

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 1

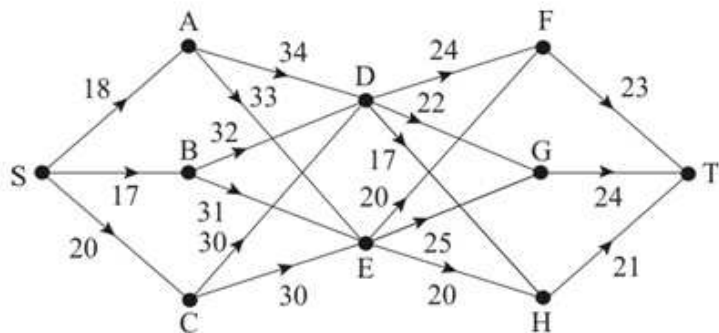
#### Question:

Use dynamic programming to find

**a** a shortest

**b** a longest

route from S to T in the network below. State the route and its length.



#### Solution:

**a Shortest**

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	24 + 23 = 47
		DG	G	22 + 24 = 46
		DH	H	17 + 21 = 38*
	E	EF	F	20 + 23 = 43
		EG	G	25 + 24 = 49
		EH	H	20 + 21 = 41*
3	A	AD	D	34 + 38 = 72*
		AE	E	33 + 41 = 74
	B	BD	D	32 + 38 = 70*
		BE	E	31 + 41 = 72
	C	CD	D	30 + 38 = 68*
		CE	E	30 + 41 = 71
4	S	SA	A	18 + 72 = 90
		SB	B	17 + 70 = 87*
		SC	C	20 + 68 = 88

Shortest route SBDHT length 87

**b Longest**

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	24 + 23 = 47*
		DG	G	22 + 24 = 46
		DH	H	17 + 21 = 38
	E	EF	F	20 + 23 = 43
		EG	G	25 + 24 = 49*
		EH	H	20 + 21 = 41
3	A	AD	D	34 + 47 = 81
		AE	E	33 + 49 = 82*
	B	BD	D	32 + 47 = 79
		BE	E	31 + 49 = 80*
	C	CD	D	30 + 47 = 77
		CE	E	30 + 49 = 79*
4	S	SA	A	18 + 82 = 100*
		SB	B	17 + 80 = 97
		SC	C	20 + 79 = 99

Longest route SAEGT length 100

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

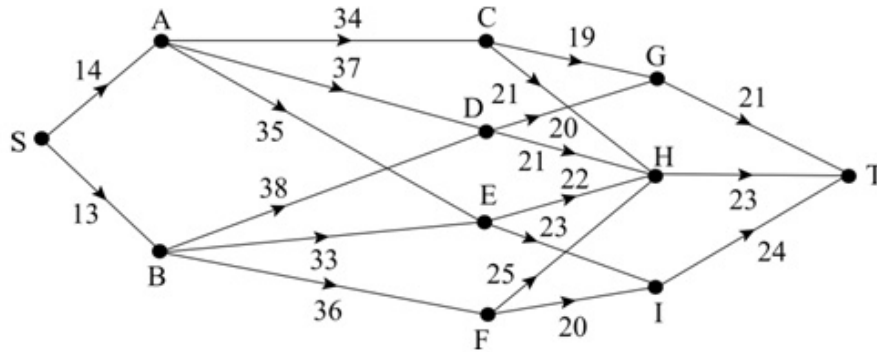
### Exercise A, Question 2

**Question:**

Use dynamic programming to find

- a a shortest
- b a longest

route from S to T in the network below. State the route and its length.



**Solution:**

**a Shortest**

Stage	State	Action	Destination	Value	
1	G	GT	T	21*	
	H	HT	T	23*	
	I	IT	T	24*	
2	C	CG	G	19+21=40*	
		CH	H	21+23=44	
		DG	G	20+21=41*	
	D	DH	H	21+23=44	
		E	EH	H	22+23=45*
		EI	I	23+24=47	
	F	FH	H	25+23=48	
		FI	I	20+24=44*	
3	A	AC	C	34+40=74*	
		AD	D	37+41=78	
		AE	E	35+45=80	
	B	BD	D	38+41=79	
		BE	E	33+45=78*	
		BF	F	36+44=80	
4	S	SA	A	14+74=88*	
		SB	B	13+78=91	

Shortest route length is 88 with route SACGT

**b Longest**

Stage	State	Action	Destination	Value	
1	G	GT	T	21*	
	H	HT	T	23*	
	I	IT	T	24*	
2	C	CG	G	19+21=40	
		CH	H	21+23=44*	
		DG	G	20+21=41	
	D	DH	H	21+23=44*	
		E	EH	H	22+23=45
		EI	I	23+24=47*	
	F	FH	H	25+23=48*	
		FI	I	20+24=44	
3	A	AC	C	34+44=78	
		AD	D	37+44=81	
		AE	E	35+47=82*	
	B	BD	D	38+44=82	
		BE	E	33+47=80	
		BF	F	36+48=84*	
4	S	SA	A	14+82=96	
		SB	B	13+84=97*	

Longest route length is 97 with route SBFHT

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 3

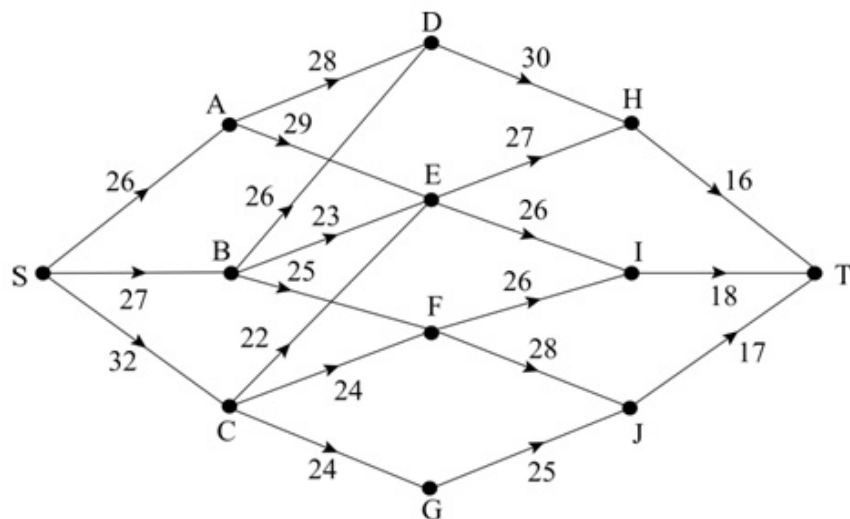
#### Question:

Use dynamic programming to find

**a** a shortest

**b** a longest

route from S to T in the network below. State the route and its length.



