Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

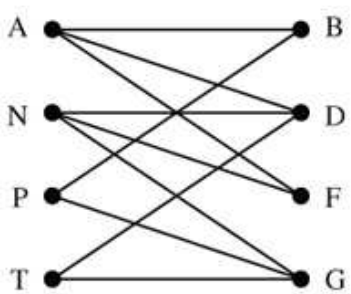
Question:

Four inspectors, Alan, Nicola, Philip and Trudy are to inspect four supermarket departments: bakery, delicatessen, fish and grocery.

Alan is qualified to inspect bakery, delicatessen and fish.
Nicola is qualified to inspect delicatessen, fish and grocery.
Philip is qualified to inspect bakery and grocery.
Trudy is qualified to inspect delicatessen and grocery.

Draw a bipartite graph to show this information.

Solution:



Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

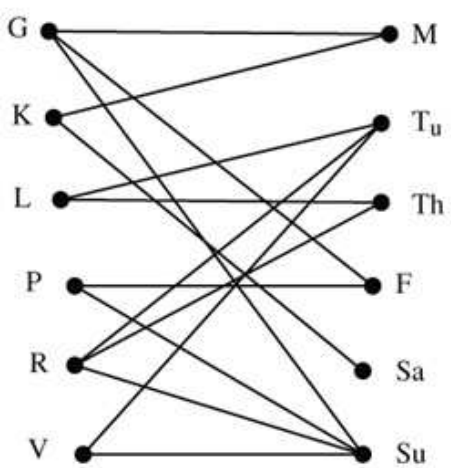
Tours of the local caves are given daily by six guides: Graham, Keith, Lethna, Preety, Rob, and Vicky. The caves are closed on Wednesdays but guides are needed for Monday, Tuesday, Thursday, Friday, Saturday and Sunday.

Next week the guides are available on the days shown in the following table.

Guide	Days available
Graham	Monday, Friday, Sunday
Keith	Monday, Saturday
Lethna	Tuesday, Thursday
Preety	Sunday, Friday
Rob	Tuesday, Sunday, Thursday
Vicky	Tuesday, Sunday

Represent this information on a bipartite graph.

Solution:



Solutionbank D1

Edexcel AS and A Level Modular Mathematics

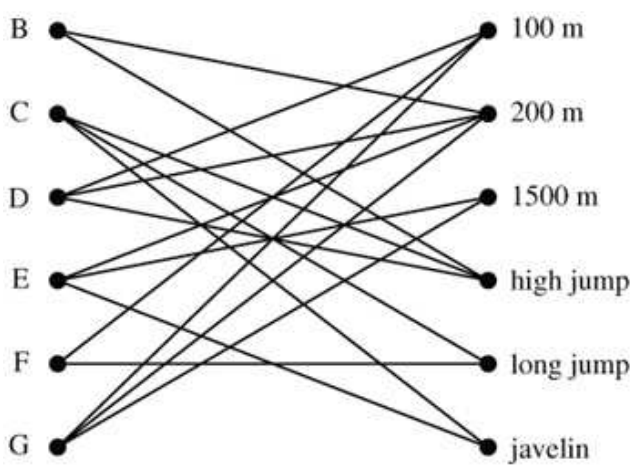
Exercise A, Question 3

Question:

An athletics team, Bill, Charley, Dara, Eun Jung, Fred and Gopan, will compete in six events: 100 m, 200 m, 1500 m, high jump, long jump and javelin. Each person may only compete in one event.

Bill prefers the high jump or 200 m.
Charley does not like the running events, but is happy to compete in either of the jumping events or the javelin.
Dara will do the 100 m, 200 m or the high jump.
Eun Jung likes competing in 200 m, 1500 m or javelin.
Fred prefers long jump or 100 m.
Gopan will do any of the running events 100 m, 200 m or 1500 m.
Draw a bipartite graph to represent this information.

Solution:



Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

The tour director of a museum needs to allocate five of his guides to parties of tourists from France, Germany, Italy, Japan and Spain. The table shows the languages spoken by the five guides.

Ruth	French	Spanish	
Steve	German	Japanese	
Tony	French	German	
Ursula	Spanish	Italian	
Victoria	Italian	Spanish	Japanese

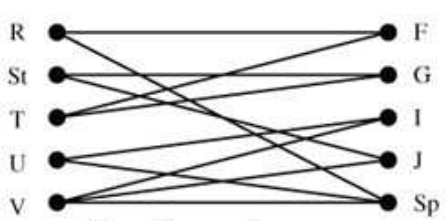
a Draw a bipartite graph to model this situation.

