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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

A chocolate manufacturer is producing two hand-made assortments, gold and silver, to commemorate 50 years in business.

It will take 30 minutes to make all the chocolates for one box of gold assortment and 20 minutes to make the chocolates for one box of silver assortment.

It will take 12 minutes to wrap and pack the chocolates in one box of gold assortment and 15 minutes for one box of silver assortment.

The manufacturer needs to make at least twice as many silver as gold assortments.

The gold assortment will be sold at a profit of 80p, and the silver at a profit of 60p.

There are 300 hours available to make the chocolates and 200 hours to wrap them. The profit is to be maximised.

Letting the number of boxes of gold assortment be x and the number of boxes of silver assortment be y, formulate this as a linear programming problem.

Solution:

Number of boxes of gold assortment $= x$
Number of boxes of silver assortment $= y$
Objective: maximise $P = 80x + 60y$

Constraints

- time to make chocolate, $30x + 20y \leq 300 \times 60$
  which simplifies to $3x + 2y \leq 1800$
- time to wrap and pack $12x + 15y \leq 200 \times 60$
  which simplifies to $4x + 5y \leq 4000$

All units now in minutes.

- 'At *least* twice as many silver as gold' $2x \leq y$
- non-negativity $x, y \geq 0$

In summary: maximise $P = 80x + 60y$
subject to

$$3x + 2y \leq 1800$$
$$4x + 5y \leq 4000$$
$$2x \leq y$$
$$x, y \geq 0$$

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Exercise A, Question 2

Question:

A floral display is required for the opening of a new building. The display must be at least 30 m long and is to be made up of two types of planted displays, type A and type B.

Type A is 1 m in length and costs £6
Type B is 1.5 m in length and costs £10

The client wants at least twice as many type A as type B, and at least 6 of type B.
The cost is to be minimised.

Letting x be the number of type A used and y be the number of type B used, formulate this as a linear programming problem.

Solution:

number of type A $= x$
number of type B $= y$
Objective: minimise $C = 6x + 10y$

Constraints

- Display must be at least 30 m long $x + 1.5y \geq 30$
  which simplifies to $2x + 3y \geq 60$
- 'At least twice as many x as y' $2y \leq x$
- At *least* six type B $y \geq 6$
- non-negativity $x, y \geq 0$

In summary: minimise $C = 6x + 10y$
subject to:

$$2x + 3y \geq 60$$
$$2y \leq x$$
$$y \geq 6$$
$$x, y \geq 0$$

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Exercise A, Question 3

Question:

A toy company makes two types of board game, Cludopoly and Trivscrab. As well as the board each game requires playing pieces and cards.

The company uses two machines, one to produce the pieces and one to produce the cards. Both machines can only be operated for up to ten hours per day.

The first machine takes 5 minutes to produce a set of pieces for Cludopoly and 8 minutes to produce a set of pieces for Trivscrab.

The second machine takes 8 minutes to produce a set of cards for Cludopoly and 4 minutes to produce a set of cards for Trivscrab.

The company knows it will sell at most three times as many games of Cludopoly as Trivscrab.

The profit made on each game of Cludopoly is £1.50 and £2.50 on each game of Trivscrab.
The company wishes to maximise its daily profit.
Let x be the number of games of Cludopoly and y the number of games of Trivscrab.
Formulate this problem as a linear programming problem.

Solution:

number of games of Cludopoly $= x$
number of games of Trivscrab $= y$
Objective: maximise $P = 1.5x + 2.5y$

Constraints

- First machine. $5x + 8y \le 10 \times 60$
  which simplifies to $5x + 8y \le 600$
- Second machine: $8x + 4y \le 10 \times 60$
  which simplifies to $2x + y \le 150$
- At *most* 3 times as many x as y $3y \ge x$
- non-negativity $x, y \ge 0$

All units now in minutes.

In summary: maximise $P = 1.5x + 2.5y$
subject to:

$$5x + 8y \le 600$$
$$2x + y \le 150$$
$$3y \ge x$$
$$x, y \ge 0$$

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Exercise A, Question 4

Question:

A librarian needs to purchase bookcases for a new library. She has a budget of £3000 and $240\,\text{m}^2$ of available floor space. There are two types of bookcase, type 1 and type 2, that she is permitted to buy.
Type 1 costs £150, needs $15\,\text{m}^2$ of floor space and has 40 m of shelving.
Type 2 costs £250, needs $12\,\text{m}^2$ of floor space and has 50 m of shelving.
She must buy at least 8 type 1 bookcases and wants at most $\frac{1}{3}$ of all the bookcases to be type 2.
She wishes to maximise the total amount of shelving.
Letting x and y be the number of type 1 and type 2 bookcases bought respectively, formulate this as a linear programming problem.

Solution:

Number of type 1 bookcases $= x$
Number of type 2 bookcases $= y$

Objective: maximise $S = 40x + 60y$

Constraints

- budget $150x + 250y \le 3000$
  which simplifies to $3x + 5y \le 60$
- floor space $15x + 12y \le 240$
  which simplifies to $5x + 4y \le 80$
- 'At most $\frac{1}{3}$ of all bookcases to be type 2' $y \le \frac{1}{3}(x + y)$
  which simplifies to $2y \le x$
- At least 8 type 1 $x \ge 8$
- non-negativity $x, y \ge 0$

In summary: maximise $S = 40x + 60y$
subject to:

$$3x + 5y \le 60$$
$$5x + 4y \le 80$$
$$2y \le x$$
$$x \ge 8$$
$$x, y \ge 0$$

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Exercise A, Question 5

Question:

A garden supplies company produces two different plant feeds, one for indoor plants and one for outdoor plants.

In addition to other ingredients, the plant feeds are made by combining three different natural ingredients A, B and C.

Each kilogram of indoor feed requires 10 g of A, 20 g of B and 20 g of C.
Each kilogram of outdoor feed requires 20 g of A, 10 g of B and 20 g of C.

The company has 5 kg of A, 5 kg of B and 6 kg of C available each week to use to make these feeds.

The company will sell at most three times as much outdoor as indoor feed, and will sell at least 50 kg of indoor feed.

The profit made on each kilogram of indoor and outdoor feed is £7 and £6 respectively. The company wishes to maximise its weekly profit.
Formulate this as a linear programming problem, defining your decision variables.

Solution:

Let $x =$ number of kg of indoor feed and $y =$ number of kg of outdoor feed
Objective: Maximise $P = 7x + 6y$
Constraints

- Amount of A $10x + 20y \le 5 \times 1000$
  which simplifies to $x + 2y \le 500$
- Amount of B $20x + 10y \le 5 \times 1000$
  which simplifies to $2x + y \le 500$
- Amount of C $20x + 20y \le 6 \times 100$
  which simplifies to $x + y \le 300$

All units now in grams.

- At *most* 3 times as much y as x $y \le 3x$
- At least 50 kg of x $x \ge 50$
- non-negativity $y \ge 0$ ($x \ge 0$ is unnecessary because of the previous constraint)

In summary:
maximise $P = 7x + 6y$
subject to

$$\begin{aligned} x + 2y &\le 500 \\ 2x + y &\le 500 \\ x + y &\le 300 \\ y &\le 3x \\ x &\ge 50 \\ y &\ge 0 \end{aligned}$$

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Exercise A, Question 6

Question:

Sam makes three types of fruit smoothies, A, B and C. As well as other ingredients all three smoothies contain oranges, raspberries, kiwi fruit and apples, but in different proportions. Sam has 50 oranges, 1000 raspberries, 100 kiwi fruit and 60 apples. The table below shows the number of these 4 fruits used to make each smoothie and the profit made per smoothie. Sam wishes to maximise the profit.