#### Solution:

**a Shortest**

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$30+16=46^*$
	E	EH	H	$27+16=43^*$
		EI	I	$26+18=44$
	F	FI	I	$26+18=44^*$
		FJ	J	$28+17=45$
	G	GJ	J	$25+17=42^*$
3	A	AD	D	$28+46=74$
		AE	E	$29+43=72^*$
	B	BD	D	$26+46=72$
		BE	E	$23+43=66^*$
		BF	F	$25+44=69$
	C	CE	E	$22+43=65^*$
		CF	F	$24+44=68$
		CG	G	$24+42=66$
4	S	SA	A	$26+72=98$
		SB	B	$27+66=93^*$
		SC	C	$32+65=97$

Shortest route length is 93 with route SBEHT

**b Longest**

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$30+16=46^*$
	E	EH	H	$27+16=43$
		EI	I	$26+18=44^*$
	F	FI	I	$26+18=44$
		FJ	J	$28+17=45^*$
	G	GJ	J	$25+17=42^*$
3	A	AD	D	$28+46=74^*$
		AE	E	$29+44=73$
	B	BD	D	$26+46=72^*$
		BE	E	$23+44=67$
		BF	F	$25+45=70$
	C	CE	E	$22+44=66$
		CF	F	$24+45=69^*$
		CG	G	$24+42=66$
4	S	SA	A	$26+74=100$
		SB	B	$27+72=99$
		SC	C	$32+69=101^*$

Longest route length is 101 with route SCFJT

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# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 4

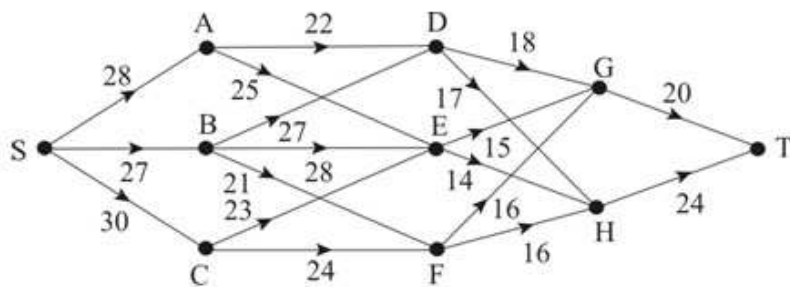
#### Question:

Use dynamic programming to find

**a** a shortest

**b** a longest

route from S to T in the network below. State the route and its length.



#### Solution:



**a Shortest**

Stage	State	Action	Destination	Value
1	G	GT	T	20*
	H	HT	T	24*
2	D	DG	G	$18 + 20 = 38^*$
		DH	H	$17 + 24 = 41$
	E	EG	G	$15 + 20 = 35^*$
		EH	H	$14 + 24 = 38$
	F	FG	G	$16 + 20 = 36^*$
		FH	H	$16 + 24 = 40$
3	A	AD	D	$22 + 38 = 60^*$
		AE	E	$25 + 35 = 60^*$
	B	BD	D	$27 + 38 = 65$
		BE	E	$28 + 35 = 63$
	C	BF	F	$21 + 36 = 57^*$
		CE	E	$23 + 35 = 58^*$
4	S	SA	A	$28 + 60 = 88$
		SB	B	$27 + 57 = 84^*$
		SC	C	$30 + 58 = 88$

Shortest route length is 84 with route SBFGT

**b Longest**

Stage	State	Action	Destination	Value
1	G	GT	T	20*
	H	HT	T	24*
2	D	DG	G	$18 + 20 = 38$
		DH	H	$17 + 24 = 41^*$
	E	EG	G	$15 + 20 = 35$
		EH	H	$14 + 24 = 38^*$
	F	FG	G	$16 + 20 = 36$
		FH	H	$16 + 24 = 40^*$
3	A	AD	D	$22 + 41 = 63^*$
		AE	E	$25 + 38 = 63^*$
	B	BD	D	$27 + 41 = 68^*$
		BE	E	$28 + 38 = 66$
	C	BF	F	$21 + 40 = 61$
		CE	E	$23 + 38 = 61$
4	S	SA	A	$28 + 63 = 91$
		SB	B	$27 + 68 = 95^*$
		SC	C	$30 + 64 = 94$

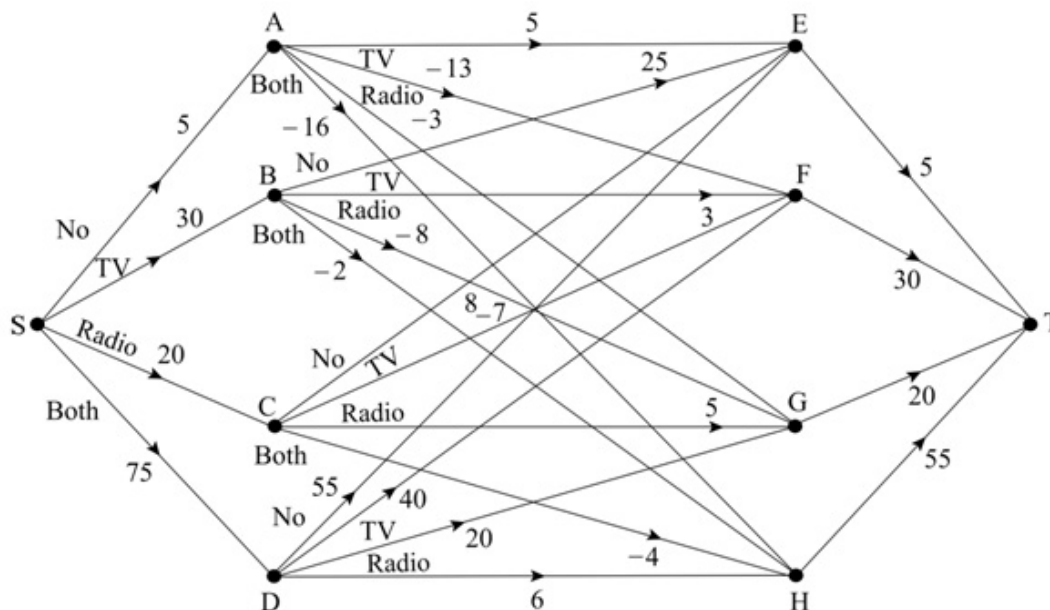
Longest route length is 95 with route SBDHT