The director allocates Ruth, Steve, Ursula and Victoria to the parties who speak the first language in their individual lists.

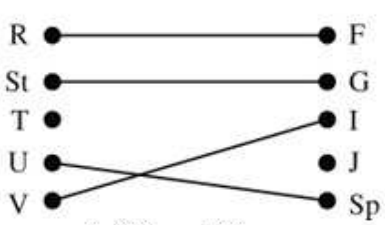
b Starting from this matching use the maximum matching algorithm to find a complete matching. Indicate clearly how the algorithm has been applied in this case. ***E***

Solution:

a



bipartite graph



initial matching

either
Alternating path: T – G=St – J
Change states T=G – St=J
Complete matching

R = F St = J T = G U = SP V = I

or
Alternating path: T – F = R – Sp = U – I = V – J
Change status T = F – R = Sp – U = I – V = J
complete matching

R = Sp St = G T = F U = I V = J

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

In order to help new A-level students to select their courses a college organises an open evening. Some students already studying A-level courses have agreed to talk about one of their A-level courses. Six of these students, Ann, Barry, David, Gemma, Jasmine and Nickos, are between them following six A-level courses in Chemistry, English, French, History, Mathematics and Physics.

The table below shows the courses being followed by each student:

Ann	English	French	History
Barry	History	English	
David	French	Chemistry	Mathematics
Gemma	Physics	Mathematics	History
Jasmine	Mathematics	Physics	Chemistry
Nickos	English	Mathematics	French

a Draw a bipartite graph to model this situation.

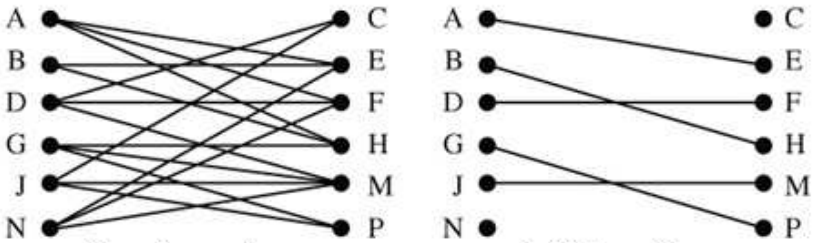
Initially Ann, Barry, David, Gemma and Jasmine are allocated to the first subject in their lists.

b Starting from this matching use the maximum matching algorithm to find a complete matching. Indicate clearly how the algorithm has been applied in this case.

c Explaining your reasoning carefully, determine whether or not your answer to **b** is unique.

Solution:

a



b Six possible alternating paths could be used, (one only is needed!)

i N – F = D – C
ii N – M = J – C
iii N – E = A – F = D – C
iv N – F = D – M = J – C
v N – E = A – F = D – M = J – C
vi N – M = J – P = G – H = B – E = A – F = D – C

Change status to give the complete matching

Person	Path (i)	Path (ii)	Path (iii)	Path (iv)	Path (v)	Path (vi)
A	E	E	F	E	F	F
B	H	H	H	H	H	E
D	C	F	C	M	M	C
G	P	P	P	P	P	H
J	M	C	M	C	C	P
N	F	M	E	F	E	M

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 3

Question:

Five coach drivers, Mihi, Pat, Robert, Sarah and Tony, have to be assigned to drive five coaches for the following school trips:

Adupgud Senior School is going to the Lake District
Brayknee Junior School is going to the seaside
Korry Stur Junior School is going to a concert
Learnalott Senior School is going to the museum (two coaches needed)

Mihi and Sarah would like to drive senior school children. Robert and Pat would like to go on the seaside trip. Pat and Tony would like to attend the concert. Robert and Pat would like to visit the museum. Pat and Tony would like to visit the Lake District.

