Solutionbank FP1
Edexcel AS and A Level Modular Mathematics

Matrix algebra
Exercise A, Question 1

Question:

Describe the dimensions of these matrices.

a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

d $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

e $\begin{pmatrix} 3 & -1 \end{pmatrix}$

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Solution:

a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$ is 2×2

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is 2×1

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ is 2×3

d $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ is 1×3

e $\begin{pmatrix} 3 & -1 \end{pmatrix}$ is 1×2

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 3×3

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Matrix algebra
Exercise A, Question 2

Question:

For the matrices

A$=\begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}$, **B**$=\begin{pmatrix}4 & 1\\-1 & -2\end{pmatrix}$, **C**$=\begin{pmatrix}6 & 0\\0 & 1\end{pmatrix}$,

find

a **A**+**C**

b **B**−**A**

c **A**+ **B**− **C** .

Solution:

a $\begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}+\begin{pmatrix}6 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}8 & -1\\1 & 4\end{pmatrix}$

b $\begin{pmatrix}4 & 1\\-1 & -2\end{pmatrix}-\begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}=\begin{pmatrix}2 & 2\\-2 & -5\end{pmatrix}$

c $\begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}+\begin{pmatrix}4 & 1\\-1 & -2\end{pmatrix}-\begin{pmatrix}6 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

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Matrix algebra
Exercise A, Question 3

Question:

For the matrices

A$=\begin{pmatrix}1\\2\end{pmatrix}$, **B**$=(1\ \ -1)$, **C**$=(-1\ \ 1\ \ 0)$,

D$=(0\ \ 1\ \ -1)$, **E**$=\begin{pmatrix}3\\-1\end{pmatrix}$, **F**$=(2\ \ 1\ \ 3)$,

find where possible:

a **A**+**B**

b **A**–**E**

c **F**– **D**+**C**

d **B**+**C**

e **F**–(**D**+**C**)

f **A**–**F**

g **C**–(**F**–**D**).

Solution:

a $A+B$ is $(2\times1)+(1\times2)$ Not possible

b $A-E=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}3\\-1\end{pmatrix}=\begin{pmatrix}-2\\3\end{pmatrix}$.

c

$$\begin{aligned}F-D+C &= (2\ \ 1\ \ 3)-(0\ \ 1\ \ -1)+(-1\ \ 1\ \ 0)\\ &= (1\ \ 1\ \ 4)\end{aligned}$$

d $B+C$ is $(1\times2)+(1\times3)$ Not possible

e

$$\begin{aligned}F-(D+C) &= (2\ \ 1\ \ 3)-[(0\ \ 1\ \ -1)+(-1\ \ 1\ \ 0)]\\ &= (2\ \ 1\ \ 3)-(-1\ \ 2\ \ -1)\\ &= (3\ \ -1\ \ 4)\end{aligned}$$

f $A-F=(2\times1)-(1\times3)$ Not possible.

g

$$\begin{aligned}C-(F-D) &= (-1\ \ 1\ \ 0)-[(2\ \ 1\ \ 3)-(0\ \ 1\ \ -1)]\\ &= (-1\ \ 1\ \ 0)-(2\ \ 0\ \ 4)\\ &= (-3\ \ 1\ \ -4)\end{aligned}$$

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Matrix algebra
Exercise A, Question 4

Question:

Given that $\begin{pmatrix} a & 2 \\ -1 & b \end{pmatrix} - \begin{pmatrix} 1 & c \\ d & -2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$, find the values of the constants *a*, *b*, *c* and *d*.

Solution:

$$\begin{aligned} a-1 &= 5 \Rightarrow a = 6 \\ 2-c &= 0 \Rightarrow c = 2 \\ -1-d &= 0 \Rightarrow d = -1 \\ b-(-2) &= 5 \Rightarrow b = 3 \end{aligned}$$

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Matrix algebra
Exercise A, Question 5

Question:

Given that $\begin{pmatrix} 1 & 2 & 0 \\ a & b & c \end{pmatrix} + \begin{pmatrix} a & b & c \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} c & 5 & c \\ c & c & c \end{pmatrix}$, find the values of a, b and c.

Solution:

$$\begin{aligned} 1+a &= c \quad ① \\ 2+b &= 5 \quad \Rightarrow \quad b=3 \\ 0+c &= c \\ a+1 &= c \\ b+2 &= c \quad ② \\ c+0 &= c \end{aligned}$$

Use $b=3$ in ② $\Rightarrow$ $c=5$

Use $c=5$ in ① $\Rightarrow$ $a=4$

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Matrix algebra
Exercise A, Question 6

Question:

Given that $\begin{pmatrix} 5 & 3 \\ 0 & -1 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ 2 & 0 \\ 1 & 4 \end{pmatrix}$, find the values of a, b, c, d, e and f.

Solution:

$5 + a = 7 \Rightarrow a = 2$
$3 + b = 1 \Rightarrow b = -2$
$0 + c = 2 \Rightarrow c = 2$
$-1 + d = 0 \Rightarrow d = 1$
$2 + e = 1 \Rightarrow e = -1$
$1 + f = 4 \Rightarrow f = 3$

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Matrix algebra
Exercise B, Question 1

Question:

For the matrices $\mathbf{A}=\begin{pmatrix}2 & 0\\4 & -6\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}1\\-1\end{pmatrix}$, find

a $3\mathbf{A}$

b $\frac{1}{2}\mathbf{A}$

c $2\mathbf{B}$.

Solution:

a $3\begin{pmatrix}2 & 0\\4 & -6\end{pmatrix}=\begin{pmatrix}6 & 0\\12 & -18\end{pmatrix}$

b $\frac{1}{2}\begin{pmatrix}2 & 0\\4 & -6\end{pmatrix}=\begin{pmatrix}1 & 0\\2 & -3\end{pmatrix}$

c $2\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}2\\-2\end{pmatrix}$

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Matrix algebra
Exercise B, Question 2

Question:

Find the value of k and the value of x so that $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} + k\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 7 \\ x & 0 \end{pmatrix}$.

Solution:

$$\begin{aligned} 1 + 2k &= 7 \\ \Rightarrow \quad 2k &= 6 \\ \Rightarrow \quad k &= 3 \\ 2 - k &= x \\ \Rightarrow \quad 2 - 3 &= x \\ \therefore \quad x &= -1 \end{aligned}$$

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Matrix algebra
Exercise B, Question 3

Question:

Find the values of a, b, c and d so that $2\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} - 3\begin{pmatrix} 1 & c \\ d & -1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -4 & -4 \end{pmatrix}$.

Solution:

$$\begin{aligned}
2a - 3 = 3 \quad &\Rightarrow \quad 2a = 6 \\
&\Rightarrow \quad a = 3 \\
0 - 3c = 3 \quad &\Rightarrow \quad c = -1 \\
2 - 3d = -4 \quad &\Rightarrow \quad -3d = -6 \\
&\Rightarrow \quad d = 2 \\
2b + 3 = -4 \quad &\Rightarrow \quad 2b = -7 \\
&\Rightarrow \quad b = -3.5
\end{aligned}$$

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Matrix algebra
Exercise B, Question 4

Question:

Find the values of a, b, c and d so that $\begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix} - 2\begin{pmatrix} c & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 9 & 1 \\ 3 & d \end{pmatrix}$.

Solution:

$$\begin{aligned} & 5 - 2c = 9 \\ \Rightarrow \quad & -4 = 2c \\ \Rightarrow \quad & c = -2 \\ & a - 4 = 1 \\ \Rightarrow \quad & a = 5 \\ & b - 2 = 3 \\ \Rightarrow \quad & b = 5 \\ & 0 + 2 = d \\ \Rightarrow \quad & d = 2 \end{aligned}$$

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Matrix algebra
Exercise B, Question 5

Question:

Find the value of k so that $\begin{pmatrix} -3 \\ k \end{pmatrix} + k\begin{pmatrix} 2k \\ 2k \end{pmatrix} = \begin{pmatrix} k \\ 6 \end{pmatrix}$.