Smoothie	Oranges	Raspberries	Kiwi fruit	Apples	Profit
A	1	10	2	2	60p
B	$\frac{1}{2}$	40	3	$\frac{1}{2}$	65p
C	2	15	1	2	55p
Total available	50	1000	100	60	

Letting x be the number of A smoothies, y the number of B smoothies and z the number of C smoothies, formulate this as a linear programming problem.

Solution:

number of A smoothies $= x$
number of B smoothies $= y$
number of C smoothies $= z$

Objective maximise $P = 60x + 65y + 55z$
Constraints

- oranges $x + \frac{1}{2}y + 2z \leq 50$
  which simplifies to $2x + y + 4z \leq 100$
- raspberries $10x + 40y + 15z \leq 1000$
  which simplifies to $2x + 8y + 3z \leq 200$
- kiwi fruit $2x + 3y + z \leq 100$
- apples $2x + \frac{1}{2}y + 2z \leq 60$
  which simplifies to $4x + y + 4z \leq 120$
- non-negativity $x, y, z \geq 0$

In summary: maximise $P = 60x + 65y + 55z$
subject to:

$$
\begin{aligned}
2x + y + 4z &\leq 100 \\
2x + 8y + 3z &\leq 200 \\
2x + 3y + z &\leq 100 \\
4x + y + 4z &\leq 120 \\
x, y, z &\geq 0
\end{aligned}
$$

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Exercise A, Question 7

Question:

A dairy manufacturer has two factories R and S. Each factory can process milk and yoghurt.
Factory R can process 1000 litres of milk and 200 litres of yoghurt per hour.
Factory S can process 800 litres of milk and 300 litres of yoghurt per hour.
It costs £300 per hour to operate factory R and £400 per hour to operate factory S. In order to safeguard jobs it has been agreed that each factory will operate for at least $\frac{1}{3}$ of the total, combined, operating time.
The manufacturer needs to process 20 000 litres of milk and 6000 litres of yoghurt. He wishes to distribute this between the 2 factories in such a way as to minimise operating costs.
Formulate this as a linear programming problem in x and y, defining your decision variables.

Solution:

Let number of hours of work for factory R $= x$
Let number of hours of work for factory S $= y$

Objective: minimise $C = 300x + 400y$

Constraints

- milk $1000x + 800y \geq 20\,000$
  which simplifies to $5x + 4y \geq 100$
- yoghurt $200x + 300y \geq 6000$
  which simplifies to $2x + 3y \geq 60$
- At *least* $\frac{1}{3}$ of total time for R $x \geq \frac{1}{3}(x + y)$
  which simplifies to $2x \geq y$
- At *least* $\frac{1}{3}$ of total time for S $y \geq \frac{1}{3}(x + y)$
  which simplifies to $2y \geq x$
- non-negativity $x, y \geq 0$

In summary minimise $C = 300x + 400y$
subject to:

$$\begin{aligned} 5x + 4y &\geq 100 \\ 2x + 3y &\geq 60 \\ 2x &\geq y \\ 2y &\geq x \\ x, y &\geq 0 \end{aligned}$$

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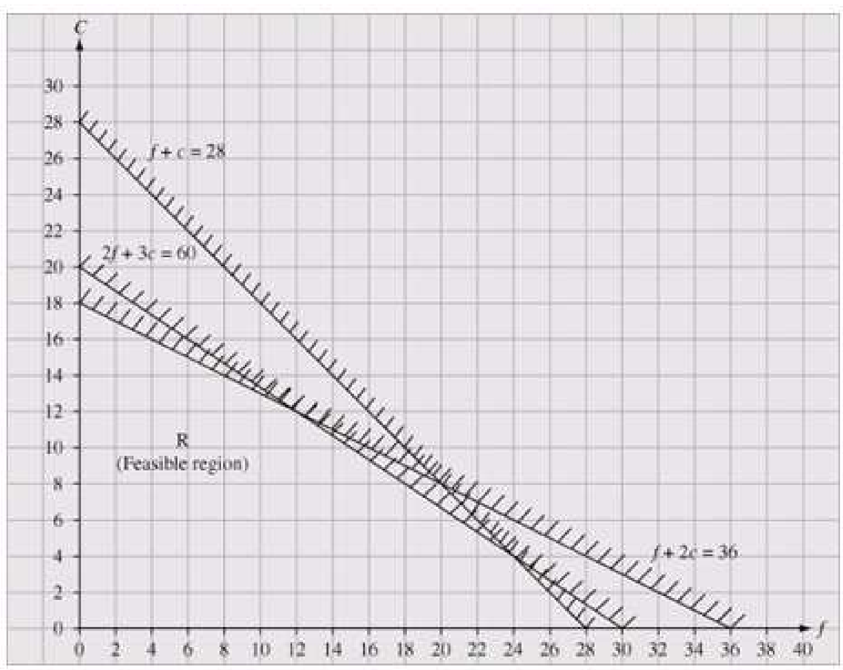
Exercise B, Question 1

Question:

Illustrate the inequalities found in Example 1 (pages 114–115) on graph paper.
Label the feasible region, R.

Solution:



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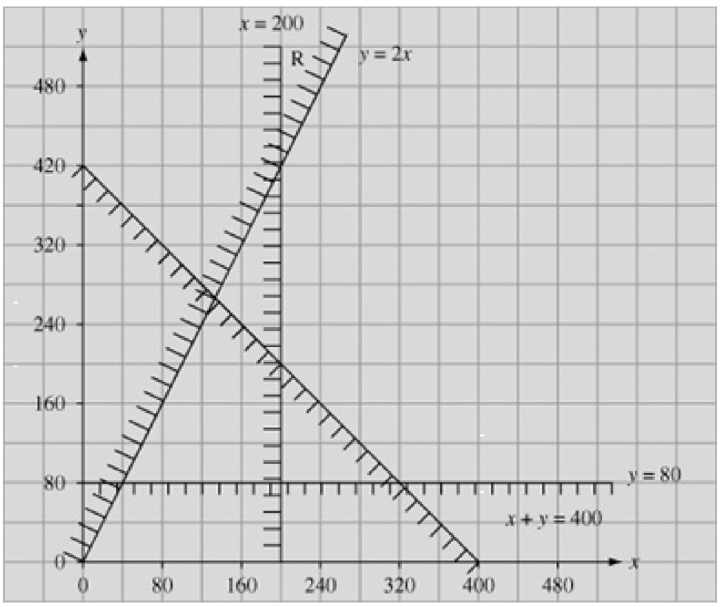
Exercise B, Question 2

Question:

Repeat Question 1 using the inequalities found in Example 2 (pages 116–117).