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 5

Question:



The diagram shows the effect on a company's profits, in £1000's, of taking various advertising decisions. The company wishes to create a two-year plan that will maximise its total profit.

Each year they must decide if they will not advertise (No), advertise through television only (TV), advertise through radio only (Radio), or advertise in both media (Both).

To determine the effectiveness of the strategy the company will estimate the value of its assets at the end of the two-year period.

Use dynamic programming to determine the advertising decisions that the directors should take.

**Solution:**

**Maximise**

Stage	State	Action	Destination	Value
1	E	ET	T	5*
Assets	F	FT	T	30*
	G	GT	T	20*
	H	HT	T	55*
2	A	AE (No)	E	$5+5=10$
		AF (TV)	F	$-13+30=17$
		AG (Radio)	G	$-3+20=17$
		AH (Both)	H	$-16+55=39^*$
	B	BE (No)	E	$25+5=30$
		BF (TV)	F	$3+30=33$
		BG (Radio)	G	$8+20=28$
		BH (Both)	H	$-2+55=53^*$
	C	CE (No)	E	$8+5=13$
		CF (TV)	F	$-7+30=23$
		CG (Radio)	G	$5+20=25$
		CH (Both)	H	$-4+55=51^*$
D	DE (No)	E	$55+5=60$	
	DF (TV)	F	$40+30=70^*$	
	DG (Radio)	G	$20+20=40$	
	DH (Both)	H	$6+55=61$	
3	S	SA (No)	A	$5+39=44$
		SB (TV)	B	$30+53=83$
	Year one	SC (Radio)	C	$20+51=71$
		SD (Both)	D	$75+70=145^*$

The maximum profit is £145 000

The maximum route is SDFT

In practical terms the company's strategy is:

Year 1 – advertise in both TV and Radio

Year 2 – advertise on TV only.

# Solutionbank D2

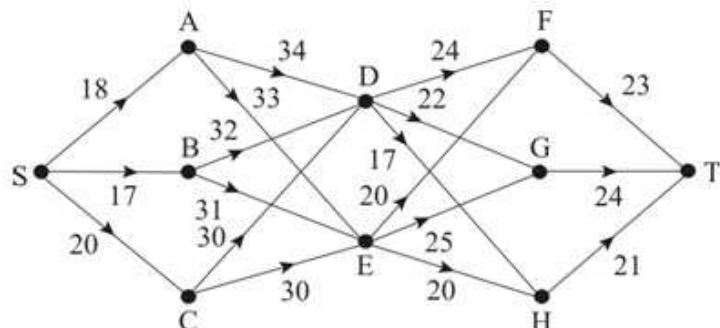
## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 1

**Question:**

Use dynamic programming to find

- a a minimax,
  - b a maximin,
- route from S to T in the network below. State the route and its length.



**Solution:**

**a Minimax**

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	Max(24, 23) = 24
		DG	G	Max(22, 24) = 24
		DH	H	Max(17, 21) = 21*
	E	EF	F	Max(20, 23) = 23
		EG	G	Max(25, 24) = 25
		EH	H	Max(20, 21) = 21*
3	A	AD	D	Max(34, 21) = 34
		AE	E	Max(33, 21) = 33*
	B	BD	D	Max(32, 21) = 32
		BE	E	Max(31, 21) = 31*
	C	CD	D	Max(30, 21) = 30*
		CE	E	Max(30, 21) = 30*
4	S	SA	A	Max(18, 33) = 33
		SB	B	Max(17, 31) = 31
		SC	C	Max(20, 30) = 30*

Minimax route SCDHT or SCEHT – both of value 30

**b Maximin**

Stage	State	Action	Destination	Value
1	F	FT	T	23*
	G	GT	T	24*
	H	HT	T	21*
2	D	DF	F	$\text{Min}(24, 23) = 23^*$
		DG	G	$\text{Min}(22, 24) = 22$
		DH	H	$\text{Min}(17, 21) = 17$
	E	EF	F	$\text{Min}(20, 23) = 20$
		EG	G	$\text{Min}(25, 24) = 24^*$
		EH	H	$\text{Min}(20, 21) = 20$
3	A	AD	D	$\text{Min}(34, 23) = 23$
		AE	E	$\text{Min}(33, 24) = 24^*$
	B	BD	D	$\text{Min}(32, 23) = 23$
		BE	E	$\text{Min}(31, 24) = 24^*$
	C	CD	D	$\text{Min}(30, 23) = 23$
		CE	E	$\text{Min}(30, 24) = 24^*$
4	S	SA	A	$\text{Min}(18, 24) = 18$
		SB	B	$\text{Min}(17, 24) = 17$
		SC	C	$\text{Min}(20, 24) = 20^*$

Maximin route SCEGT of value 20

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 2

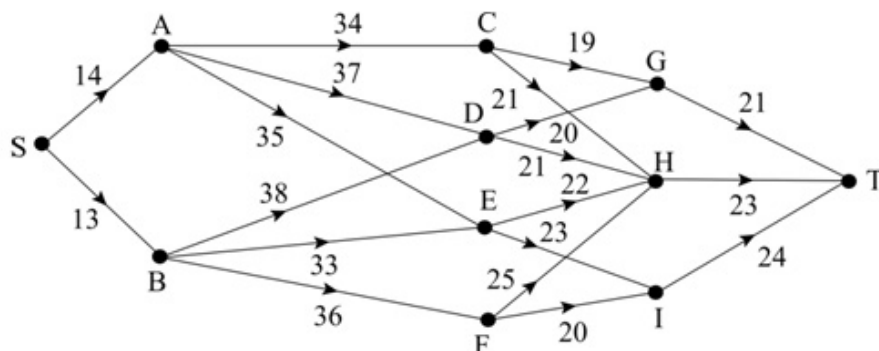
#### Question:

Use dynamic programming to find

a a minimax,

b a maximin,

route from S to T in the network below. State the route and its length.