Solution:

$$-3 + 2k^2 = k$$
$$\Longrightarrow \quad 2k^2 - k - 3 = 0$$
$$(2k-3)(k+1) = 0$$
$$\therefore \quad k = \frac{3}{2} \text{ or } -1$$

AND $k + 2k^2 = 6$
$$\Longrightarrow \quad 2k^2 + k - 6 = 0$$
$$(2k-3)(k+2) = 0$$
$$\therefore \quad k = \frac{3}{2} \text{ or } -2$$

So common value is $k = \frac{3}{2}$

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Matrix algebra
Exercise C, Question 1

Question:

Given the dimensions of the following matrices:

Matrix	**A**	**B**	**C**	**D**	**E**
Dimension	2×2	1×2	1×3	3×2	2×3

Give the dimensions of these matrix products.

a BA

b DE

c CD

d ED

e AE

f DA

Solution:

a $(1 \times 2) \cdot (2 \times 2) = 1 \times 2$

b $(3 \times 2) \cdot (2 \times 3) = 3 \times 3$

c $(1 \times 3) \cdot (3 \times 2) = 1 \times 2$

d $(2 \times 3) \cdot (3 \times 2) = 2 \times 2$

e $(2 \times 2) \cdot (2 \times 3) = 2 \times 3$

f $(3 \times 2) \cdot (2 \times 2) = 3 \times 2$

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Matrix algebra
Exercise C, Question 2

Question:

Find these products.

a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

b $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ -1 & -2 \end{pmatrix}$

Solution:

a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

b $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -4 & 7 \end{pmatrix}$

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Matrix algebra
Exercise C, Question 3

Question:

The matrix **A**= $\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}$ and the matrix B= $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

Find

$\mathbf{A}^2$ means $\mathbf{A} \times \mathbf{A}$

a AB

b $\mathbf{A}^2$

Solution:

a $\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & -2 & -1 \\ 3 & 3 & 0 \end{pmatrix}$

b $\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 0 & 9 \end{pmatrix}$

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Matrix algebra

Exercise C, Question 4

Question:

The matrices **A, B** and **C** are given by

$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} -3 & -2 \end{pmatrix}$

Determine whether or not the following products are possible and find the products of those that are.

a AB

b AC

c BC

d BA

e CA

f CB

Solution:

a AB is $(2 \times 1) \cdot (2 \times 2)$ Not possible

b $\mathbf{AC} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}\begin{pmatrix} -3 & -2 \end{pmatrix} = \begin{pmatrix} -6 & -4 \\ -3 & -2 \end{pmatrix}$

c BC is $(2 \times 2) \cdot (1 \times 2)$ Not possible

d $\mathbf{BA} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

e $\mathbf{CA} = \begin{pmatrix} -3 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \end{pmatrix}$.

f $\mathbf{CB} = \begin{pmatrix} -3 & -2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -7 & -7 \end{pmatrix}$

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Matrix algebra
Exercise C, Question 5

Question:

Find in terms of $a \begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 6-a & 2a \\ 1 & 4 & -2 \end{pmatrix}$$

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Matrix algebra
Exercise C, Question 6

Question:

Find in terms of $x \begin{pmatrix} 3 & 2 \\ -1 & x \end{pmatrix}\begin{pmatrix} x & -2 \\ 1 & 3 \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 3 & 2 \\ -1 & x \end{pmatrix}\begin{pmatrix} x & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3x+2 & 0 \\ 0 & 3x+2 \end{pmatrix}$$

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Matrix algebra
Exercise C, Question 7

Question:

The matrix $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Find

a $\mathbf{A}^2$

b $\mathbf{A}^3$

c Suggest a form for $\mathbf{A}^k$.

You might be asked to prove this formula for $\mathbf{A}^k$ in FP1 using induction from Chapter 6.

Solution:

a $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

b $A^3 = AA^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$

Note $A^2 = \begin{pmatrix} 1 & 2 \times 2 \\ 0 & 1 \end{pmatrix}$

$A^3 = \begin{pmatrix} 1 & 2 \times 3 \\ 0 & 1 \end{pmatrix}$

Suggests $A^k = \begin{pmatrix} 1 & 2 \times k \\ 0 & 1 \end{pmatrix}$

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Matrix algebra
Exercise C, Question 8

Question:

The matrix $\mathbf{A}=\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$.

a Find, in terms of a and b, the matrix $\mathbf{A}^2$.

Given that $\mathbf{A}^2 = 3\mathbf{A}$

b find the value of a.

Solution:

a $A^2 = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} = \begin{pmatrix} a^2 & 0 \\ ab & 0 \end{pmatrix}$

b $A^2 = 3\,A \Rightarrow \begin{pmatrix} a^2 & 0 \\ ab & 0 \end{pmatrix} = \begin{pmatrix} 3a & 0 \\ 3b & 0 \end{pmatrix}$

$\Rightarrow \quad a^2 = 3a \quad \Rightarrow \quad a = 3 \quad (\text{or } 0)$
and $\quad ab = 3b \quad \Rightarrow \quad a = 3$
$\therefore \quad a = 3$

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Matrix algebra
Exercise C, Question 9

Question:

$\mathbf{A} = \begin{pmatrix} -1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}$.

Find **a BAC**

b $\mathbf{AC}^2$

Solution:

a

$$\text{BAC} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} -4 & -7 \end{pmatrix}$$
$$= \begin{pmatrix} -8 & -14 \\ -4 & -7 \\ 0 & 0 \end{pmatrix}$$

b

$$\text{AC}^2 = \begin{pmatrix} -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} -4 & -7 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} -16 & 29 \end{pmatrix}$$

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Matrix algebra
Exercise C, Question 10

Question:

A= $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, **B**= $(3 \quad -2 \quad -3)$.

Find **a ABA**

b BAB

Solution:

a

$$\text{ABA} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}(3 \quad -2 \quad -3)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}(-1)$$
$$= \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

b

$$\text{BAB} = (3 \quad -2 \quad -3)\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}(3 \quad -2 \quad -3)$$
$$= (-1)(3 \quad -2 \quad -3)$$
$$= (-3 \quad 2 \quad 3)$$

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Matrix algebra
Exercise D, Question 1

Question:

Which of the following are not linear transformations?

a **P**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x \\ y+1 \end{pmatrix}$

b **Q**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x^2 \\ y \end{pmatrix}$

c **R**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x+y \\ x+xy \end{pmatrix}$

d **S**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 3y \\ -x \end{pmatrix}$

e **T**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y+3 \\ x+3 \end{pmatrix}$

f **U**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x \\ 3y-2x \end{pmatrix}$

Solution:

a P is not $\because$ $(0,0) \to (0,1)$

b Q is not $\because$ $x \to x^2$ is not linear

c R is not $\because$ $y \to x + xy$ is not linear

d S is linear

e T is not $\because$ $(0,0) \to (3,3)$

f U is linear.

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Matrix algebra
Exercise D, Question 2

Question:

Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a **S**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2x - y \\ 3x \end{pmatrix}$

b **T**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2y + 1 \\ x - 1 \end{pmatrix}$

c **U**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} xy \\ 0 \end{pmatrix}$

d **V**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2y \\ -x \end{pmatrix}$

e **W**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ x \end{pmatrix}$

Solution:

a S is represented by $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$

b T is not linear $\because$ $(0,0) \to (1, -1)$

c U is not linear $\because$ $x \to xy$ is not linear

d V is represented by $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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Matrix algebra
Exercise D, Question 3

Question:

Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a **S**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$

b **T**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$

c **U**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x-y \\ x-y \end{pmatrix}$

d **V**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

e **W**: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

a S is not linear $\because$ $x \to x^2$ and $y \to y^2$ are not linear

b T is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

c U is represented by $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

d V is represented by $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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Matrix algebra
Exercise D, Question 4

Question:

Find matrix representations for these linear transformations.

a $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y+2x \\ -y \end{pmatrix}$

b $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x+2y \end{pmatrix}$

Solution:

a $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y+2x \\ -y \end{pmatrix} = \begin{pmatrix} 2x+y \\ 0x-y \end{pmatrix}$ is represented by $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

b $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 0-y \\ x+2y \end{pmatrix}$ is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

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Matrix algebra
Exercise D, Question 5

Question:

The triangle T has vertices at $(-1,\ 1)$, $(2,\ 3)$ and $(5,\ 1)$.

Find the vertices of the image of T under the transformations represented by these matrices.

a $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

b $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$

c $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Solution:

a $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -5 \\ 1 & 3 & 1 \end{pmatrix}$

$\therefore$ vertices of image of T are at $(1,1)$; $(-1,3)$; $(-5,1)$

b $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 14 & 9 \\ -2 & -6 & -2 \end{pmatrix}$

$\therefore$ vertices of image of T are at $(3,-2)$; $(14,-6)$; $(9,-2)$

c $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -6 & -2 \\ -2 & 4 & 10 \end{pmatrix}$

$\therefore$ vertices of image of T are at $(-2,-2)$; $(-6,4)$; $(-2,10)$

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Matrix algebra
Exercise D, Question 6

Question:

The square S has vertices at $(-1,\ 0)$, (0, 1), (1, 0) and $(0\ ,\ -1)$.