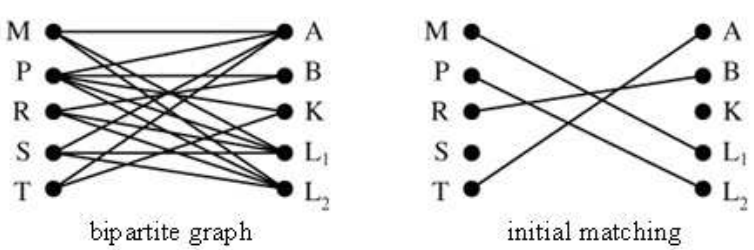
The driver manager wishes to assign each driver to a trip they would like to do.

a Draw a bipartite graph to show the trips that the drivers would like to make.
Initially Mihi and Pat are assigned to Learnalott Senior School, Robert is assigned to Brayknee Junior School and Tony is assigned to Adupgad Senior School.

b Starting from this matching use the maximum matching algorithm to find a complete matching. You must indicate clearly how the algorithm has been applied in this case. State your alternating path and the final matching. ***E***

Solution:

a



bipartite graph initial matching

b Four possible alternating paths – one only is needed
i S – L = P – K
ii S – L = M – A = T – K
iii S – A = T – K
iv S – L = P – A = T – K
change status to give a complete matching

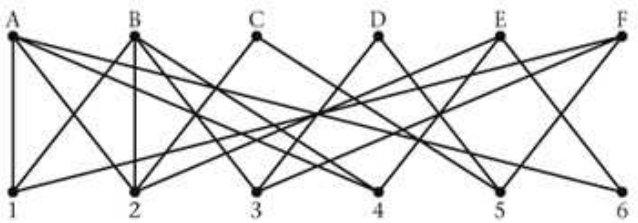
Person	Path (i)	Path (ii)	Path (iii)	Path (iv)
M	L	A	L	L
P	K	L	L	A
R	B	B	B	B
S	L	L	A	L
T	A	K	K	K

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

Question:



The bipartite graph above shows a mapping between six volunteers, A, B, C, D, E and F, and six tasks, 1, 2, 3, 4, 5 and 6. The lines indicate which tasks each volunteer is qualified to do.

The initial matching is $A-2, B-1, C-5, D-3$ and $E-4$.

a Starting from this matching, use the maximum matching algorithm to find a complete matching. You must indicate clearly how the algorithm has been applied in this case. State your alternating path and your final matching.

Volunteer E now insists on doing task 2.

b State the changes that need to be made to the initial model to accommodate this. ***E***

Solution:

a 12 possible alternating paths – one only is needed

i F – 1 = B – 2 = A – 6

ii F – 1 = B – 2 = A – 4 = E – 6

iii F – 1 = B – 3 = D – 5 = C – 2 = A – 6

iv F – 1 = B – 3 = D – 5 = C – 2 = A – 4 = E – 6

v F – 1 = B – 4 = E – 6

vi F – 1 = B – 4 = E – 2 = A – 6

vii F – 5 = C – 2 = A – 1 = B – 4 = E – 6

viii F – 5 = C – 2 = A – 4 = E – 6

ix F – 5 = C – 2 = A – 6

x F – 3 = D – 5 = C – 2 = A – 6

xi F – 3 = D – 5 = C – 2 = A – 4 = E – 6

xii F – 3 = D – 5 = C – 2 = A – 1 = B = 4 – E = 6

Change status to give complete matching

Volunteer	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)
A	6	4	6	4	2	6	1	4	6	6	4	1
B	2	2	3	3	4	4	4	1	1	1	1	4
C	5	5	2	2	5	5	2	2	2	2	2	2
D	3	3	5	5	3	3	3	3	3	5	5	5
E	4	6	4	6	6	2	6	6	4	4	6	6
F	1	1	1	1	1	1	5	5	5	3	3	3

b Remove E and 2 and all arcs attached to each of them. Then run the algorithm as usual on the reduced problem.

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 5

Question:

At a school fete six teachers, A, B, C, D, E and F, are to run six stalls, R, S, T, U, V and W.

A would prefer to run T but is willing to run R
B would prefer to run S but is willing to run R or W
C would prefer to run U but is willing to run S
D would prefer to run V but is willing to run R
E is willing to run T or V
F is willing to run V

a Draw a bipartite graph to model this situation.

Initially, A, B, C and D are matched to their preferred choices.

b Indicate this initial matching in a distinctive way on the bipartite graph drawn in **a**.

c Use the maximum matching algorithm to find a maximum matching, listing clearly your alternating path.