Solution:



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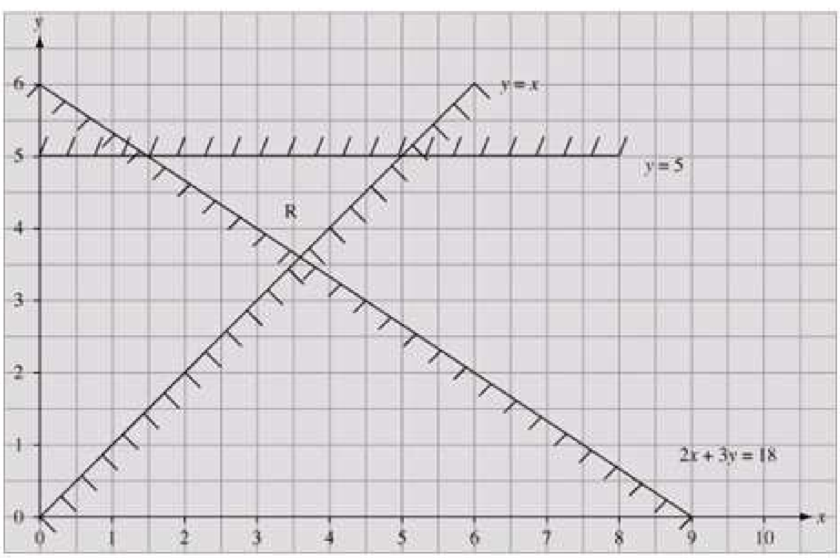
Exercise B, Question 3

Question:

Represent the following inequalities graphically. Label the region bounded by these inequalities R.

$$2x + 3y > 18$$
$$y > x$$
$$y \leq 5$$
$$x, y \geq 0$$

Solution:



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Exercise B, Question 4

Question:

The following inequalities were found when solving a linear programming problem.

$2x \leq 3y$

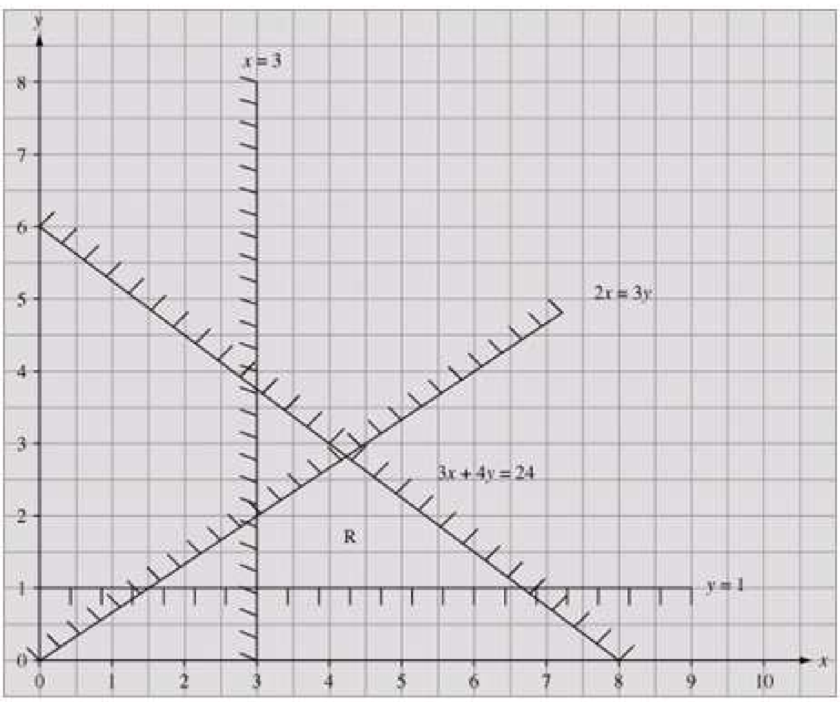
$3x + 4y \leq 24$

$x \geq 3$

$y \geq 1$

Represent these inequalities on a graph.
Indicate the feasible region by labelling it R.

Solution:



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Exercise B, Question 5

Question:

Region R is bounded by the following inequalities

$x+y \le 20$

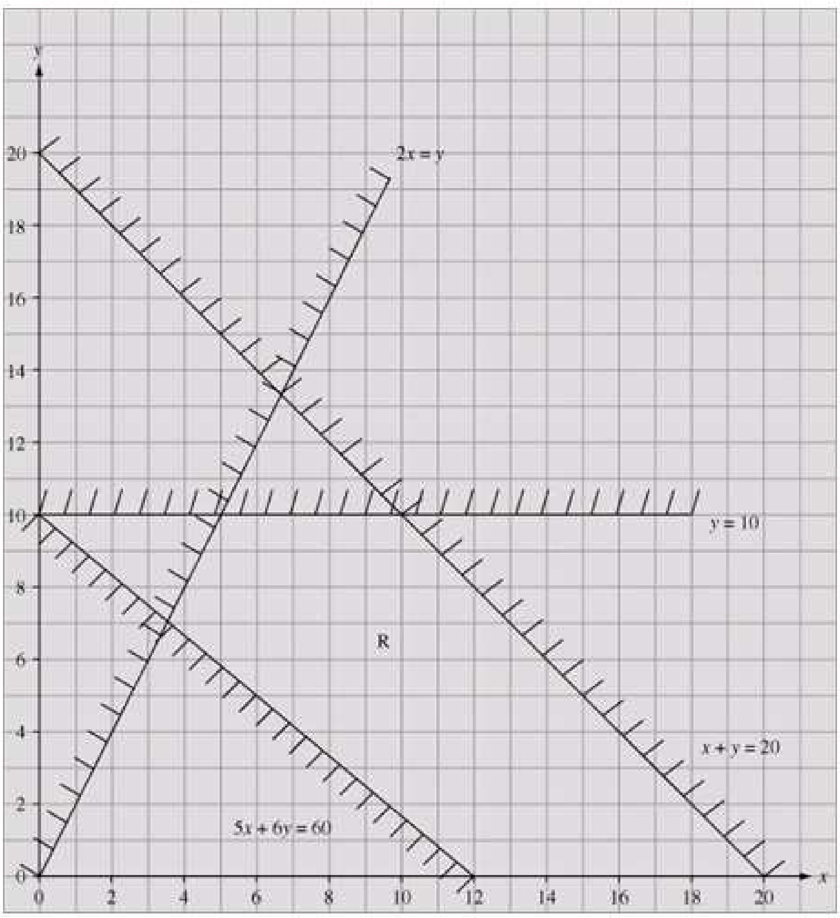
$5x+6y \ge 60$

$2x \ge y$

$y \le 10$

By drawing suitable straight lines, draw a graph to show region R.

Solution:



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Exercise B, Question 6

Question:

Indicate on a graph the region, R, for which

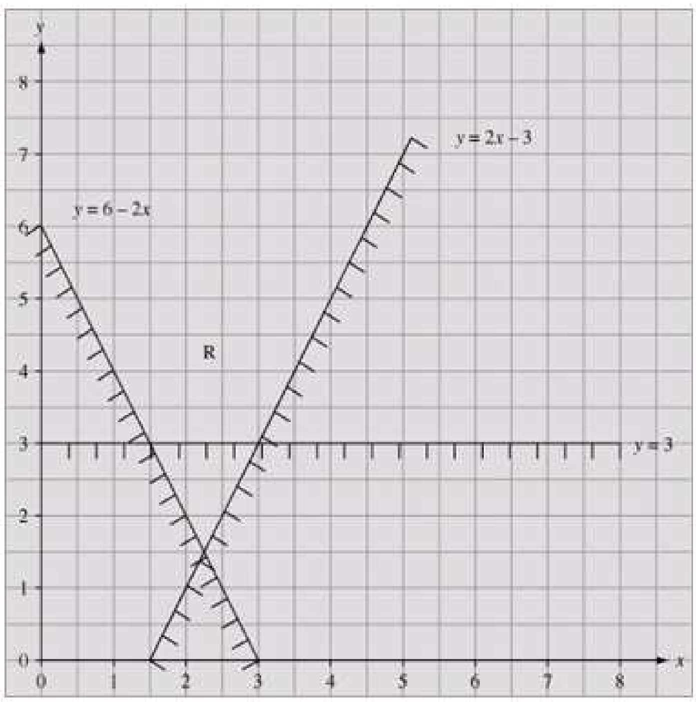
$2x-3 < y$

$y > 3$

$y > 6-2x$

$x \geq 0$

Solution:



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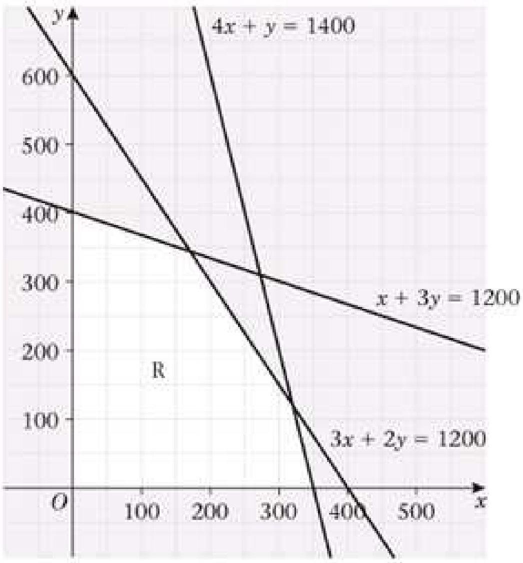
Exercise C, Question 1

Question:



The diagram shows a feasible region, R. Find the optimal point and the optimal value, using

a the objective line method, with the objective 'maximise $M = 2x + y$',
b the objective line method, with the objective 'maximise $N = x + 4y$',
c the vertex testing method, with the objective 'maximise $P = x + y$',
d the vertex testing method, with the objective 'maximise $Q = 6x + y$'.

Solution:

a Need intersection of $4x + y = 1400$ and $3x + 2y = 1200$

Objective line passes through (200, 0) and (0, 400).

(320, 120) $m = 760$

b (0, 400) $N = 1600$

Objective line passes through (400,0) and (100,0).

c Need intersection of $x + 3y = 1200$ and $3x + 2y = 1200$

Objective line passes through (200,0) and (0, 200).

$(171\frac{3}{7}, 342\frac{6}{7})$ $P = 514\frac{2}{7}$

d (350, 0) $Q = 2100$

Objective line passes through (100, 0) and (0, 600).

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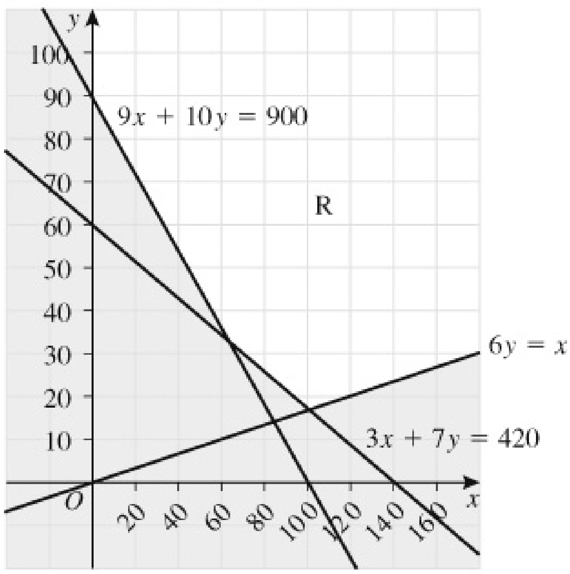
Exercise C, Question 2

Question:



The diagram shows a feasible region, R.

Find the optimal point and the optimal value, using

a the vertex testing method, with the objective 'minimise $E = 2x + y$',

b the vertex testing method, with the objective 'minimise $F = x + 4y$',

c the objective line method, with the objective 'minimise $G = 3x + 4y$',

d the objective line method, with the objective 'minimise $H = x + 6y$',

Solution:

a $(0, 90)$ $E = 90$

b Need intersection of $6y = x$ and $3x + 7y = 420$
$(100.8, 16.8)$ $F = 168$

c Need intersection of $9x + 10y = 900$
$3x + 7y = 420$
$\left(63\frac{7}{11}, 32\frac{8}{11}\right)$ $G = 321\frac{9}{11}$

Objective line passes through (80, 0) and (0, 60).

d Same intersection as in **b** $(100.8, 16.8)$ $H = 201.6$

Objective line passes through (120, 0) and (0, 20).

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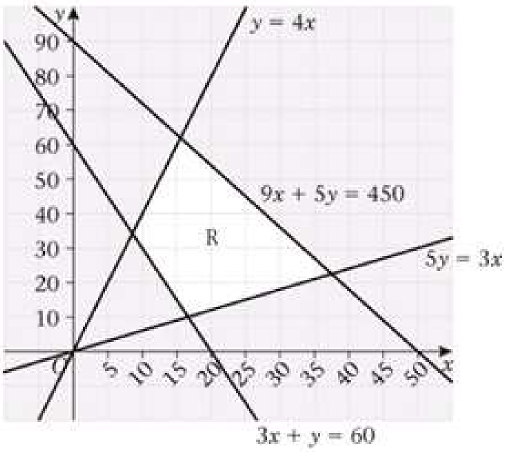
Exercise C, Question 3

Question:



The diagram shows a feasible region, R.
Find the optimal point and the optimal value, using

a the vertex testing method, with the objective 'minimise $J = x + 4y$',

b the vertex testing method, with the objective 'maximise $K = x + y$',

c the objective line method, with the objective 'minimise $L = 6x + y$',

d the objective line method, with the objective 'maximise $M = 2x + y$'.

Solution:

a Need intersection of $3x + y = 60$ and $5y = 3x$

$\left(16\frac{2}{3}, 10\right)$ $J = 56\frac{2}{3}$

b Need intersection of $y = 4x$ and $9x + 5y = 450$

$\left(15\frac{15}{29}, 62\frac{2}{29}\right)$ $K = 77\frac{17}{29}$

c Need intersection of $3x + y = 60$ and $y = 4x$ ← Objective line passes through (10, 0) and (0, 60).

$\left(8\frac{4}{7}, 34\frac{2}{7}\right)$ $L = 85\frac{5}{7}$

d Need intersection of $9x + 5y = 450$ and $5y = 3x$ ← Objective line passes through (40, 0) and (0, 80).