Find the vertices of the image of S under the transformations represented by these matrices.

a $\begin{pmatrix}2 & 0\\0 & 3\end{pmatrix}$

b $\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$

c $\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$

Solution:

a $\begin{pmatrix}2 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-2 & 0 & 2 & 0\\0 & 3 & 0 & -3\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-2,0)$: $(0,3)$; $(2,0)$; $(0\ ,-3)$

b $\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-1 & -1 & 1 & 1\\-1 & 1 & 1 & -1\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-1\ ,-1)$; $(-1,1)$; $(1,1)$; $(1\ ,-1)$

c $\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-1 & 1 & 1 & -1\\-1 & -1 & 1 & 1\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-1\ ,-1)$; $(1\ ,-1)$; $(1,1)$; $(-1,1)$

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Matrix algebra
Exercise E, Question 1

Question:

Describe fully the geometrical transformations represented by these matrices.

a $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

b $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

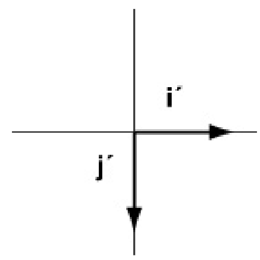
c $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Solution:

a

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

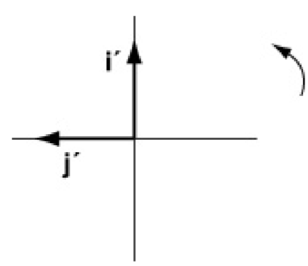


Reflection is x-axis (or line $y = 0$)

b

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

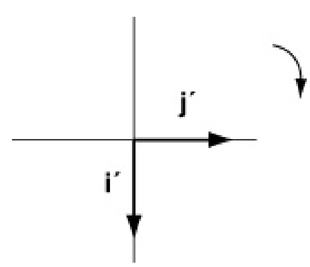


Rotation 90° anticlockwise about (0,0)

c

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



Rotation 90° clockwise (or 270° anticlockwise) about (0,0)

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Matrix algebra
Exercise E, Question 2

Question:

Describe fully the geometrical transformations represented by these matrices.

a $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

b $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

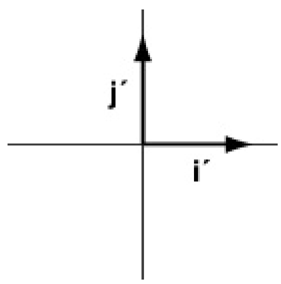
c $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Solution:

a

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$

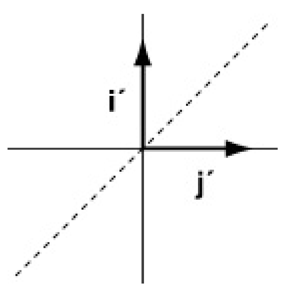


Enlargement - scale factor $\frac{1}{2}$ centre (0,0)

b

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

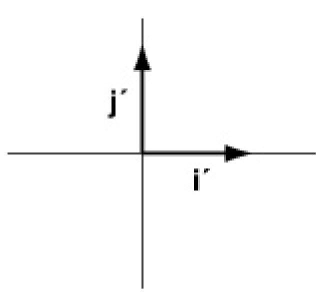


Reflection in line $y = x$

c

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



No change so this is the Identity matrix.

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Matrix algebra
Exercise E, Question 3

Question:

Describe fully the geometrical transformations represented by these matrices.

a $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

b $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

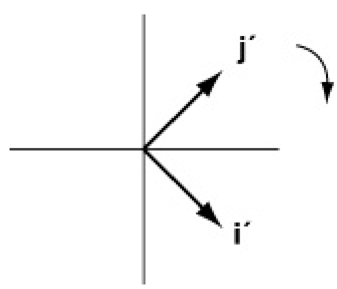
c $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$

Solution:

a

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

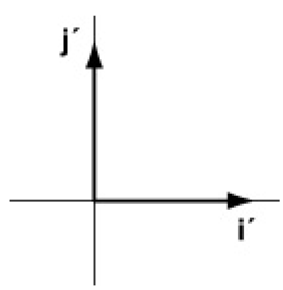


Rotation 45° clockwise about (0,0)

b

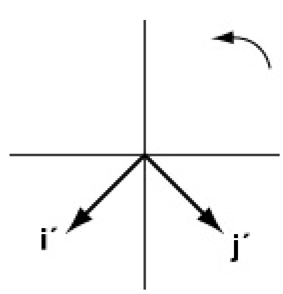
$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 4 \end{pmatrix}$



Enlargement Scale factor 4 centre (0,0)

c



$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}-\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}$$

Rotation 225° anti-clockwise about (0,0) or 135° clockwise

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Matrix algebra
Exercise E, Question 4

Question:

Find the matrix that represents these transformations.

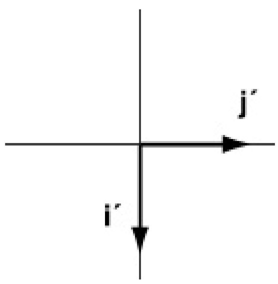
a Rotation of 90° clockwise about (0, 0).

b Reflection in the *x*-axis.

c Enlargement centre (0, 0) scale factor 2.

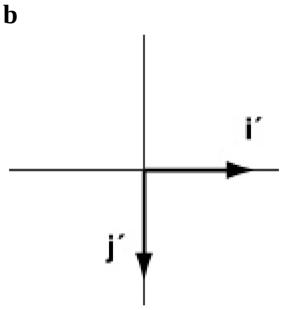
Solution:

a



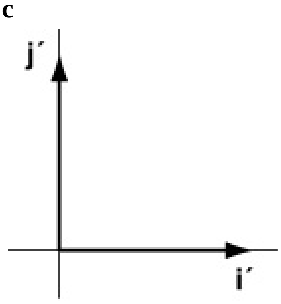
$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix} \quad \therefore \text{ Matrix is } \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$$

b



$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix} \quad \therefore \text{ Matrix is } \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

c



$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}2\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\2\end{pmatrix} \quad \therefore \text{ Matrix is } \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$$

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Matrix algebra
Exercise E, Question 5

Question:

Find the matrix that represents these transformations.

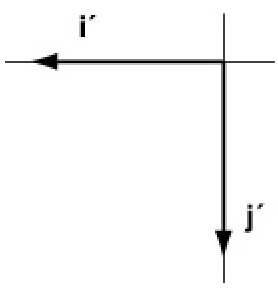
a Enlargement scale factor −4 centre (0, 0).

b Reflection in the line $y = x$.

c Rotation about (0, 0) of 135° anticlockwise.

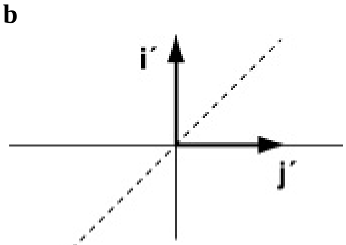
Solution:

a



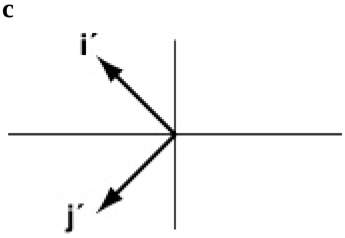
$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}-4\\0\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\-4\end{pmatrix}$$
$\therefore$ Matrix is $\begin{pmatrix}-4 & 0\\0 & -4\end{pmatrix}$

b



$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\1\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$$
$\therefore$ Matrix is $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

c



$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}-\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}$$
$\therefore$ Matrix is $\begin{pmatrix}-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$

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Matrix algebra
Exercise F, Question 1

Question:

A= $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, **B**= $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, **C**= $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Find these matrix products and describe the single transformation represented by the product.

a AB

b BA

c AC

d A^2

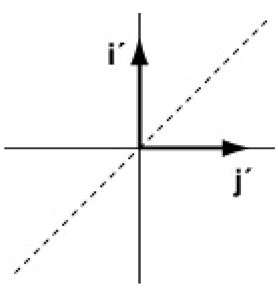
e C^2

Solution:

a $AB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



Reflection in $y = x$

b $BA = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in $y = x$

c $AC = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ Enlargement scale facter -2 centre $(0,0)$

d $A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity (No transformation)

[This can be thought of as a rotation of $180^\circ + 180^\circ = 360^\circ$]

e $C^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

Enlargement scale facter 4 centre (0,0)

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Matrix algebra
Exercise F, Question 2

Question:

A = rotation of 90° anticlockwise about (0, 0)
B = rotation of 180° about (0, 0)
C = reflection in the x-axis
D = reflection in the y-axis

a Find matrix representations of each of the four transformations A, B, C and D.

b Use matrix products to identify the single geometric transformation represented by each of these combinations.

i Reflection in the x-axis followed by a rotation of 180° about (0, 0).

ii Rotation of 180° about (0, 0) followed by a reflection in the x-axis.

iii Reflection in the y-axis followed by reflection in the x-axis.

iv Reflection in the y-axis followed by rotation of 90° about (0, 0).

v Rotation of 180° about (0, 0) followed by a second rotation of 180° about (0, 0).

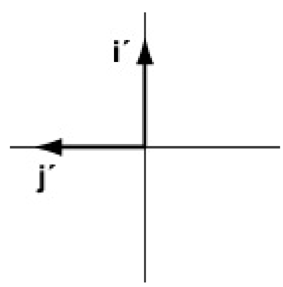
vi Reflection in the x-axis followed by rotation of 90° about (0, 0) followed by a reflection in the y-axis.

vii Reflection in the y-axis followed by rotation of 180° about (0, 0) followed by a reflection in the x-axis.