#### Solution:

##### a Minimax

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$\text{Max}(19, 21) = 21^*$
		CH	H	$\text{Max}(21, 23) = 23$
	D	DG	G	$\text{Max}(20, 21) = 21^*$
		DH	H	$\text{Max}(21, 23) = 23$
	E	EH	H	$\text{Max}(22, 23) = 23^*$
		EI	I	$\text{Max}(23, 24) = 24$
	F	FH	H	$\text{Max}(25, 23) = 25$
		FI	I	$\text{Max}(20, 24) = 24^*$
3	A	AC	C	$\text{Max}(34, 21) = 34^*$
		AD	D	$\text{Max}(37, 21) = 37$
		AE	E	$\text{Max}(35, 23) = 35$
	B	BD	D	$\text{Max}(38, 21) = 38$
		BE	E	$\text{Max}(33, 23) = 33^*$
		BF	F	$\text{Max}(36, 24) = 36$
4	S	SA	A	$\text{Max}(14, 34) = 34$
		SB	B	$\text{Max}(13, 33) = 33^*$

Minimax route SBEHT of value 33

**b Maximin**

Stage	State	Action	Destination	Value
1	G	GT	T	21*
	H	HT	T	23*
	I	IT	T	24*
2	C	CG	G	$\text{Min}(19, 21) = 19$
		CH	H	$\text{Min}(21, 23) = 21^*$
	D	DG	G	$\text{Min}(20, 21) = 20$
		DH	H	$\text{Min}(21, 23) = 21^*$
	E	EH	H	$\text{Min}(22, 23) = 22$
		EI	I	$\text{Min}(23, 24) = 23^*$
	F	FH	H	$\text{Min}(25, 23) = 23^*$
		FI	I	$\text{Min}(20, 24) = 20$
3	A	AC	C	$\text{Min}(34, 21) = 34$
		AD	D	$\text{Min}(37, 21) = 37^*$
		AE	E	$\text{Min}(35, 23) = 35$
	B	BD	D	$\text{Min}(38, 21) = 38^*$
		BE	E	$\text{Min}(33, 23) = 33$
		BF	F	$\text{Min}(36, 24) = 36$
4	S	SA	A	$\text{Min}(14, 37) = 37$
		SB	B	$\text{Min}(13, 38) = 38^*$

Maximin route SBDHT of value 38

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

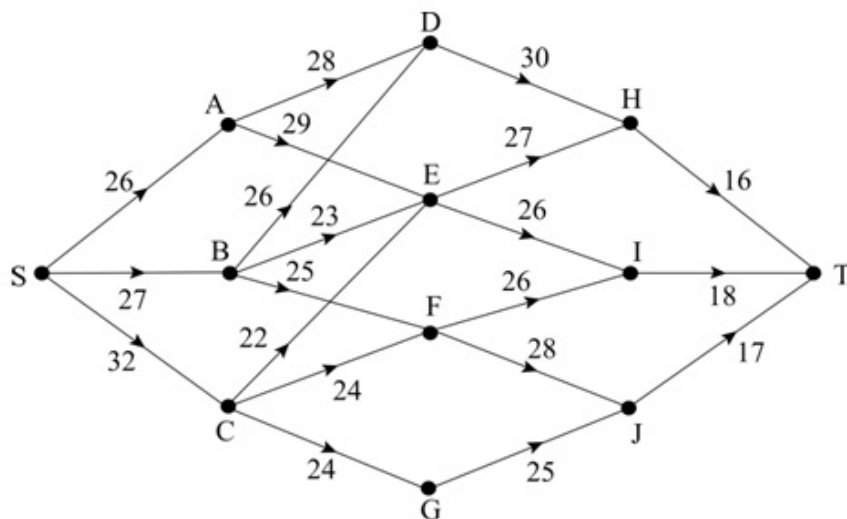
### Exercise B, Question 3

**Question:**

Use dynamic programming to find

- a a minimax,
- b a maximin,

route from S to T in the network below. State the route and its length.



**Solution:**



## a Minimax

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$\text{Max}(30,16) = 30^*$
	E	EH	H	$\text{Max}(27,16) = 27$
		EI	I	$\text{Max}(26,18) = 26^*$
	F	FI	I	$\text{Max}(26,18) = 26^*$
		FJ	J	$\text{Max}(28,17) = 28$
	G	GJ	J	$\text{Max}(25,17) = 25^*$
3	A	AD	D	$\text{Max}(28,30) = 30$
		AE	E	$\text{Max}(29,26) = 29^*$
	B	BD	D	$\text{Max}(26,30) = 30$
		BE	E	$\text{Max}(23,26) = 26^*$
		BF	F	$\text{Max}(25,26) = 26^*$
	C	CE	E	$\text{Max}(22,26) = 26$
		CF	F	$\text{Max}(24,26) = 26$
		CG	G	$\text{Max}(24,25) = 25^*$
4	S	SA	A	$\text{Max}(26,29) = 29$
		SB	B	$\text{Max}(27,26) = 27^*$
		SC	C	$\text{Max}(32,25) = 32$