$(37.5, 22.5)$ $m = 97.5$

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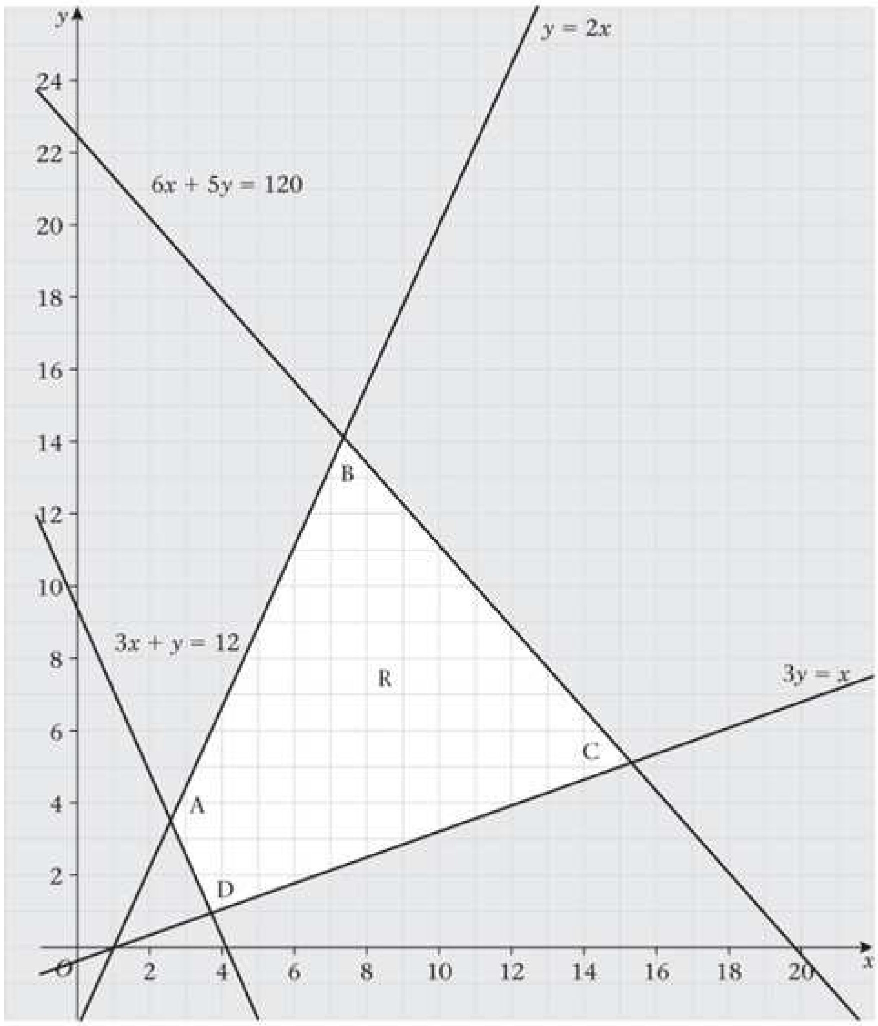
Exercise C, Question 4

Question:



The diagram shows a feasible region, R, which is defined by

$3x + y \geq 12$

$y \leq 2x$

$3y \geq x$

$6x + 5y \leq 120.$

Determine which vertex, A, B, C or D, is the optimal point for each of the following objectives.

a maximise x
b minimise x
c maximise y
d minimise y
e maximise $6x+y$
f minimise $6x+y$
g maximise $2x+5y$
h minimise $2x+5y$
i maximise $3x+2y$
j minimise $3x+2y$.

Solution:

a C

b A

c B

d D

e C

f A

g B

h D

i C

j D

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Exercise C, Question 5

Question:

A linear programming problem is given as

minimise $C = 3x + 2y$

subject to $2x + y \geq 160$

$x + y \geq 120$

$x + 3y \geq 180$

and $x \geq 0, y \geq 0$

a Draw a graph to illustrate the feasible, R.
(Take the values $0 \leq x \leq 180$ and $0 \leq y \leq 160$ for your axes.)

b Use the vertex testing method, on the four vertices, to identify the optimal point and optimal value.

Given that the objective changes to

minimise $C_1 = 2x + 3y$,

c draw a suitable objective line and use it to identify the optimal point and optimal value.

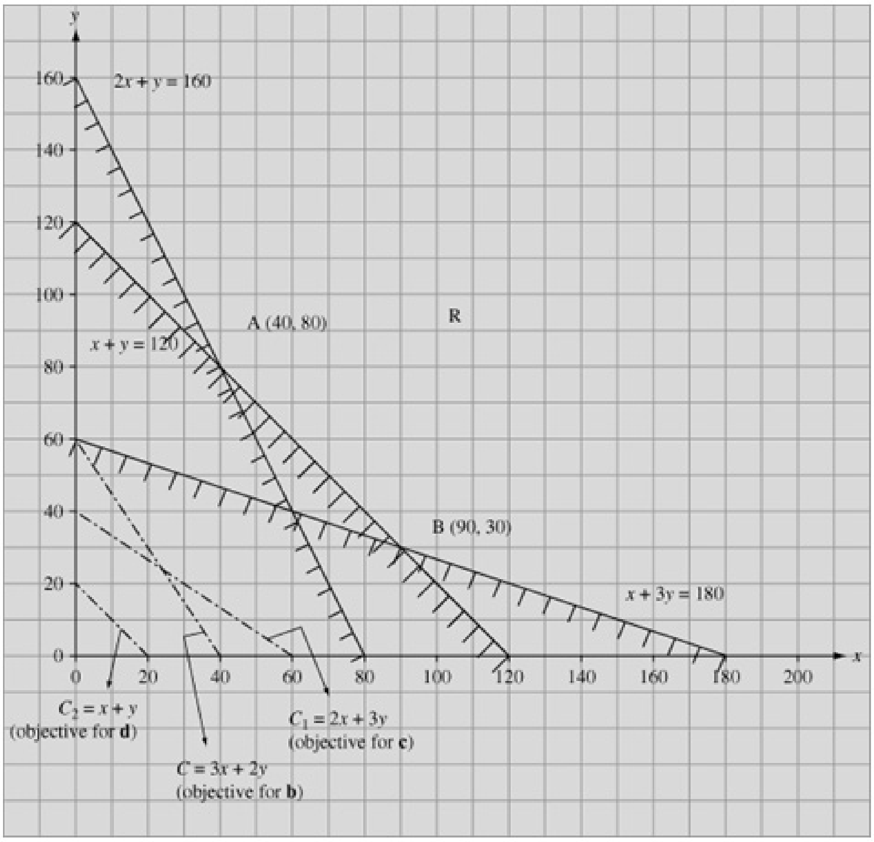
Given that the objective changes to

minimise $C_2 = x + y$,

d explain why there is more than one solution to the problem.

Solution:

a



b

Vertices	$C = 3x + 2y$
(0, 160)	320
(40, 80)	280
(90, 30)	330
(180, 0)	540

so minimum is (40, 80) value of $C = 280$

c (90, 30) $C_1 = 270$

d C_2 is parallel to $x + y = 120$ so all points from A to B are optimal points.

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Exercise C, Question 6

Question:

A feasible region, R, is defined by

$$y \le 5x$$
$$14x + 9y \le 630$$
$$2y \ge x$$
$$4x + y \ge 60$$

a Draw a graph to illustrate the feasible region, R.
(Take the values $0 \le x \le 45$ and $0 \le y \le 70$ for your axes.)

b Use the objective line method to determine the optimal point and the optimal value given the objective

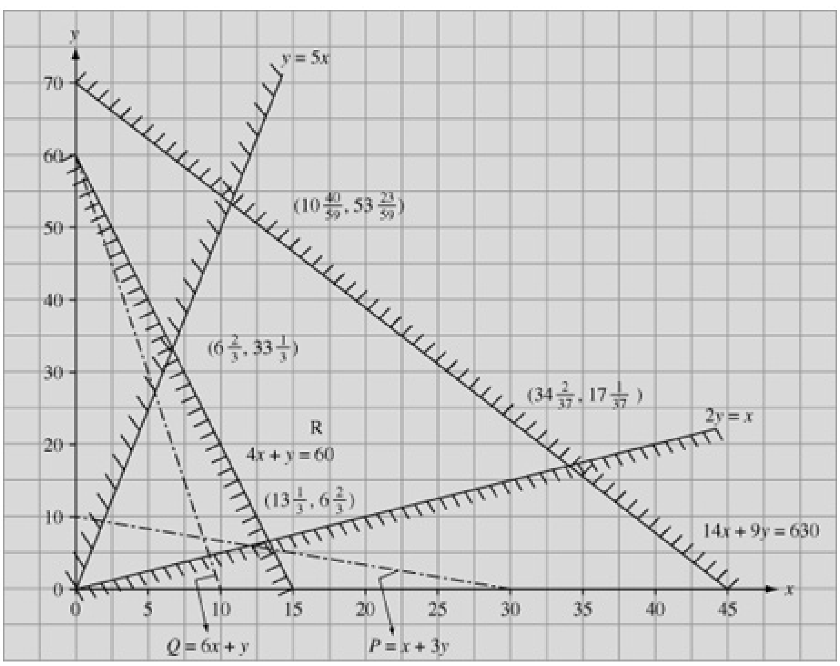
i minimise $P = x + 3y$,

ii maximise $Q = 6x + y$.