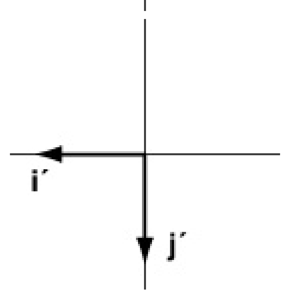
Solution:

a

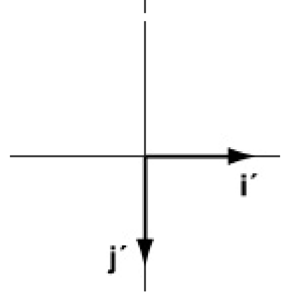
Rotation of 90° anticlockwise $A=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$



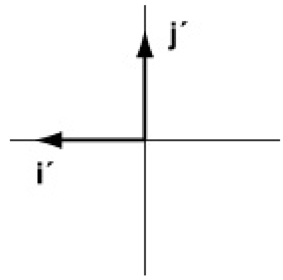
Rotation of 180° about (0,0) $B=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$



Reflection in x-axis $C=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$



Reflection in *y*-axis $\quad D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$



b

i $BC = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (=D)

Reflection in *y*-axis

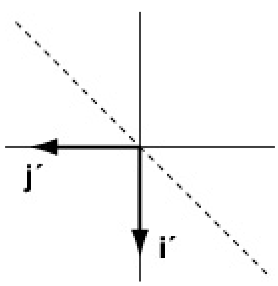
ii $CB = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (=D)

Reflection in *y*-axis

iii $CD = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (=B)

Rotation of 180° about (0,0)

iv $AD = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$



Reflection in line $y = -x$

v $BB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Rotation of 360° about (0, 0) or Identity

vi

$$\begin{aligned} DAC &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (= A) \end{aligned}$$

Rotation of 90° anticlockwise about (0, 0)

vii

$$\text{CBD} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

Identity - no transformation

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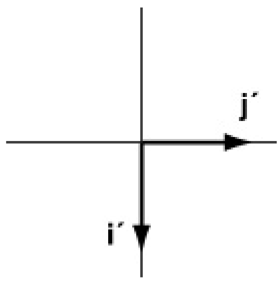
Matrix algebra
Exercise F, Question 3

Question:

Use a matrix product to find the single geometric transformation represented by a rotation of 270° anticlockwise about (0, 0) followed by a refection in the *x*-axis.

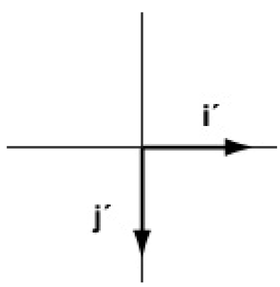
Solution:

Rotation of 270° about (0,0)



$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\-1\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \therefore \text{ Matrix is } \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}.$$

Reflection is *x*-axis

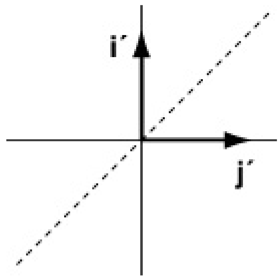


$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0\\-1\end{pmatrix} \therefore \text{ Matrix is } \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}.$$

Rotation of 270 followed by reflection in *x*-axis is:

$$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix}$$
$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$$



Reflection is $y = x$

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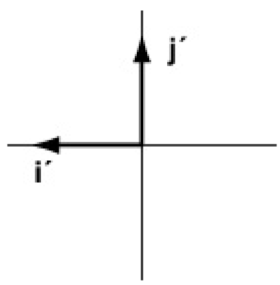
Matrix algebra
Exercise F, Question 4

Question:

Use matrices to show that a refection in the y-axis followed by a reflection in the line $y = -x$ is equivalent to a rotation of 90° anticlockwise about (0, 0).

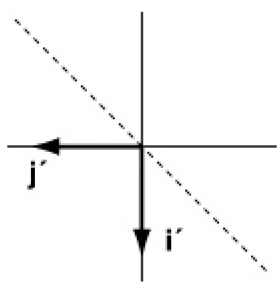
Solution:

Reflection in y-axis



$\therefore$ Matrix is $Y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Reflection in $y = -x$

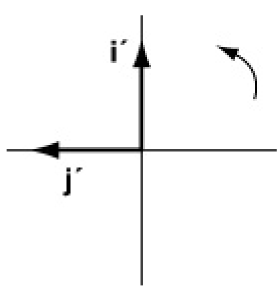


$\therefore$ Matrix $R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$RY = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$



i.e. Rotation of 90° anticlockwise about (0, 0).

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Matrix algebra
Exercise F, Question 5

Question:

The matrix **R** is given by $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

a Find $\mathbf{R}^2$.

b Describe the geometric transformation represented by $\mathbf{R}^2$.

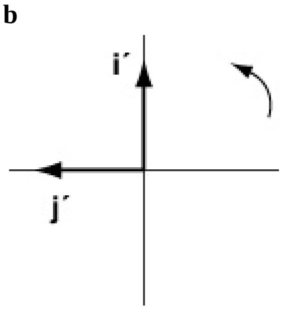
c Hence describe the geometric transformation represented by **R**.

d Write down $\mathbf{R}^8$.

Solution:

a $R^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

b



i.e. R^2 represents rotation of 90° anticlockwise about (0, 0)

c R represents a rotation of 45° anticlockwise about (0, 0)

d R^8 will represent rotation of $8 \times 45^\circ = 360^\circ$

This is equivalent to no transformation

$\therefore \quad R^8 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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Matrix algebra
Exercise F, Question 6

Question:

P= $\begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$, **Q**= $\begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix}$

The transformation represented by the matrix **R** is the result of the transformation represented by the matrix **P** followed by the transformation represented by the matrix **Q**.

a Find **R**.

b Give a geometrical interpretation of the transformation represented by **R**.

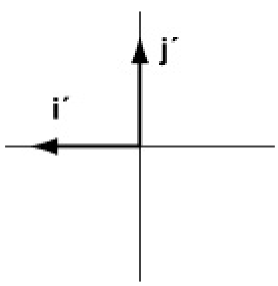
Solution:

a R= QP = $\begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

b

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



Reflection in *y*-axis

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Matrix algebra
Exercise F, Question 7

Question:

A= $\begin{pmatrix} 5 & -7 \\ 7 & -10 \end{pmatrix}$, **B**= $\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$, **C**= $\begin{pmatrix} -2 & 1 \\ -1 & 1 \end{pmatrix}$

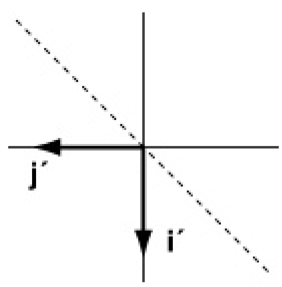
Matrices **A, B** and **C** represent three transformations. By combining the three transformations in the order **B**, followed by **A**, followed by **C** a single transformation is obtained.

Find a matrix representation of this transformation and interpret it geometrically.

Solution:

$$\text{CAB} = \begin{pmatrix} -2 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & -7 \\ 7 & -10 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -3 & 4 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$



Reflection in the line $y = -x$

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Matrix algebra
Exercise F, Question 8

Question:

P= $\begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}$, **Q**= $\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$, **R**= $\begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$

Matrices **P, Q** and **R** represent three transformations. By combining the three transformations in the order **R**, followed by **Q**, followed by **P** a single transformation is obtained.

Find a matrix representation of this transformation and interpret it geometrically.

Solution:

$$\begin{aligned} \text{PQR} &= \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \end{aligned}$$

Enlargement scale factor 8

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Matrix algebra
Exercise G, Question 1

Question:

Determine which of these matrices are singular and which are non-singular. For those that are non-singular find the inverse matrix.

a $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

b $\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}$

c $\begin{pmatrix} 2 & 5 \\ 0 & 0 \end{pmatrix}$

d $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$

e $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$

f $\begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$

Solution:

a

$$\det\begin{vmatrix} 3 & -1 \\ -4 & 2 \end{vmatrix} = 6-(-4)\times(-1)$$
$$= 6-4$$
$$= 2 \quad \neq 0$$

$\therefore$ the Matrix is non-singular

So inverse is $\frac{1}{2}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

or $\begin{pmatrix} 1 & 0.5 \\ 2 & 1.5 \end{pmatrix}$

b

$$\det\begin{vmatrix} 3 & 3 \\ -1 & -1 \end{vmatrix} = -3-(-1)\times 3$$
$$= -3+3$$
$$= 0$$

$\therefore$ Matrix is singular.

c

$$\det\begin{vmatrix} 2 & 5 \\ 0 & 0 \end{vmatrix} = 0-0$$
$$= 0$$

$\therefore$ Matrix is singular

d

$$\det\begin{vmatrix}1 & 2\\3 & 5\end{vmatrix} = 5-6$$
$$= -1 \quad \neq 0$$

$\therefore$ Matrix is non-singular

Inverse is $\frac{1}{-1}\begin{pmatrix}5 & -2\\-3 & 1\end{pmatrix} = \begin{pmatrix}-5 & 2\\3 & -1\end{pmatrix}$

e

$$\det\begin{vmatrix}6 & 3\\4 & 2\end{vmatrix} = 12-12$$
$$= 0$$

$\therefore$ Matrix is singular

f

$$\det\begin{vmatrix}4 & 3\\6 & 2\end{vmatrix} = 8-18$$
$$= -10 \quad \neq 0$$

$\therefore$ Matrix is non-singular

Inverse is $\frac{1}{-10}\begin{pmatrix}2 & -3\\-6 & 4\end{pmatrix}$

$$= \begin{pmatrix}-0.2 & 0.3\\0.6 & -0.4\end{pmatrix}$$

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Matrix algebra
Exercise G, Question 2