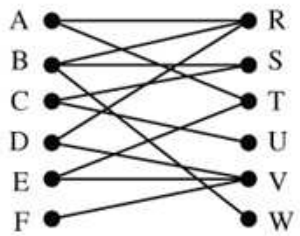
d Explain why it is not possible to find a complete matching. You should make specific reference to individual stalls and teachers.

Teacher A now offers to run stall S.

e Draw a new bipartite graph. Hence, using the previous initial matching and the maximum matching algorithm, determine if it is now possible to obtain a complete matching. If it is possible, give the matching, stating clearly your alternating path; if it is still not possible explain why.

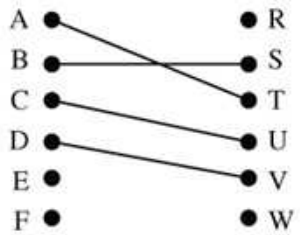
Solution:

a



bipartite graph

b



initial matching

c There are 3 alternating paths, that could be used

i E – V = D – R or
ii E – T = A – R or
iii F – V = D – R

Change status to give

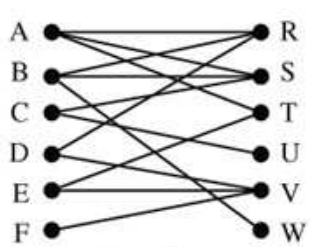
i E = V – D = R or
ii E = T – A = R or
iii F = V – D = R

Maximum matching

Person	Path (i)	Path (ii)	Path (iii)
A	T	R	T
B	S	S	S
C	U	U	U
D	R	V	R
E	V	T	?
F	?	?	V

d For example C must do U and B must do W, but B and C are the only people who can do S. So a complete matching is not possible.

e



new bipartite graph

Depending on the path chosen in **c**

either E – V = D – R and F – V = E – T = A – S = B – W

or E – T = A – S = B – W and F – V = D – R

or E – T = A – S = B – R and F – V = D – R = B – W

Final matching

A = S B = W C = U E = T D = R F = V

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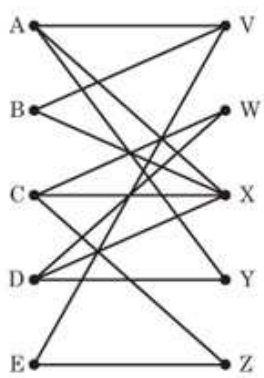
Edexcel AS and A Level Modular Mathematics

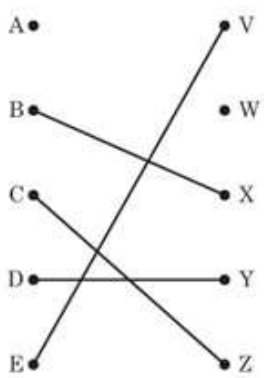
Exercise B, Question 6

Question:

a



b



Five people A, B, C, D and E are to be matched to five tasks V, W, X, Y and Z. A bipartite graph showing the possible matching is shown in **a**, and an initial matching is shown in **b**.

There are a number of distinct alternating paths that can be generated from this initial matching. Two such paths are

A – Y – D – W and A – X – B – V – E – Z – C – W.

a Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

b Using the maximum matching algorithm and the given initial matching,

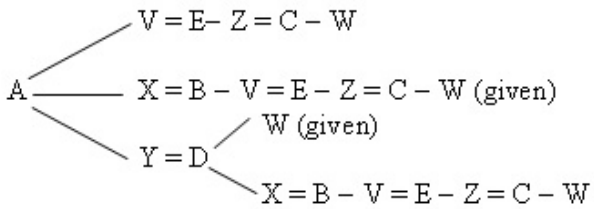
i find two further distinct alternating paths, making your reasoning clear,

ii write down the complete matchings they generate. ***E***

Solution:

a A = Y B = X C = Z D = W E = V
A = X B = V C = W D = Y E = Z

b



i Changing status in the two new paths
A = V – E = Z – C = W and
A = Y – D = X – B = V – E = Z – C = W

ii Giving matchings
A = V B = X C = W D = Y E = Z and
A = Y B = V C = W D = X E = Z

Solutionbank D1

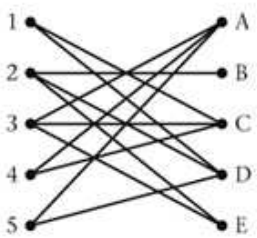
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

A college has five vacant jobs, A, B, C, D and E. There are five applicants who are labelled 1, 2, 3, 4 and 5. The applicants are only qualified for certain jobs and the following table summarises this information. The diagram shows a bipartite graph modelling this information.