Minimax route SBEIT or SBFIT – both of value 27

**b Maximin**

Stage	State	Action	Destination	Value
1	H	HT	T	16*
	I	IT	T	18*
	J	JT	T	17*
2	D	DH	H	$\text{Min}(30, 16) = 16^*$
	E	EH	H	$\text{Min}(27, 16) = 16$
		EI	I	$\text{Min}(26, 18) = 18^*$
	F	FI	I	$\text{Min}(26, 18) = 18^*$
		FJ	J	$\text{Min}(28, 17) = 17$
	G	GJ	J	$\text{Min}(25, 17) = 17^*$
3	A	AD	D	$\text{Min}(28, 16) = 16$
		AE	E	$\text{Min}(29, 18) = 18^*$
	B	BD	D	$\text{Min}(26, 16) = 16$
		BE	E	$\text{Min}(23, 18) = 18^*$
		BF	F	$\text{Min}(25, 18) = 18^*$
	C	CE	E	$\text{Min}(22, 18) = 18^*$
		CF	F	$\text{Min}(24, 18) = 18^*$
		CG	G	$\text{Min}(24, 17) = 17$
4	S	SA	A	$\text{Min}(26, 18) = 18^*$
		SB	B	$\text{Min}(27, 18) = 18^*$
		SC	C	$\text{Min}(32, 18) = 18^*$

Maximin routes SAEIT, SBEIT, SBFIT, SCEIT, SCFIT all of value 18

# Solutionbank D2

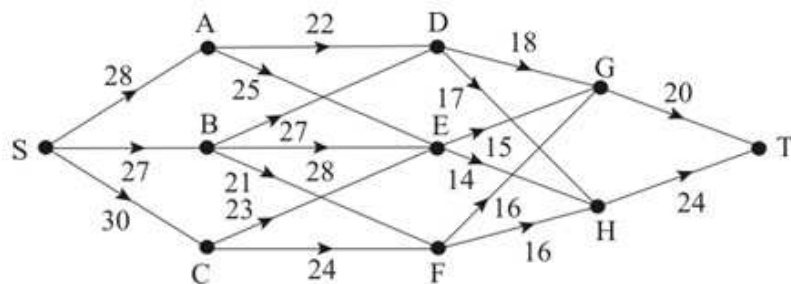
## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 4

#### Question:

Use dynamic programming to find

- a minimax,  
route from S to T in the network below. State the route and its length.



#### Solution:

##### a Minimax

Stage	State	Action	Destination	Value
1	G	GT	T	20*
	H	HT	T	24*
2	D	DG	G	$\text{Max}(18, 20) = 20^*$
		DH	H	$\text{Max}(17, 24) = 24$
	E	EG	G	$\text{Max}(15, 20) = 20^*$
		EH	H	$\text{Max}(14, 24) = 24$
	F	FG	G	$\text{Max}(16, 20) = 20^*$
		FH	H	$\text{Max}(16, 24) = 24$
3	A	AD	D	$\text{Max}(22, 20) = 22^*$
		AE	E	$\text{Max}(25, 20) = 25$
		AF	F	$\text{Max}(28, 20) = 28$
	B	BD	D	$\text{Max}(27, 20) = 27$
		BE	E	$\text{Max}(28, 20) = 28$
		BF	F	$\text{Max}(21, 20) = 21^*$
C	CE	E	$\text{Max}(23, 20) = 23^*$	
	CF	F	$\text{Max}(24, 20) = 24$	
4	S	SA	A	$\text{Max}(28, 22) = 28$
		SB	B	$\text{Max}(27, 21) = 27^*$
		SC	C	$\text{Max}(30, 23) = 30$

Minimax route SBFGT of value 27

**b Maximin**

Stage	State	Action	Destination	Value
1	G	GT	T	20*
	H	HT	T	24*
2	D	DG	G	$\text{Min}(18, 20) = 18^*$
		DH	H	$\text{Min}(17, 24) = 17$
	E	EG	G	$\text{Min}(15, 20) = 15^*$
		EH	H	$\text{Min}(14, 24) = 14$
	F	FG	G	$\text{Min}(16, 20) = 16^*$
		FH	H	$\text{Min}(16, 24) = 16^*$
3	A	AD	D	$\text{Min}(22, 18) = 18^*$
		AE	E	$\text{Min}(25, 15) = 15$
	B	BD	D	$\text{Min}(27, 18) = 18^*$
		BE	E	$\text{Min}(28, 15) = 15$
		BF	F	$\text{Min}(21, 16) = 16$
	C	CE	E	$\text{Min}(23, 15) = 15$
CF		F	$\text{Min}(24, 16) = 16^*$	
4	S	SA	A	$\text{Min}(28, 18) = 18^*$
		SB	B	$\text{Min}(27, 18) = 18^*$
		SC	C	$\text{Min}(30, 16) = 16$

Maximin routes SADGT and SBDGT both of value 18

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 1

#### Question:

A company is created to sell holidays on an island. There are three new resorts A, B and C being created on the island and the company decides to introduce one new resort to its catalogue each year over the next three years. The costs of introducing each resort will be influenced by the number of resorts listed in the catalogue. The more resorts the company has listed, the smaller the cost of adding another resort. The estimates of annual costs are shown in the table below, in hundreds of pounds.

Resorts listed	A	B	C
None	60	60	55
A	–	50	60
B	40	–	55
C	35	50	–
A and B	–	–	50
A and C	–	45	–
B and C	30	–	–

For funding reasons the company needs to choose the order in which the resorts are introduced so that the greatest annual cost is as small as possible.

Dynamic programming will be used to determine the order in which the resorts are introduced.

- Explain the meaning of Stage, State and Action in this context.
- Find the order in which the resorts should be added and the greatest annual cost.

#### Solution:

- a Stage – time, in years, remaining  
 State – resorts already created  
 Action – resort to be opened
- b We require the route that gives the minimax value

Stage	State	Action	Destination	Value
1	AB	C	ABC	50*
	AC	B	ABC	45*
	BC	A	ABC	30*
2	A	B	AB	$\text{Max}(50, 50) = 50^*$
		C	AC	$\text{Max}(60, 45) = 60$
	B	A	AB	$\text{Max}(40, 50) = 50^*$
		C	BC	$\text{Max}(55, 30) = 55$
	C	A	AC	$\text{Max}(35, 45) = 45^*$
		B	BC	$\text{Max}(50, 30) = 50$
3	none	A	A	$\text{Max}(60, 50) = 60$
		B	B	$\text{Max}(60, 50) = 60$
		C	C	$\text{Max}(55, 45) = 55^*$

The minimax route is CAB with a value of £5500  
 The order in which the resorts should be built is C then A then B

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 2

#### Question:

A house renovation project is to be completed in 6 weeks (30 working days). The work is in four phases: clearance, repairing, modernisation and decorating, which must be undertaken in that order. The cost, in £1000, of each stage depends on the time taken to do it. These are shown in the table.