In each case you must draw, and label, an objective line, and find *exact* values.

Solution:

a



b **i** $\left(13\frac{1}{3}, 6\frac{2}{3}\right)$ $P = 33\frac{1}{3}$

ii $\left(34\frac{2}{37}, 17\frac{1}{37}\right)$ $Q = 221\frac{13}{37}$

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Exercise C, Question 7

Question:

A feasible region, R, is defined by

$y \geq 10x$

$x \leq 120$

$2y - x \geq 100$

$2x + y \leq 400$

a Draw a graph to illustrate R.

Given that the objective function is $z = 5x + y$,

b determine the exact value of z, if z is to be

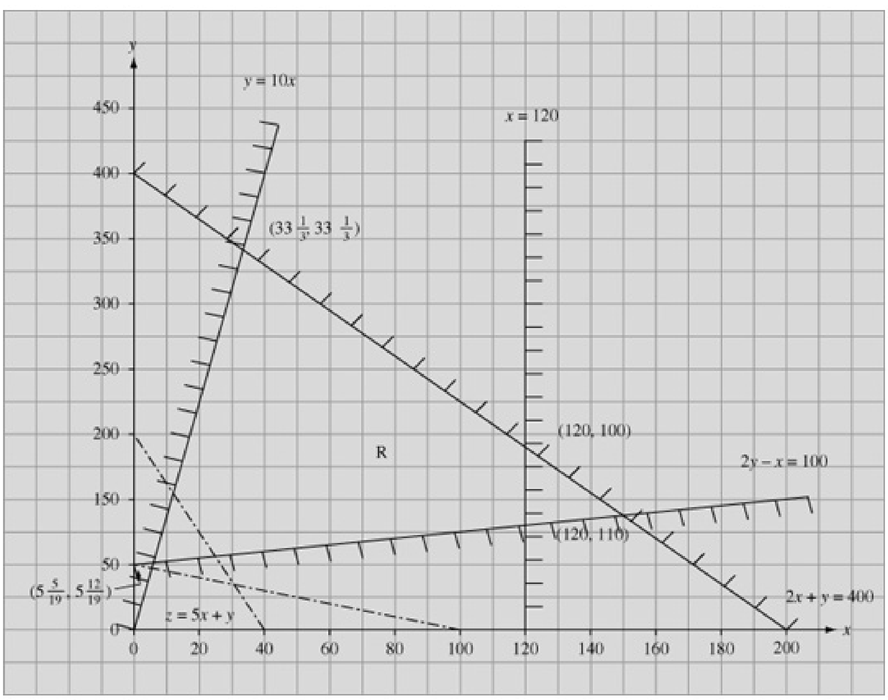
i maximised,

ii minimised.

c Determine the optimal value of $x + 2y$. Give your answer as an exact value.

Solution:

a



b **i** (120, 160) $z = 760$

ii $\left(5\frac{5}{19}, 52\frac{12}{19}\right)$ $z = 78\frac{18}{19}$

c Optimal point $\left(33\frac{1}{3}, 333\frac{1}{3}\right)$ optimal value of $x + 2y = 700$

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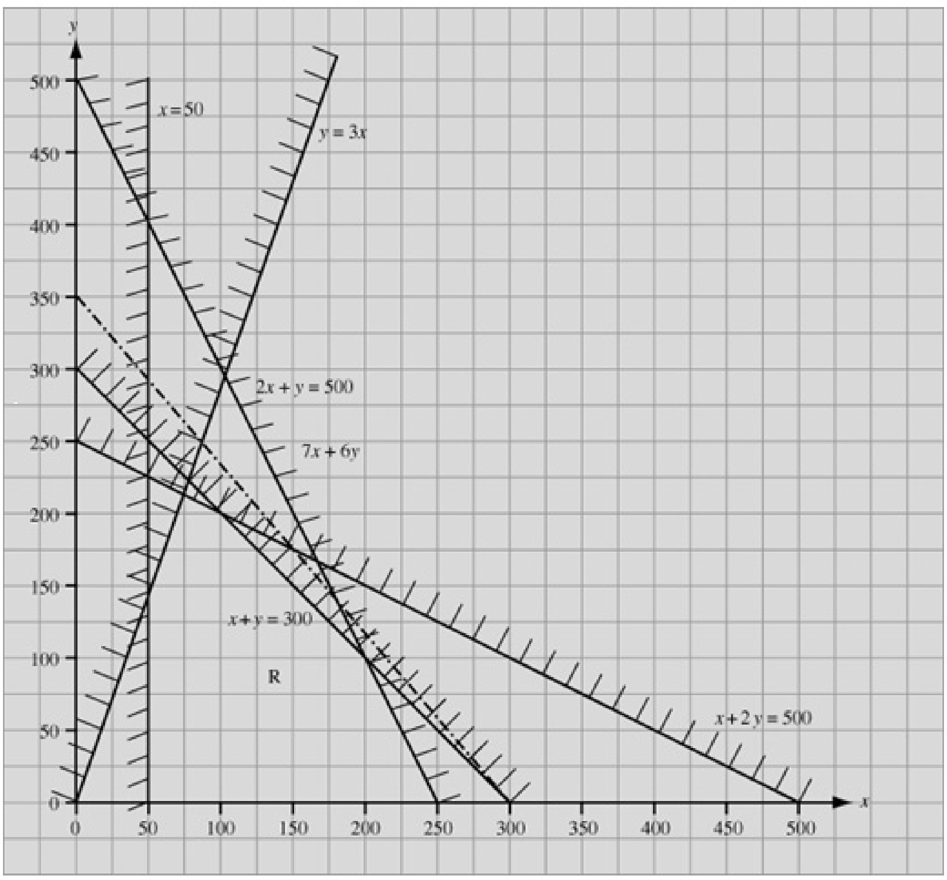
Exercise C, Question 8

Question:

Solve the linear programming problem posed in Exercise 6A, Question 5 (page 120).

Solution:



Objective line passes through (0, 350) and (300, 0). Maximum point is (200, 100).
P = 2000.

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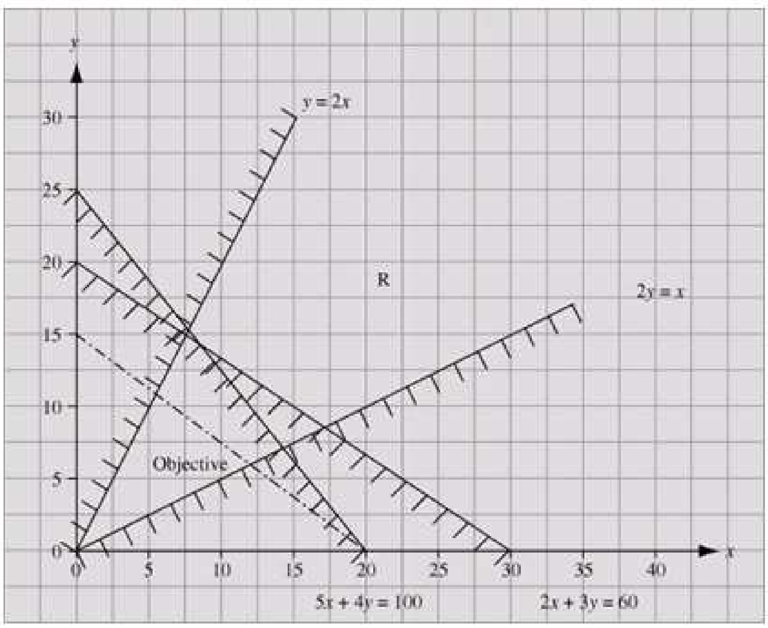
Exercise C, Question 9

Question:

Solve the linear programming problem posed in Exercise 6A, Question 7.