Applicant	Jobs qualified for
1	C, D
2	B, D, E
3	A, C, E
4	A, C
5	A, D



Initially Applicant 2 is allocated to job B, 3 is attached to C and 5 is allocated to A.

a **i** Show this matching clearly on a diagram.

ii Starting from this matching, use the maximum matching algorithm to obtain an improved matching. State clearly your alternating path and show this improved matching on Diagram 2.

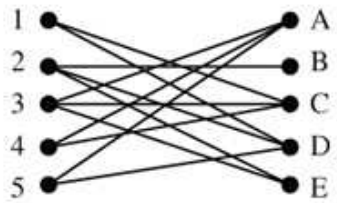
iii Hence obtain a complete matching. State clearly your alternating path and this complete matching.

The interviewing committee decides that applicant 3 is to be appointed to job C.

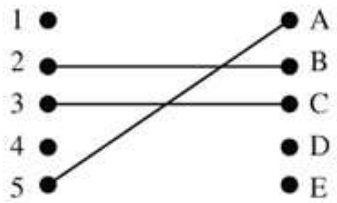
b If this appointment is made, explain why it is not possible to fill all the other jobs with the remaining applicants. ***E*** (adapted)

Solution:

a **i**

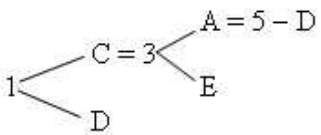


bipartite graph

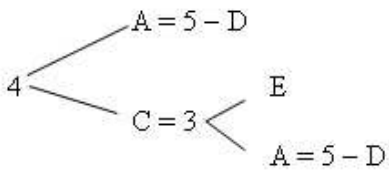


initial matching

ii



or



So six possible alternating paths – one only needs to be used changing status

i 1 = D
ii 1 = C – 3 = E
iii 1 = C – 3 = A – 5 = D
iv 4 = A – 5 = D
v 4 = C – 3 = E
vi 4 = C – 3 = A – 5 = D

Giving improved matchings

Applicant	(i)	(ii)	(iii)	(iv)	(v)	(vi)
1	D	C	C	?	?	?
2	B	B	B	B	B	B
3	C	E	A	C	E	A
4	?	?	?	A	C	C
5	A	A	D	D	A	D

iii The next path depends on the previous path choice.

Previously chosen path	Possible next alternating path
i	4 – C = 3 – E
ii	4 – C = 1 – D
iii	4 – C = 1 – D = 5 – A = 3 – E
iv	1 – C = 3 – E
v	1 – D
vi	1 – D = 5 – A = 3 – E

Changing status leads to the following complete matching

Applicant	(i)	(ii)	(iii)	(iv)	(v)	(vi)
1	D	D	D	C	D	D
2	B	B	B	B	B	B
3	E	E	E	E	E	E
4	C	C	C	A	C	C
5	A	A	A	D	A	A

b If 3 matches with C
2 must now do E since no one else can but 2 is the only person who can do B too so a complete matching is not possible.

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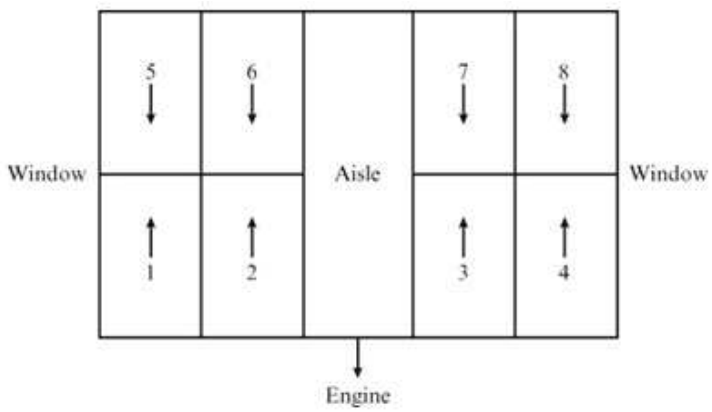
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 8

Question:



The diagram above represents eight seats in a railway carriage, numbered 1, 2, 3, 4, 5, 6, 7 and 8. These are the last eight seats available on a special sightseeing trip. The booking clerk has to arrange the seating for the final customers. Six customers make the following requests:

Ms A wants an aisle seat facing the engine (6 or 7)
Mr B wants a window seat (1, 4, 5 or 8)
Rev C wants a seat with his back to the engine (1, 2, 3 or 4)
Mrs D wants an aisle seat (2, 3, 6 or 7)
Miss E wants a seat facing the engine (5, 6, 7 or 8)
Dr F wants a window seat with her back to the engine (1 or 4)

Initially the clerk assigns the seats as follows:
A to 6, B to 5, C to 4, D to 2, E to 7 and F to 1
The day before departure Mr and Mrs G join the trip. They ask to sit next to each other (1 and 2, 3 and 4, 5 and 6 or 7 and 8). The clerk reassigns the seats, using as far as possible the original seat assignments as the initial matching.

a Choose two seats for Mr and Mrs G and, using a bipartite graph, model the possible seat allocations of the other customers.

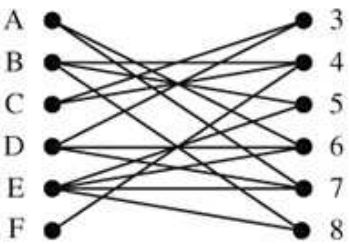
b Indicate, in a distinctive way, those elements of the clerk's original matching that are still possible.

c Using your answer to part **b** as the initial matching, apply the maximum matching algorithm. You must state your alternating path and your final maximum matching. ***E***

Solution:

Mr + Mrs G must be assigned to two seats next to each other.
1+2 or 3+4 or 5+6 or 7+8
Depending on what you change we get

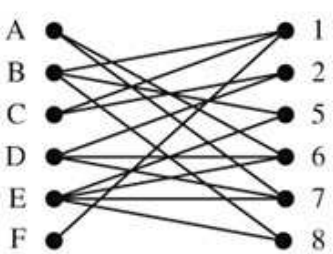
Alternative 1 (1 and 2 assigned to Mr + Mrs G)



Initially
A = 6
B = 5
C = 4
E = 7
For example, alternating paths

D − 7 = E − 8 and F − 4 = C − 3
Change status D = 7 − E = 8 and F = 4 − C = 3
A = 6 C = 3 E = 8
B = 5 D = 7 F = 4
Mr + Mrs G in 1 and 2

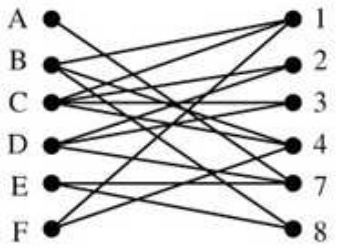
Alternative 2 (3+4 assigned to Mr + Mrs G)



Initially
A = 6
B = 5
D = 2
E = 7
For example, alternating path

C − 2 = D − 7 = E − 8
Change status C = 2 − D = 7 − E = 8
A = 6 C = 2 E = 8
B = 5 D = 7 F = 1
Mr + Mrs G in 3 and 4

Alternative 3 (5 and 6 assigned to Mr + Mrs G)



Initially

$C = 4$

$D = 2$

$E = 7$

$F = 1$

For example, alternating path

$A - 7 = E - 8$ and $B - 4 = C - 3$

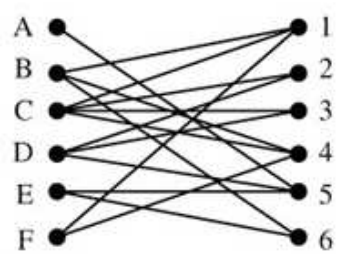
Change status $A = 7 - E = 8$ and $B = 4 - C = 3$

$A = 7$ $C = 3$ $E = 8$

$B = 4$ $D = 2$ $F = 1$

Mr + Mrs G in 5 and 6

Alternative 4 (7 and 8 assigned to Mr + Mrs G)



Initially

$A = 6$

$B = 5$

$C = 4$

$D = 2$

$F = 1$

For example, alternating path

$E - 5 = B - 4 = C - 3$

Change status $E = 5 - B = 4 - C = 3$

$A = 6$ $C = 3$ $E = 5$

$B = 4$ $D = 2$ $F = 1$

Mr + Mrs G in 7 and 8