Time for stage (days)	Clearance	Repairing	Modernisation	Decorating
5	15	24	22	14
10	13	20	19	12
15	8	15	15	9
20	5	10	11	4
25	2	6	7	2

Dynamic programming will be used to solve this problem.

- Define the terms Stage, State, Action, Destination and Value in this context.
- Determine the number of days that should be allocated to each stage in order to minimise costs.

#### Solution:

- Stage – phase being considered  
State – number of days remaining  
Action – number of days allocated  
Destination – number of days remaining  
Value – total costs

**b**

Stage	State	Action	Destination	Value
Decorating	5	5	0	14*
	10	10	0	12*
Modernisation	10	5	5	$22 + 14 = 36^*$
	15	10	5	$19 + 14 = 33^*$
		5	10	$22 + 12 = 34$
Repairing	15	5	10	$24 + 36 = 60^*$
	20	10	10	$20 + 36 = 56^*$
		5	15	$24 + 33 = 57$
Clearance	25	10	15	$13 + 60 = 73$
		5	20	$15 + 56 = 71^*$

The minimum cost is £71 000. The time should be allocated as follows

Activity	Clearance	Repairing	Modernisation	Decorating
Number of days	5	10	5	5

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 3

#### Question:

A company makes aircraft. The order book over the next four months is shown in the table below.

Month	March	April	May	June
Number of aircraft ordered	1	2	3	2

The aircraft are delivered to customers at the end of each month.

Up to three aircraft can be made in any month, but if more than two are made in any one month additional equipment will need to be hired at £20 000 per month.

If any work is done in a month the overhead costs are £50 000.

Up to two aircraft can be held in secure hangers at a cost of £10 000 per aircraft per month.

There are no aircraft in store at the beginning of March and there should be no aircraft in store after the June delivery.

Use dynamic programming to determine the production schedule that minimises the costs.

#### Solution:



Stage – Month

State – number in storage

Action – number to be made

Stage	State	Action	Destination	Value (in £10000)
June	2	0	0	$2 = 2^*$
(2)	1	1	0	$5+1=6^*$
	0	2	0	$5=5^*$
May	2	1	0	$5+2+5=12$
(3)		2	1	$5+2+6=13$
		3	2	$2+5+2+2=11^*$
	1	2	0	$5+1+5=11^*$
		3	1	$2+5+1+6=14$
	0	3	0	$2+5+1+2=10^*$
April	2	0	0	$2+10=12^*$
(2)		1	1	$5+2+11=18$
		2	2	$5+2+11=18$
	1	1	0	$5+1+10=16^*$
		2	1	$5+1+11=17$
		3	2	$2+5+1+11=19$
	0	2	0	$5+10=15^*$
		3	1	$2+5+11=18$
March	0	1	0	$5+15=20$
(1)		2	1	$5+16=21$
		3	2	$2+5+12=19^*$

The minimum cost is £190 000, the aircraft should be built as follows.

Month	March	April	May	June
Number of aircraft built in each month	3	0	3	2

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 4

#### Question:

A salesman will visit four shops in the next four days to demonstrate a new product. He will start at home and travel to the first shop and spend the first day there, then travel directly to the second shop for day 2, onto the third shop for day 3, then to the fourth shop for day 4 and then travel home.

Table 1 shows the shops he could visit on each day.

Table 2 shows the anticipated profit, in £100, from sales at each shop.

Table 3 shows the travelling expenses, in £100, that will be incurred.

The company employing the salesman wishes to maximise the income, after subtracting the travel costs, generated by the salesman's visits. Find his optimum route.

Table 1

Monday	Tuesday	Wednesday	Thursday
A, B, C	D, E	F, G	H, I, J

Table 2

Shop	A	B	C	D	E	F	G	H	I	J
Profit	8	9	8	12	14	10	11	14	13	11

Table 3

	A	B	C	D	E	F	G	H	I	J
Home	2	2	3					6	4	3
A				3	4					
B				4	6					
C				4	4					
D						5	5			
E						4	7			
F								5	4	4
G								5	5	4

#### Solution:

Stage – day

State – shop being visited

Action – next journey to be undertaken

Stage	State	Action	Destination	Value, in £ 100
Thursday	H	H-home	home	$14 - 6 = 8^*$
	I	I-home	home	$13 - 4 = 9^*$
	J	J-home	home	$11 - 3 = 8^*$
Wednesday	F	FH	H	$10 - 5 + 8 = 13$
		FI	I	$10 - 4 + 9 = 15^*$
		FJ	J	$10 - 4 + 8 = 14$
	G	GH	H	$11 - 5 + 8 = 14$
		GI	I	$11 - 5 + 9 = 15^*$
		GJ	J	$11 - 4 + 8 = 15^*$
Tuesday	D	DF	F	$12 - 5 + 15 = 22^*$
		DG	G	$12 - 5 + 15 = 22^*$
	E	EF	F	$14 - 4 + 15 = 25^*$
		EG	G	$14 - 7 + 15 = 22$
Monday	A	AD	D	$8 - 3 + 22 = 27$
		AE	E	$8 - 4 + 25 = 29^*$
	B	BD	D	$9 - 4 + 22 = 27$
		BE	E	$9 - 6 + 25 = 28^*$
	C	CD	D	$8 - 4 + 22 = 26$
		CE	E	$8 - 4 + 25 = 29^*$
Sunday	Home	Home – A		$-2 + 29 = 27$
		Home – B		$-2 + 28 = 26^*$
		Home – C		$-3 + 29 = 26^*$

There are two possible minimum routes.