Question:

Find the value of a for which these matrices are singular.

a $\begin{pmatrix} a & 1+a \\ 3 & 2 \end{pmatrix}$

b $\begin{pmatrix} 1+a & 3-a \\ a+2 & 1-a \end{pmatrix}$

c $\begin{pmatrix} 2+a & 1-a \\ 1-a & a \end{pmatrix}$

Solution:

a

$$\det \begin{vmatrix} a & 1+a \\ 3 & 2 \end{vmatrix} = 2a - 3(1+a)$$
$$= 2a - 3 - 3a$$
$$= -3 - a$$

Matrix is singular for $a = -3$

b

$$\text{Let A} = \begin{pmatrix} 1+a & 3-a \\ a+2 & 1-a \end{pmatrix}$$
$$\det \text{A} = (1+a)(1-a) - (3-a)(a+2)$$
$$= 1 - a^2 - (-a^2 + a + 6)$$
$$= 1 - a^2 + a^2 - a - 6$$
$$= -a - 5$$
$$\det \text{A} = 0 \quad \Rightarrow \quad a = -5$$

c

$$\text{Let B} = \begin{pmatrix} 2+a & 1-a \\ 1-a & a \end{pmatrix}$$
$$\det \text{B} = 2a + a^2 - (1-a)^2$$
$$= 2a + a^2 - 1 + 2a - a^2$$
$$= 4a - 1$$
$$\det \text{B} = 0 \quad \Rightarrow \quad a = \frac{1}{4}$$

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Matrix algebra
Exercise G, Question 3

Question:

Find inverses of these matrices.

a $\begin{pmatrix} a & 1+a \\ 1+a & 2+a \end{pmatrix}$

b $\begin{pmatrix} 2a & 3b \\ -a & -b \end{pmatrix}$

Solution:

a

$$\begin{aligned}
\text{Let A} &= \begin{pmatrix} a & 1+a \\ 1+a & 2+a \end{pmatrix} \\
\det \text{A} &= 2a + a^2 - (1+a)^2 \\
&= 2a + a^2 - 1 - 2a - a^2 \\
&= -1 \\
\text{A}^{-1} &= \frac{1}{-1}\begin{pmatrix} 2+a & -(1+a) \\ -(1+a) & a \end{pmatrix} = \begin{pmatrix} -[2+a] & (1+a) \\ (1+a) & -a \end{pmatrix}
\end{aligned}$$

b

$$\begin{aligned}
\text{Let B} &= \begin{pmatrix} 2a & 3b \\ -a & -b \end{pmatrix} \\
\det \text{B} &= -2ab - (-a) \times 3b \\
&= -2ab + 3ab \\
&= ab \\
\text{B}^{-1} &= \frac{1}{ab}\begin{pmatrix} -b & -3b \\ a & 2a \end{pmatrix} \\
&= \begin{pmatrix} -\frac{1}{a} & -\frac{3}{a} \\ \frac{1}{b} & \frac{2}{b} \end{pmatrix} \quad \text{provided that } ab \neq 0
\end{aligned}$$

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Matrix algebra
Exercise G, Question 4

Question:

a Given that $\mathbf{ABC} = \mathbf{I}$, prove that $\mathbf{B}^{-1} = \mathbf{CA}$.

b Given that $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}$, find $\mathbf{B}$.

Solution:

a

$$\begin{aligned} ABC &= I \\ \Rightarrow\quad A^{-1}ABC &= A^{-1}I \\ \Rightarrow\quad BC &= A^{-1} \\ \Rightarrow\quad BCC^{-1} &= A^{-1}C^{-1} \\ \Rightarrow\quad B &= A^{-1}C^{-1} = (CA)^{-1} \\ \therefore\quad B^{-1} &= CA \end{aligned}$$

b

$$\begin{aligned} CA &= \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix} \\ \therefore\quad (CA)^{-1} &= \frac{1}{-3+4}\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \\ \therefore\quad B &= \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \end{aligned}$$

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Matrix algebra
Exercise G, Question 5

Question:

a Given that **AB** =**C**, find an expression for **B**.

b Given further that **A**= $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ and **C**= $\begin{pmatrix} 3 & 6 \\ 1 & 22 \end{pmatrix}$, find **B**.

Solution:

a

$$\begin{aligned} AB &= C \\ \Rightarrow\ A^{-1}AB &= A^{-1}C \\ \Rightarrow\ B &= A^{-1}C \end{aligned}$$

b

$$A = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} \quad \Rightarrow \det A = 6 - -4 = 10$$

$$\therefore\ A^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$$

$$\begin{aligned} \therefore\ B &= A^{-1}C \\ &= \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ 1 & 22 \end{pmatrix} \\ &= \frac{1}{10}\begin{pmatrix} 10 & 40 \\ -10 & 20 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 4 \\ -1 & 2 \end{pmatrix} \end{aligned}$$

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Matrix algebra
Exercise G, Question 6

Question:

a Given that **BAC** =**B**, where **B** is a non-singular matrix, find an expression for **A**.

b When **C**= $\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, find **A**.

Solution:

a

$$\begin{aligned} & \text{BAC} = \text{B} \\ \Rightarrow\ & \text{B}^{-1}\text{BAC} = \text{B}^{-1}\text{B} \\ \Rightarrow\ & \text{AC} = \text{I} \\ \Rightarrow\ & \text{A} = \text{C}^{-1} \end{aligned}$$

b

$$\text{C} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$\det \text{C} = 10 - 9 = 1$$

$$\therefore \quad \text{C}^{-1} = \frac{1}{1}\begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

$$\therefore \quad \text{A} = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$$

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Matrix algebra
Exercise G, Question 7

Question:

The matrix $\mathbf{A}=\begin{pmatrix}2 & -1\\ -4 & 3\end{pmatrix}$ and $\mathbf{AB}=\begin{pmatrix}4 & 7 & -8\\ -8 & -13 & 18\end{pmatrix}$. Find the matrix **B**.

Solution:

$$\mathrm{A} = \begin{pmatrix}2 & -1\\ -4 & 3\end{pmatrix} \quad \Rightarrow \quad \det \mathrm{A} = 6-(-4)\times(-1)=2$$

$$\therefore \quad \mathrm{A}^{-1} = \frac{1}{2}\begin{pmatrix}3 & 1\\ 4 & 2\end{pmatrix}$$

$$\mathrm{AB} = \begin{pmatrix}4 & 7 & -8\\ -8 & -13 & 18\end{pmatrix} \quad (\times \text{ on left by} \mathrm{A}^{-1})$$

$$\Rightarrow \quad \mathrm{B} = \frac{1}{2}\begin{pmatrix}3 & 1\\ 4 & 2\end{pmatrix}\begin{pmatrix}4 & 7 & -8\\ -8 & -13 & 18\end{pmatrix}$$

$$\mathrm{B} = \frac{1}{2}\begin{pmatrix}4 & 8 & -6\\ 0 & 2 & 4\end{pmatrix}$$

$$\mathrm{B} = \begin{pmatrix}2 & 4 & -3\\ 0 & 1 & 2\end{pmatrix}$$

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Matrix algebra
Exercise G, Question 8

Question:

The matrix **B**= $\begin{pmatrix} 5 & -4 \\ 2 & 1 \end{pmatrix}$ and **AB** = $\begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix}$. Find the matrix **A**.

Solution:

$$B = \begin{pmatrix} 5 & -4 \\ 2 & 1 \end{pmatrix} \quad \Rightarrow \det B = 5 + 8 = 13$$

$$B^{-1} = \frac{1}{13}\begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix}$$

$$AB = \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix} \quad (\times \text{ on right by } B^{-1})$$

$$\Rightarrow \quad ABB^{-1} = \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix}B^{-1}$$

$$\therefore \quad A = \frac{1}{13}\begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix}$$

$$= \frac{1}{13}\begin{pmatrix} 13 & 39 \\ -26 & 13 \\ 0 & -13 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 0 & -1 \end{pmatrix}$$

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Matrix algebra
Exercise G, Question 9

Question:

The matrix **A**= $\begin{pmatrix} 3a & b \\ 4a & 2b \end{pmatrix}$, where a and b are non-zero constants.

a Find $\mathbf{A}^{-1}$.

The matrix **B**= $\begin{pmatrix} -a & b \\ 3a & 2b \end{pmatrix}$ and the matrix **X** is given by **B**= **XA**.

b Find **X**.