Solution:



Objective line passes through (0, 15) and (20, 0).
Intersection of $2x + 3y = 60$
$5x + 4y = 100$
$\left(8\frac{4}{7}, 14\frac{2}{7}\right)$
$\text{value} = 8285\frac{5}{7}$

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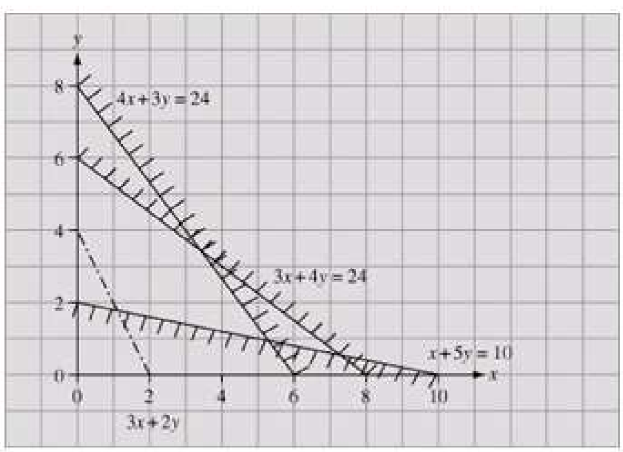
Exercise D, Question 1

Question:

Solve the following linear programming problems, given that integer values are required for the decision variables.

$$\begin{aligned} &\text{Maximise } 3x + 2y \\ &\text{subject to } x+5y \geq 10 \\ &3x+4y \leq 24 \\ &4x+3y \leq 24 \\ &x, y \geq 0 \end{aligned}$$

Solution:



Maximum integer value (5, 1)
$3x+2y=17$

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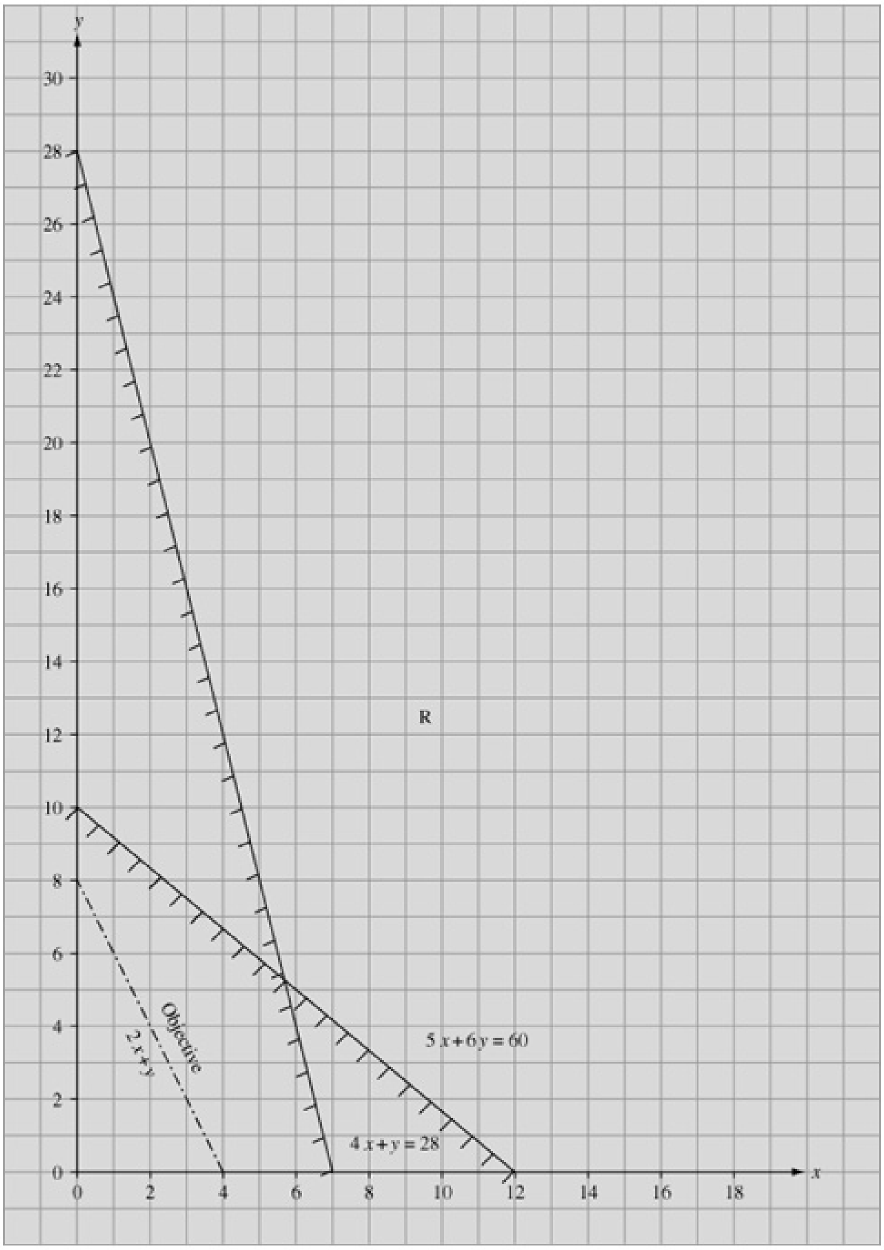
Exercise D, Question 2

Question:

Solve the following linear programming problems, given that integer values are required for the decision variables.

$$\begin{aligned} &\text{Minimise } 2x + y \\ &\text{subject to } 5x + 6y \geq 60 \\ &\qquad 4x + y \geq 28 \\ &\qquad x, y \geq 0 \end{aligned}$$

Solution:



Minimum integer values (6, 6)

$2x + y = 18$

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Exercise D, Question 3

Question:

Solve the following linear programming problem, given that integer values are required for the decision variables.