Home – B – E – F – I – Home

Home – C – E – F – I – Home

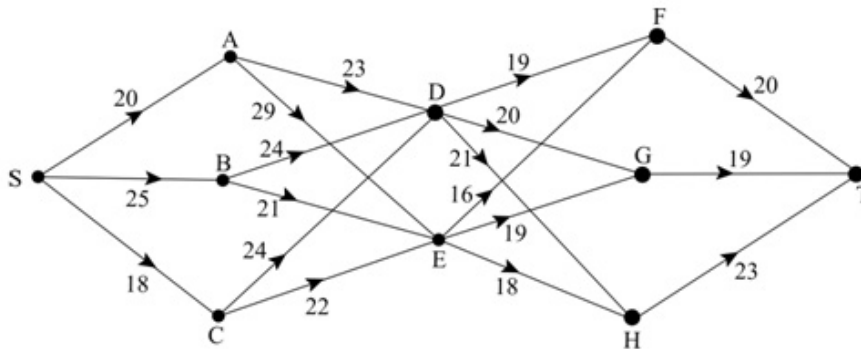
Each has a value of £2600

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 1

Question:



Use dynamic programming to find the minimax route from S to T in the network above.

Solution:

Minimax

Stage	State	Action	Destination	Value
1	F	FT	T	20*
	G	GT	T	19*
	H	HT	T	23*
2	D	DF	F	Max(19, 20) = 20*
		DG	G	Max(20, 19) = 20*
		DH	H	Max(21, 23) = 23
	E	EF	F	Max(16, 20) = 20
		EG	G	Max(19, 19) = 19*
		EH	H	Max(18, 23) = 23
	A	AD	D	Max(23, 20) = 23*
		AE	E	Max(29, 19) = 29
	B	BD	D	Max(24, 20) = 24
		BE	E	Max(21, 19) = 21*
	C	CD	D	Max(24, 20) = 24
		CE	E	Max(22, 19) = 22*
3	S	SA	A	Max(20, 23) = 23
		SB	B	Max(25, 21) = 25
		SC	C	Max(18, 22) = 22*

Minimax route is SCEGT value 22

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 2

#### Question:

Using the same diagram as in question 1, use dynamic programming to find the maximin route.

#### Solution:

Maximin

Stage	State	Action	Destination	Value
1	F	FT	T	20*
	G	GT	T	19*
	H	HT	T	23*
2	D	DF	F	$\text{Min}(19, 20) = 19$
		DG	G	$\text{Min}(20, 19) = 19$
		DH	H	$\text{Min}(21, 23) = 21^*$
	E	EF	F	$\text{Min}(16, 20) = 16$
		EG	G	$\text{Min}(19, 19) = 19^*$
		EH	H	$\text{Min}(18, 23) = 18$
	A	AD	D	$\text{Min}(23, 21) = 21^*$
		AE	E	$\text{Min}(29, 19) = 19$
	B	BD	D	$\text{Min}(24, 21) = 21^*$
		BE	E	$\text{Min}(21, 19) = 19$
	C	CD	D	$\text{Min}(24, 21) = 21^*$
		CE	E	$\text{Min}(22, 19) = 19$
3	S	SA	A	$\text{Min}(20, 21) = 20$
		SB	B	$\text{Min}(25, 21) = 21^*$
		SC	C	$\text{Min}(18, 21) = 18$

The maximin route is SBDHT of value 21

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 3

#### Question:

A dairy manufacturer can make butter, cheese and yoghurt. Up to five units of milk can be processed and the profits from the various allocations are shown in the table.

Number of units	1	2	3	4	5
Butter	14	25	34	41	47
Cheese	12	30	40	45	49
Yoghurt	10	20	30	40	50

The manufacturer wishes to maximise his profit.

- Use dynamic programming to find an optimal solution and state the profit.
- Show that there is a second optimal solution.

#### Solution:

Maximum

Stage	State	Action	Destination	Value
Yoghurt	5	5	0	50*
	4	4	0	40*
	3	3	0	30*
	2	2	0	20*
	1	1	0	10*
	0	0	0	0*
Cheese	5	0	5	$0+50=50$
		1	4	$12+40=52$
		2	3	$30+30=60^*$
		3	2	$40+20=60^*$
		4	1	$45+10=55$
		5	0	$49+0=49$
	4	0	4	$0+40=40$
		1	3	$12+30=42$
		2	2	$30+20=50^*$
		3	1	$40+10=50^*$
		4	0	$45+0=45$
	3	0	3	$0+30=30$
		1	2	$12+20=32$
		2	1	$30+10=40^*$
		3	0	$40+0=40^*$
	2	0	2	$0+20=20$
		1	1	$12+10=22$
		2	0	$30+0=30^*$
	1	0	1	$0+10=10$
		1	0	$12+0=12^*$
	0	0	0	$0+0=0^*$
Butter	5	0	5	$0+60=60$
		1	4	$14+50=64$
		2	3	$25+40=65^*$
		3	2	$34+30=64$
		4	1	$41+12=53$
		5	0	$47+0=47$

There are two possible courses of action each of value £65

Product	Butter	Cheese	Yoghurt
Units to be used	2	2	1
Product	Butter	Cheese	Yoghurt
Units to be used	2	3	0

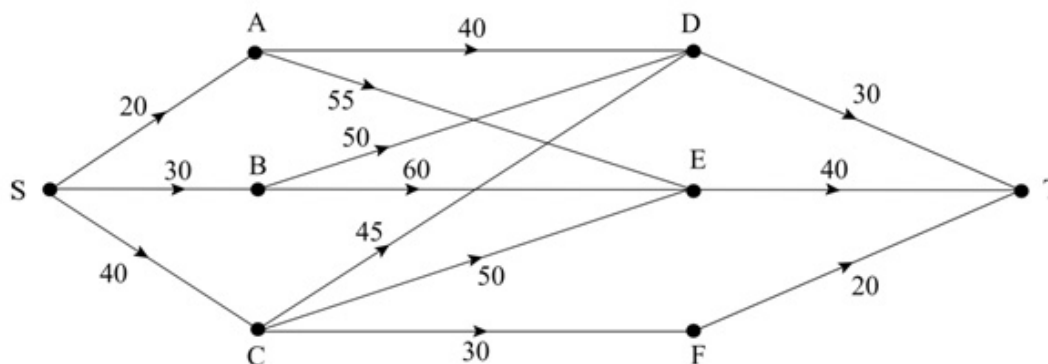
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 4

**Question:**

Jenny wishes to travel from S to T. There are several routes available. She wishes to choose the route on which the maximum altitude, above sea level, is as small as possible. This is called the minimax route.