Solution:

a

$$A = \begin{pmatrix} 3a & b \\ 4a & 2b \end{pmatrix} \quad \Rightarrow \quad \det A = 6ab - 4ab = 2ab$$

$$\therefore \quad A^{-1} = \frac{1}{2ab}\begin{pmatrix} 2b & -b \\ -4a & 3a \end{pmatrix}$$

b

$$B = XA$$
$$\Rightarrow \quad BA^{-1} = XAA^{-1}$$
$$\therefore \quad X = BA^{-1}$$
$$\text{So} \quad X = \begin{pmatrix} -a & b \\ 3a & 2b \end{pmatrix}\begin{pmatrix} 2b & -b \\ -4a & 3a \end{pmatrix} \times \frac{1}{2ab}$$
$$= \frac{1}{2ab}\begin{pmatrix} -6ab & 4ab \\ -2ab & 3ab \end{pmatrix}$$
$$\therefore \quad X = \begin{pmatrix} -3 & 2 \\ -1 & 3/2 \end{pmatrix}$$

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Matrix algebra
Exercise G, Question 10

Question:

The matrix $\mathbf{A}=\begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix}$ and the matrix $\mathbf{B}=\begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix}$.

a Find det (**A**) and det (**B**).

b Find **AB**.

Solution:

a

$$\text{A} = \begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix} \Rightarrow \det \text{A} = 2ab - 2ab = 0$$

$$\text{B} = \begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix} \Rightarrow \det \text{B} = 2ab - 2ab = 0$$

b

$$\begin{aligned} \text{AB} &= \begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix}\begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix} \\ &= \begin{pmatrix} 2ab - 2ab & -2a^2 + 2a^2 \\ 2b^2 - 2b^2 & -2ab + 2ab \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

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Matrix algebra
Exercise G, Question 11

Question:

The non-singular matrices **A** and **B** are commutative (i.e. $\mathbf{AB} = \mathbf{BA}$) and $\mathbf{ABA} = \mathbf{B}$.

a Prove that $\mathbf{A}^2 = \mathbf{I}$.

Given that $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, by considering a matrix **B** of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

b show that $a = d$ and $b = c$.

Solution:

a

Given $\quad AB = BA$

and $\quad ABA = B$

$\Rightarrow \quad A(AB) = B$

$\Rightarrow \quad A^2 B = B$

$\Rightarrow \quad A^2 BB^{-1} = BB^{-1}$

$\Rightarrow \quad A^2 = I$

b

$$\mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ a & b \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} b & a \\ d & c \end{pmatrix}$$

$\mathbf{AB} = \mathbf{BA} \Rightarrow \quad b = c$

$\qquad d = a$

i.e. $a = d$ and $b = c$

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Matrix algebra
Exercise H, Question 1

Question:

The matrix $\mathbf{R}=\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

a Give a geometrical interpretation of the transformation represented by **R**.

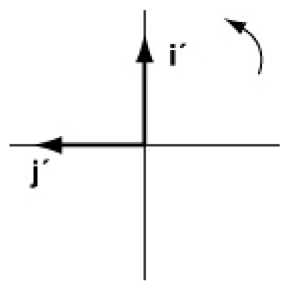
b Find $\mathbf{R}^{-1}$.

c Give a geometrical interpretation of the transformation represented by $\mathbf{R}^{-1}$.

Solution:

a

$$(1,0) \to (0,1)$$
$$(0,1) \to (-1,0)$$



R represents a rotation of 90° anticlockwise about (0, 0)

b

$$\det \mathbf{R} = 0 - -1 = 1$$
$$\therefore \quad \mathbf{R}^{-1} = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$$

c $\mathbf{R}^{-1}$represents a rotation of −90° anticlockwise about (0,0)

(or … 90° clockwise … or … 270° anticlockwise …)

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Matrix algebra
Exercise H, Question 2

Question:

a The matrix $\mathbf{S} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

i Give a geometrical interpretation of the transformation represented by **S**.

ii Show that $\mathbf{S}^2 = \mathbf{I}$.

iii Give a geometrical interpretation of the transformation represented by $\mathbf{S}^{-1}$.

b The matrix $\mathbf{T} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

i Give a geometrical interpretation of the transformation represented by **T**.

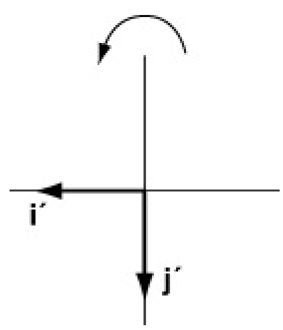
ii Show that $\mathbf{T}^2 = \mathbf{I}$.

iii Give a geometrical interpretation of the transformation represented by $\mathbf{T}^{-1}$.

c Calculate det(**S**) and det(**T**) and comment on their values in the light of the transformations they represent.

Solution:

a i $(1,0) \rightarrow (-1,0)$
$(0,1) \rightarrow (0,-1)$



S represents a rotation of 180° about (0,0)

ii $\mathbf{S}^2$ will be a rotation of $180 + 180 = 360^\circ$ about (0,0) $\therefore \mathbf{S}^2 = \mathbf{I}$

or $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$

iii $\mathbf{S}^{-1} = \mathbf{S} =$ rotation of 180° about (0,0)

b i

$(1,0) \rightarrow (0,-1)$
$(0,1) \rightarrow (-1,0)$

T represents a reflection in the line $y = -x$

ii $\mathbf{T}^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$

iii $\mathbf{T}^{-1} = \mathbf{T}$ = reflection in the line $y = -x$

c

det $\mathbf{S}$ $= 1 - 0 = 1$
det $\mathbf{T}$ $= 0 - 1 = -1$

For both **S** and **T**, area is unaltered

T represents a reflection and $\therefore$ has a negative determinant. Orientation is reversed

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Matrix algebra
Exercise H, Question 3

Question:

The matrix **A** represents a reflection in the line $y = x$ and the matrix **B** represents a rotation of 270° about (0, 0).

a Find the matrix **C**= **BA** and interpret it geometrically.

b Find $\mathbf{C}^{-1}$ and give a geometrical interpretation of the transformation represented by $\mathbf{C}^{-1}$.

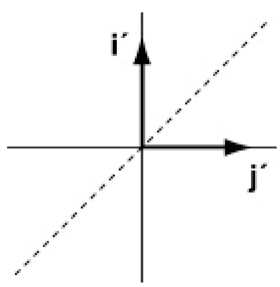
c Find the matrix **D**= **AB** and interpret it geometrically.

d Find $\mathbf{D}^{-1}$ and give a geometrical interpretation of the transformation represented by $\mathbf{D}^{-1}$.

Solution:

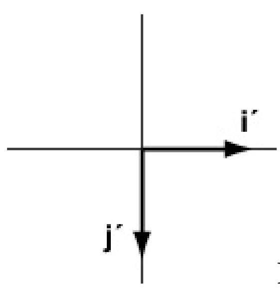
a

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



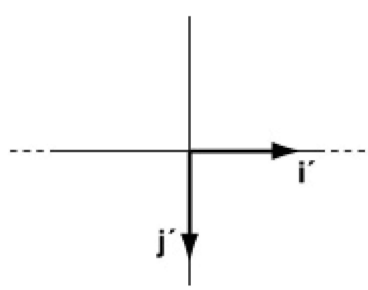
Reflection in $y = x$

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



Rotation of 270° (about (0,0))

$$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

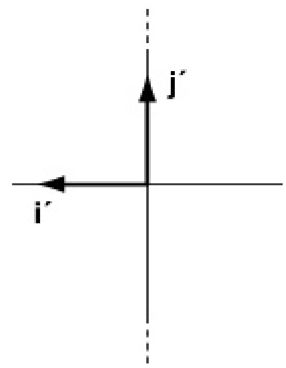


C represents a reflection in the line $y = 0$ (or the x-axis)

b $\mathbf{C}^{-1} = \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is a reflection in the line $y = 0$

c

$$\mathbf{D} = \mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



D represents a reflection in the line $x = 0$ (or the y-axis)

d $\mathbf{D}^{-1} = \mathbf{D} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ is a reflection in the line $x = 0$

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Matrix algebra
Exercise I, Question 1

Question:

The matrix **A**= $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ is used to transform the rectangle *R* with vertices at the points (0, 0), (0, 1), (4, 1) and (4, 0).

a Find the coordinates of the vertices of the image of *R*.

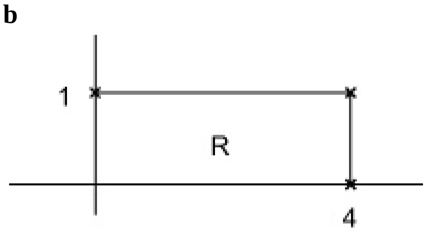
b Calculate the area of the image of *R*.

Solution:

a $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 0 & 0 & 4 & 4 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 7 & 8 \\ 0 & 3 & 19 & 16 \end{pmatrix}$

Coordinates of image are: (0,0); (–1,3); (7,19); (8,16)

b



Area of R= $4 \times 1 = 4$

det $\mathbf{A} = 6 - -4 = 10$

$\therefore$ Area of image $= 10 \times 4$
$= 40$.

