Solution:



Let the normal reaction, the friction and the tension be RN, FN and TN respectively.

Consider the 3 m kg particle.

R(↑)

$$T - 3mg = 0$$
$$\therefore T = 3mg$$

Consider the 2 m kg ring.

R(→)

$$R - T\sin 30° = 0$$
$$\therefore R = T\sin 30°$$
$$\therefore R = 3mg\sin 30°\text{, as } T = 3mg$$
$$\therefore R = \frac{3mg}{2}$$

R(↑)

$$T\cos 30° - F - 2mg = 0$$
$$\therefore F = T\cos 30° - 2mg$$
$$= 3mg\frac{\sqrt{3}}{2} - 2mg = 0.598mg$$

As friction is limiting: $F = \mu R$

$$\therefore \mu = \frac{F}{R} = \frac{0.598mg}{1.5mg}$$
$$= 0.399 \ (3 \text{ s.f.})$$

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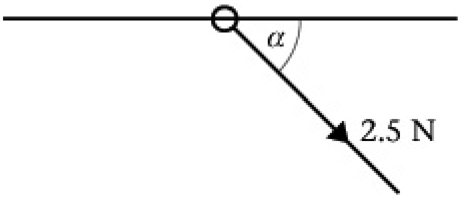
Statics of a particle
Exercise D, Question 23

Question:



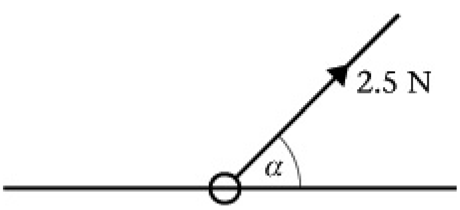
A ring of mass 0.3 kg is threaded on a fixed, rough horizontal curtain pole. A light inextensible string is attached to the ring. The string and the pole lie in the same vertical plane. The ring is pulled downwards by the string which makes an angle α to the horizontal, where $\tan\alpha = \frac{1}{4}$ as shown in the diagram.

The tension in the string is 2.5 N.

Given that, in this position, the ring is in limiting equilibrium,

a find the coefficient of friction between the ring and the pole.

The direction of the string is now altered so that the ring is pulled upwards. The string lies in the same vertical plane as before and again makes an angle α with the horizontal, as shown in the diagram below.



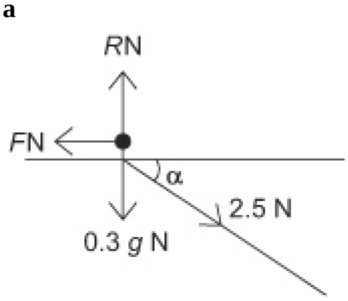
The tension in the string is again 2.5 N.

b Find the normal reaction exerted by the pole on the ring.

c State whether the ring is in equilibrium in the position shown in the second figure, giving a brief justification for your answer. You need make no further detailed calculation of the forces acting.

Solution:

a



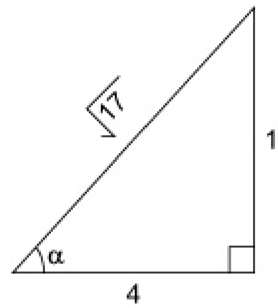
Let the normal reaction and friction force be RN and FN respectively.

R(→)

$2.5 \cos \alpha - F = 0$

$\therefore F = 2.5 \cos \alpha.$

As $\tan \alpha = \frac{1}{4}$, $\sin \alpha = \frac{1}{\sqrt{17}}$ and $\cos \alpha = \frac{4}{\sqrt{17}}$ from Pythagoras' Theorem.



$\therefore F = 2.5 \times \frac{4}{\sqrt{17}} = 2.425$

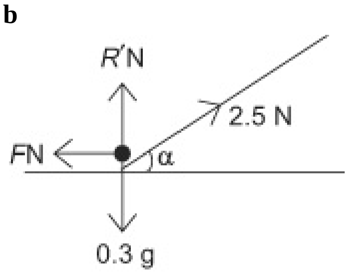
R(↑)

$$R - 0.3g - 2.5 \sin \alpha = 0$$
$$\therefore R = 0.3g + 2.5 \times \frac{1}{\sqrt{17}}$$
$$= 3.546$$

As the friction is limiting $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$
$$= \frac{2.425}{3.546}$$
$$= 0.684 \ (3 \text{ s.f.})$$

b



Let the new normal reaction be R' N, the friction remains FN.

R(↑)

$$R' + 2.5 \sin \alpha = 0.3g$$
$$\therefore R' = 0.3g - 2.5 \times \frac{1}{\sqrt{17}}$$
$$= 2.33$$

c To maintain equilibrium F would need to be 2.425 N (as in part **a**)

But the maximum value F can take is μR

i.e. : 0.684 × 2.33 (1.596 N)

As 2.425 > 1.596 the ring is not in equilibrium.