The diagram gives the possible routes and the weights on the edges give the maximum altitude on the road (in units of 100 feet).

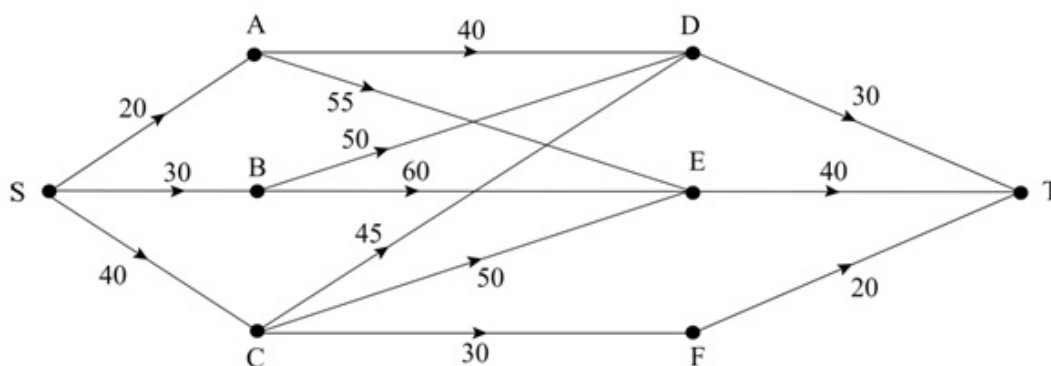
Use dynamic programming, carefully defining the stages and states, to determine the route or routes Jenny should take. You should show your calculations in tabular form, using a table with columns labelled as shown below.

Stage	Initial state	Action	Final state	Value
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[E]

**Solution:**





The states are the vertices.

Stage	Initial state	Action	Final state	Value
1	D	DT	T	30
	E	ET	T	40
	F	FT	T	20
2	A	AD	D	$\max(40, 30) = 40^*$
		AE	E	$\max(55, 40) = 55$
	B	BD	D	$\max(50, 30) = 50^*$
		BE	E	$\max(60, 40) = 60$
	C	CD	D	$\max(45, 30) = 45$
		CE	E	$\max(50, 40) = 50$
		CF	F	$\max(30, 20) = 30^*$
3	S	SA	A	$\max(40, 20) = 40^*$
		SB	B	$\max(50, 30) = 50$
		SC	C	$\max(40, 30) = 40^*$

Tracing back there are two routes

SC, CF, FT,  $\Rightarrow$  SCFT

SA, AD, DT,  $\Rightarrow$  SADT

Maximum altitude on these routes is  $40 (\times 100 \text{ ft}) = 4000 \text{ ft}$ .

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

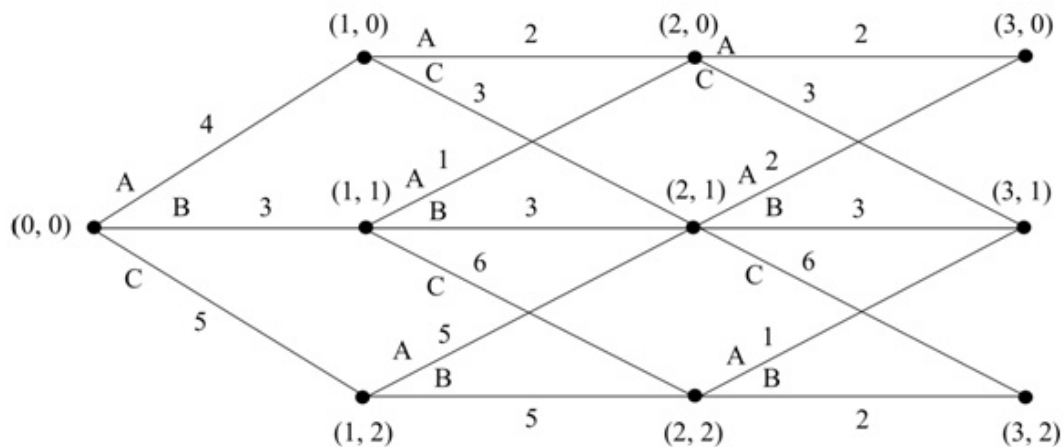
### Exercise D, Question 5

**Question:**

At the beginning of each month an advertising manager must choose one of 3 adverts:  
 the previous advert;  
 the current advert;  
 a new advert.

She therefore has 3 options:  
 A: use the previous advert;  
 B: use the current advert;  
 C: run a new advert.

The possible choices are shown in the network below together with (stage, state) variables at the vertices and the expected profits, in thousands of pounds, on the arcs.



September	October	November
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The manager wants to maximise her profits for the three-month period.

- a Complete the table on the worksheet.
- b Hence obtain the sequence of decisions she should make to obtain the maximum profit. State the maximum profit.

**[E]**

**Solution:**

a

Stage	State	Action	Cost	Total Cost
2	0	A	2	2
		C	3	3*
	1	A	2	2
		B	3	3
		C	6	6*
	2	A	1	1
		B	2	2*
1	0	A	2	$2 + 3 = 5$
		C	3	$3 + 6 = 9^*$
	1	A	1	$1 + 3 = 4$
		B	3	$3 + 6 = 9^*$
		C	6	$6 + 2 = 8$
	2	A	5	$5 + 6 = 11^*$
		B	5	$5 + 2 = 7$
0	0	A	4	$4 + 9 = 13$
		B	3	$3 + 9 = 12$
		C	5	$5 + 11 = 16$

b Hence maximum profit is 16

Tracing back through calculations the optimal strategy is CAC