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Matrix algebra
Exercise I, Question 2

Question:

The triangle T has vertices at the points $(-3.5,\ 2.5)$, $(-16,\ 10)$ and $(-7,\ 4)$.

a Find the coordinates of the vertices of T under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix}$.

b Show that the area of the image of T is 7.5.

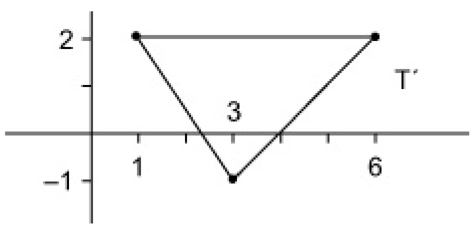
c Hence find the area of T.

Solution:

a $\begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} -3.5 & -16 & -7 \\ 2.5 & 10 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & 3 \\ 2 & 2 & -1 \end{pmatrix}$

Coordinates of T′ are (1,2); (6,2); (3 ,– 1)

b



Area of $T' = \frac{1}{2} \times 5 \times 3 = 7.5$

c

$$\det \mathbf{M} = -5 + 3 = -2$$
$$\therefore \text{Area of } T \times |-2| = \text{Area of } T'$$
$$\Rightarrow \text{Area of } T = \frac{7.5}{2} = 3.75$$

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Matrix algebra
Exercise I, Question 3

Question:

The rectangle R has vertices at the points $(-1, 0)$, $(0, -3)$, $(4, 0)$ and $(3, 3)$.

The matrix $\mathbf{A} = \begin{pmatrix} -2 & 3-a \\ 1 & a \end{pmatrix}$, where a is a constant.

a Find, in terms of a, the coordinates of the vertices of the image of R under the transformation given by **A**.

b Find det(**A**), leaving your answer in terms of a.

Given that the area of the image of R is 75

c find the positive value of a.

Solution:

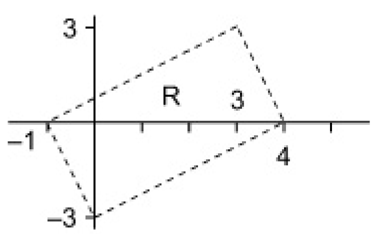
a $\begin{pmatrix} -2 & 3-a \\ 1 & a \end{pmatrix}\begin{pmatrix} -1 & 0 & 4 & 3 \\ 0 & -3 & 0 & 3 \end{pmatrix} = \begin{pmatrix} +2 & 3a-9 & -8 & 3-3a \\ -1 & -3a & 4 & 3+3a \end{pmatrix}$

Image of R is : $(+2, -1)$; $(3a-9, -3a)$; $(-8, 4)$; $(3-3a, 3+3a)$

b

$$\det \mathbf{A} = -2a - 3 + a$$
$$= -a - 3$$

$$\text{Area of R} = \left(\frac{1}{2} \times 5 \times 3\right) \times 2.$$
$$= 15$$



c

$$\text{Area of R} \times |\det \mathbf{A}| = 75$$
$$\therefore \quad |\det \mathbf{A}| = \frac{75}{15} = 5$$
$$\text{So} \quad |-a-3| = 5$$
$$\Rightarrow \quad -a-3 = 5 \text{ or } a+3 = 5$$

$\therefore$ positive value of $a = 2$

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Matrix algebra
Exercise I, Question 4

Question:

P= $\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$, **Q**= $\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$, **R**= $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

A rectangle of area 5 cm^2 is transformed by the matrix **X**. Find the area of the image of the rectangle when **X** is:

a P

b Q

c R

d RQ

e QR

f RP

Solution:

a $\det \mathbf{P} = 2 + 12 = 14$ $\therefore$ area of image is 70 cm^2

b $\det \mathbf{Q} = 4 + 2 = 6$ $\therefore$ area of image is 30 cm^2

c $\det \mathbf{R} = 1 - 4 = -3$ $\therefore$ area of image is 15 cm^2

d $\det \mathbf{RQ} = \det \mathbf{R} \times \det \mathbf{Q} = -18$ $\therefore$ area of image is 90 cm^2

e $\det \mathbf{QR} = \det \mathbf{Q} \times \det \mathbf{R} = -18$ $\therefore$ area of image is 90 cm^2

f $\det \mathbf{RP} = \det \mathbf{R} \times \det \mathbf{P} = -42$ $\therefore$ area of image is 210 cm^2

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Matrix algebra
Exercise I, Question 5

Question:

The triangle T has area 6 cm^2 and is transformed by the matrix $\begin{pmatrix} a & 3 \\ 3 & a+2 \end{pmatrix}$, where a is a constant, into triangle T'.

a Find det(**A**) in terms of a.

Given that the area of T' is 36 cm^2

b find the possible values of a.

Solution:

a

$$\begin{aligned} \det \mathbf{A} &= a(a+2)-9 \\ &= a^2+2a-9 \end{aligned}$$

b

$$\begin{aligned} \text{Area of T}\times|\det \mathbf{A}| &= \text{Area of T}' \\ \therefore 6\times|\det \mathbf{A}| &= 36 \\ \therefore \det \mathbf{A} &= \pm 6 \\ \Rightarrow a^2+2a-9 &= 6 \\ a^2+2a-15 &= 0 \\ (a+5)(a-3) &= 0 \\ \therefore a = 3 \text{ or } -5 \end{aligned}$$

or

$$\begin{aligned} \Rightarrow a^2+2a-9 &= -6 \\ a^2+2a-3 &= 0 \\ (a+3)(a-1) &= 0 \\ a &= 1 \text{ or } -3 \end{aligned}$$

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Matrix algebra
Exercise J, Question 1

Question:

Use inverse matrices to solve the following simultaneous equations

a $7x + 3y = 6$

$-5x - 2y = -5$

b $4x - y = -1$

$-2x + 3y = 8$

Solution:

a $\begin{pmatrix} 7 & 3 \\ -5 & -2 \end{pmatrix} = \mathbf{A} \quad \Rightarrow \quad \det \mathbf{A} = -14 + 15 = 1$

$\therefore \mathbf{A}^{-1} = \frac{1}{1}\begin{pmatrix} -2 & -3 \\ 5 & 7 \end{pmatrix}$

$\therefore \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 6 \\ -5 \end{pmatrix}$

$\therefore \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 6 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -12 + 15 \\ 30 - 35 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$\therefore \quad x = 3, \ y = -5$

b $\mathbf{B} = \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix} \quad \Rightarrow \quad \det \mathbf{B} = 12 - (-2)(-1) = 10$

$\therefore \quad \mathbf{B}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$

$\therefore \quad \mathbf{B}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \quad \Rightarrow \quad \mathbf{B}^{-1}\begin{pmatrix} -1 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

So $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 8 \end{pmatrix}$

$= \frac{1}{10}\begin{pmatrix} -3 + 8 \\ -2 + 32 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 3 \end{pmatrix}$

$\therefore \quad x = 0.5, \ y = 3$

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Matrix algebra
Exercise J, Question 2

Question:

Use inverse matrices to solve the following simultaneous equations

a $4x - y = 11$

$3x + 2y = 0$

b $5x + 2y = 3$

$3x + 4y = 13$

Solution:

a $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} \Rightarrow \det \mathbf{A} = 8 + 3 = 11$

$\therefore \mathbf{A}^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$

So $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 11 \\ 0 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 11 \\ 0 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 22 \\ -33 \end{pmatrix}$

$\therefore \quad x = 2, \ y = -3$

b $\mathbf{B} = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow \det \mathbf{B} = 20 - 6 = 14$

$\therefore \mathbf{B}^{-1} = \frac{1}{14}\begin{pmatrix} 4 & -2 \\ -3 & 5 \end{pmatrix}$

So $\mathbf{B}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{B}^{-1}\begin{pmatrix} 3 \\ 13 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 4 & -2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 13 \end{pmatrix}$

$= \frac{1}{14}\begin{pmatrix} 12 - 26 \\ -9 + 65 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} -14 \\ 56 \end{pmatrix}$

$\therefore \quad x = -1, \ y = 4$

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Matrix algebra
Exercise K, Question 1

Question:

The matrix $\mathbf{A}=\begin{pmatrix}3 & 1\\4 & 2\end{pmatrix}$ transforms the triangle *PQR* into the triangle with coordinates $(6\,,\,-2)$, (4, 4), (0, 8).

Find the coordinates of *P*, *Q* and *R*.

Solution:

$\mathbf{A}=\begin{pmatrix}3 & 1\\4 & 2\end{pmatrix} \Rightarrow \text{ det } \mathbf{A}=6-4=2.$

$\therefore \quad \mathbf{A}^{-1}=\frac{1}{2}\begin{pmatrix}2 & -1\\-4 & 3\end{pmatrix}$

$\mathbf{A}(\Delta PQR)=\begin{pmatrix}6 & 4 & 0\\-2 & 4 & 8\end{pmatrix}$

$\therefore \ \Delta PQR \text{ given by } \frac{1}{2}\begin{pmatrix}2 & -1\\-4 & 3\end{pmatrix}\begin{pmatrix}6 & 4 & 0\\-2 & 4 & 8\end{pmatrix}$

$=\frac{1}{2}\begin{pmatrix}14 & 4 & -8\\-30 & -4 & 24\end{pmatrix}$

$=\begin{pmatrix}7 & 2 & -4\\-15 & -2 & 12\end{pmatrix}$

$\therefore\ P$ is $(7\,,\,-15)$, Q is $(2\,,\,-2)$, R is $(-4,\ 12)$

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Matrix algebra
Exercise K, Question 2

Question:

The matrix $\mathbf{A}=\begin{pmatrix}1 & -3\\ 2 & 1\end{pmatrix}$ and $\mathbf{AB}=\begin{pmatrix}4 & 1 & 9\\ 1 & 9 & 4\end{pmatrix}$.

Find the matrix **B**.

Solution:

$\mathbf{A}=\begin{pmatrix}1 & -3\\ 2 & 1\end{pmatrix} \Rightarrow \det \mathbf{A} = 1+6=7$

$\therefore \quad \mathbf{A}^{-1}=\frac{1}{7}\begin{pmatrix}1 & 3\\ -2 & 1\end{pmatrix}$

$\mathbf{A}^{-1}(\mathbf{AB}) = \frac{1}{7}\begin{pmatrix}1 & 3\\ -2 & 1\end{pmatrix}\begin{pmatrix}4 & 1 & 9\\ 1 & 9 & 4\end{pmatrix}$

$\therefore \quad \mathbf{B} = \frac{1}{7}\begin{pmatrix}7 & 28 & 21\\ -7 & 7 & -14\end{pmatrix}$

$\therefore \quad \mathbf{B} = \begin{pmatrix}1 & 4 & 3\\ -1 & 1 & -2\end{pmatrix}$

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Matrix algebra
Exercise K, Question 3

Question:

A= $\begin{pmatrix} -2 & 1 \\ 7 & -3 \end{pmatrix}$, **B**= $\begin{pmatrix} 4 & 1 \\ -5 & -1 \end{pmatrix}$, **C**= $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$.

The matrices **A**, **B** and **C** represent three transformations. By combining the three transformations in the order **A**, followed by **B**, followed by **C**, a simple single transformation is obtained which is represented by the matrix **R**.

a Find **R**.

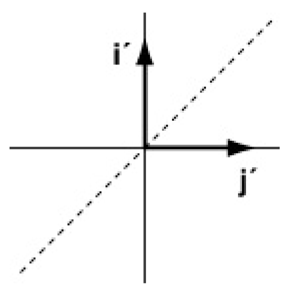
b Give a geometrical interpretation of the transformation represented by **R**.

c Write down the matrix $\mathbf{R}^2$.

Solution:

a

$$\mathbf{R} = \mathbf{C\,B\,A}$$
$$= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ -5 & -1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 7 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$$



$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

b **R** represents a reflection in the line $y = x$

c $\mathbf{R}^2 = \mathrm{I}$

Since repeating a reflection twice returns an object to its original position.

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Matrix algebra
Exercise K, Question 4

Question:

The matrix **Y** represents a rotation of 90° about (0, 0).

a Find **Y**.

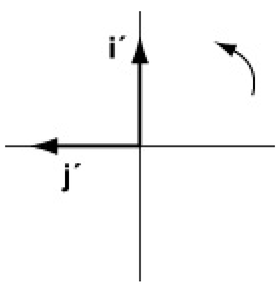
The matrices **A** and **B** are such that **AB** =**Y**. Given that **B**= $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

b find **A**.

c Simplify **ABABABAB**.

Solution:

a



$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

b $\mathbf{AB} = \mathbf{Y} \Rightarrow \mathbf{A} = \mathbf{YB}^{-1}$.

$$\mathbf{B} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \Rightarrow \det \mathbf{B} = 3 - 4 = -1$$

$$\therefore \quad \mathbf{B}^{-1} = \frac{1}{-1}\begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

$$\therefore \quad \mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix}$$

c $\mathbf{ABABABAB} = \mathbf{Y}^4$

$= \text{rotation of } 4 \times 90 = 360^\circ \text{ about } (0,\ 0)$

$= \mathrm{I}$

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Matrix algebra
Exercise K, Question 5

Question:

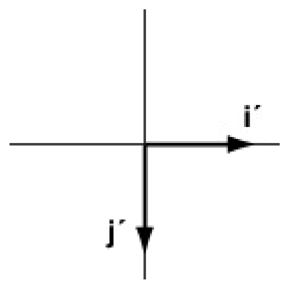
The matrix **R** represents a reflection in the *x*-axis and the matrix **E** represents an enlargement of scale factor 2 centre (0, 0).

a Find the matrix **C**= **ER** and interpret it geometrically.

b Find $\mathbf{C}^{-1}$ and give a geometrical interpretation of the transformation represented by $\mathbf{C}^{-1}$.

Solution:

Reflection in *x*-axis



$$\Rightarrow \mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Enlargement S.F. 2 centre (0, 0)

$$\Rightarrow \mathrm{E} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

a

$$\mathbf{C} = \mathbf{ER} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

Reflection in the *x*-axis and enlargement SF 2. Centre (0, 0)

b $$\mathbf{C}^{-1} = \frac{1}{-4}\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

Reflection in the *x*-axis and enlargement scale factor $\frac{1}{2}$. Centre (0, 0)

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Matrix algebra
Exercise K, Question 6

Question:

The quadrilateral R of area 4cm^2 is transformed to R' by the matrix $\mathbf{P}=\begin{pmatrix}1+p & p\\ 2-p & p\end{pmatrix}$, where p is a constant.

a Find det(**P**) in terms of p.

Given that the area of $R' = 12\text{cm}^2$

b find the possible values of p.

Solution:

a

$$\mathbf{P}=\begin{pmatrix}1+p & p\\ 2-p & p\end{pmatrix} \Rightarrow \quad \det \mathbf{P} = p(1+p)-p(2-p)$$
$$= p+p^2-2p+p^2$$
$$= 2p^2-p.$$

b

$$\begin{aligned}\text{Area of R}\times|\det p| &= \text{Area of } R^1\\ \therefore\quad 4\times|\det p| &= 12\\ \therefore\quad \det p &= \pm 3\\ \text{So}\quad 2p^2-p &= 3\\ \Rightarrow\quad 2p^2-p-3 &= 0\\ (2p-3)(p+1) &= 0\end{aligned}$$

$$p=-1 \text{ or } \frac{3}{2}$$

$$\begin{aligned}\text{or}\quad 2p^2-p &= -3\\ \Rightarrow\quad 2p^2-p+3 &= 0\end{aligned}$$

Discrimininat is $(-1)^2-4\times 3\times 2=-23$
< 0
$\therefore$ no solutions

so $p=-1$ or $\frac{3}{2}$ are the only solutions

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Matrix algebra
Exercise K, Question 7

Question:

The matrix $\mathbf{A}=\begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix}$, where a and b are non-zero constants.

a Find $\mathbf{A}^{-1}$.

The matrix $\mathbf{Y}=\begin{pmatrix} a & 2b \\ 2a & b \end{pmatrix}$ and the matrix **X** is given by **XA** =**Y**.

b Find **X**.

Solution:

a

$$\mathbf{A} = \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} \quad \Rightarrow \quad \det \mathbf{A} = 3ab - 2ab = ab$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{ab}\begin{pmatrix} 3b & -b \\ -2a & a \end{pmatrix} = \begin{pmatrix} \frac{3}{a} & -\frac{1}{a} \\ \frac{-2}{b} & \frac{1}{b} \end{pmatrix}$$

b

$$\mathbf{XA} = \mathbf{Y} \quad \Rightarrow \quad \mathbf{X} = \mathbf{YA}^{-1}$$

$$\therefore \quad \mathbf{X} = \begin{pmatrix} a & 2b \\ 2a & b \end{pmatrix}\begin{pmatrix} \frac{3}{a} & -\frac{1}{a} \\ -\frac{2}{b} & \frac{1}{b} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix}$$

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Matrix algebra

Exercise K, Question 8

Question:

The 2×2, non-singular matrices, **A**, **B** and **X** satisfy $\mathbf{XB} = \mathbf{BA}$.

a Find an expression for **X**.

b Given that $\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 0 & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$, find **X**.

Solution:

a

$$\mathbf{XB} = \mathbf{BA}$$

$$\therefore (\mathbf{XB})\mathbf{B}^{-1} = \mathbf{BAB}^{-1}$$

$$\text{i.e.} \quad \mathbf{X} = \mathbf{BAB}^{-1} \qquad (\because \mathbf{BB}^{-1} = \mathbf{I})$$

b

$$\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \Rightarrow \det \mathbf{B} = -2 - (-1) = -1$$

$$\therefore \quad \mathbf{B}^{-1} = \frac{1}{-1}\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\therefore \quad \mathbf{X} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 4 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} 6 & 2 \\ -4 & -3 \end{pmatrix}$$

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