Maximise $5x + 2y$

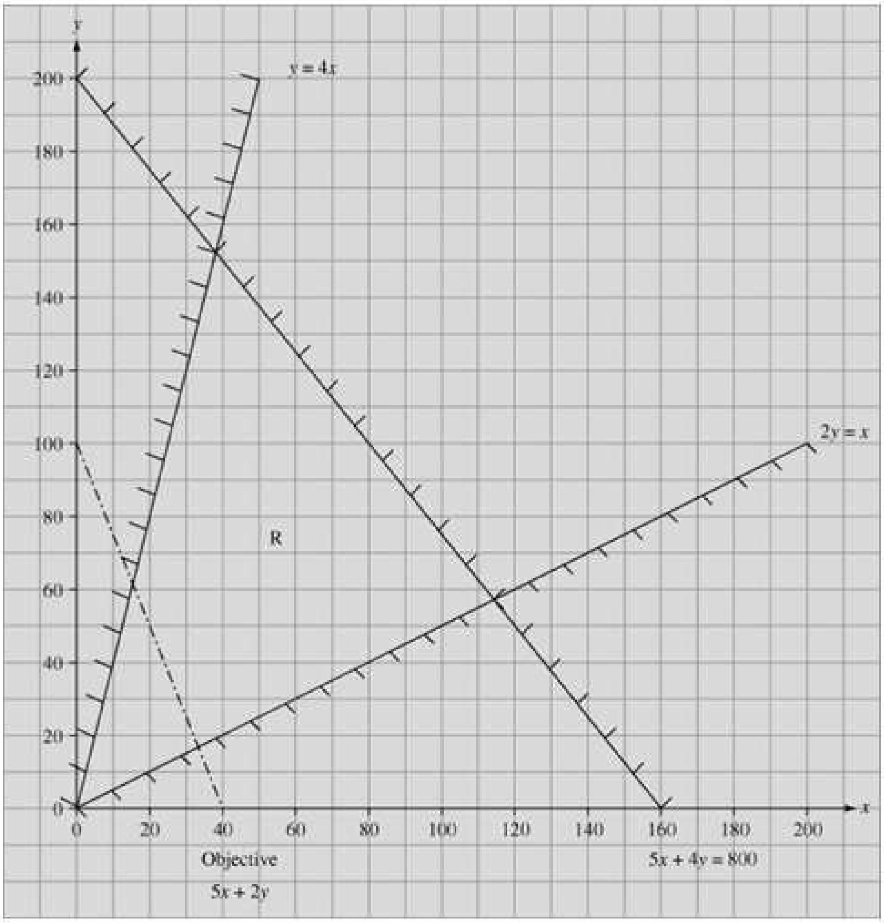
subject to $2y \geq x$

$5x + 4y \leq 800$

$y \leq 4x$

$x, y \geq 0$

Solution:



Solving $2y = x$ and $5x + 4y = 800$ simultaneously gives $\left(114\frac{2}{7}, 57\frac{1}{7}\right)$

Test integer values nearby.

Point	$2y \geq x$	$5x+4y \leq 800$	$5x+2y$
(114, 57)	✓	✓	684
(114, 58)	✓	X	–
(115, 57)	X	X	–
(115, 58)	✓	X	–

so optimal point is (114, 57) value 684

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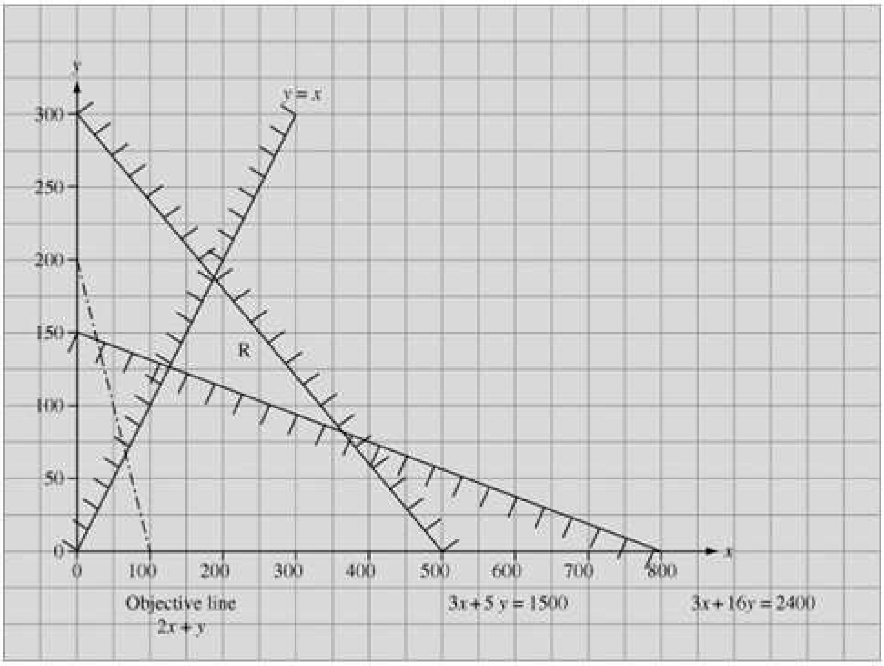
Exercise D, Question 4

Question:

$$\begin{aligned} \text{Maximise } & 2x + y \\ \text{subject to } & 3x + 5y \leq 1500 \\ & 3x + 16y \geq 2400 \\ & y \leq x \\ & x, y \geq 0 \end{aligned}$$

Solve this problem.

Solution:



Solving $3x + 16y = 2400$
$3x + 5y = 1500$
simultaneously gives
$\left(363\frac{7}{11}, 81\frac{9}{11}\right)$

Taking integer point

Point	$3x+16y \geq 2400$	$3x+5y \leq 1500$	$2x+y$
(363, 81)	✓	X	–
(363, 82)	✓	✓	808
(364, 81)	✓	X	–
(364, 82)	X	✓	–

So optimal integer point is (363, 82)

value 808

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Exercise D, Question 5

Question:

Solve this problem.

A chocolate manufacturer is producing two hand-made assortments, gold and silver, to commemorate 50 years in business.

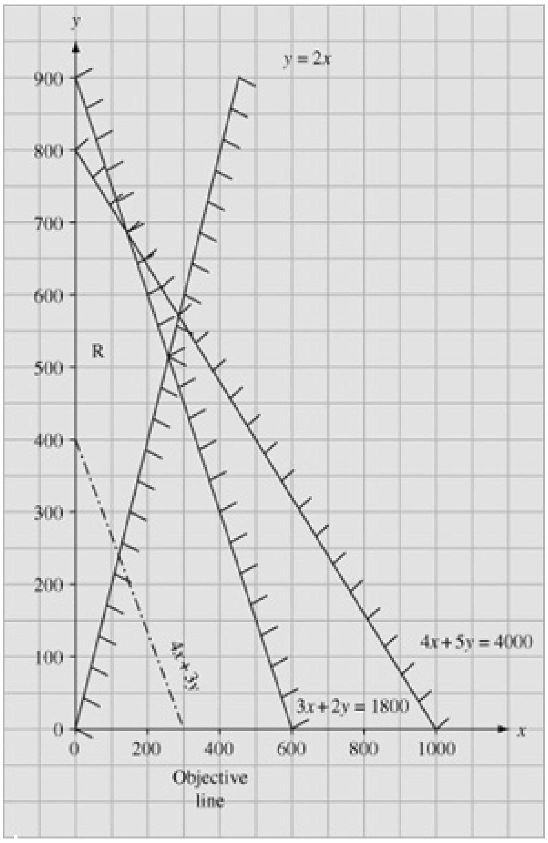
It will take 30 minutes to make all the chocolates for one box of gold assortment and 20 minutes to make the chocolates for one box of silver assortment.

It will take 12 minutes to wrap and pack the chocolates in one box of gold assortment and 15 minutes for one box of silver assortment.

The manufacturer needs to make at least twice as many silver as gold assortments. The gold assortment will be sold at a profit of 80p, and the silver at a profit of 60p.

There are 300 hours available to make the chocolates and 200 hours to wrap them. Maximise the profit, P.

Solution:



Intersection of $4x+5y=4000$ and $3x+2y=1800$

giving $\left(142\frac{6}{7}, 685\frac{5}{7}\right)$

Testing nearby integer points

Point	$4x+5y \leq 4000$	$3x+2y \leq 1800$	$80x+60y$
(142, 685)	√	√	52460
(142, 686)	√	√	52520
(143, 685)	√	√	52540
(143, 686)	X	X	

so maximum integer solution is 52540 pennies at (143, 685).

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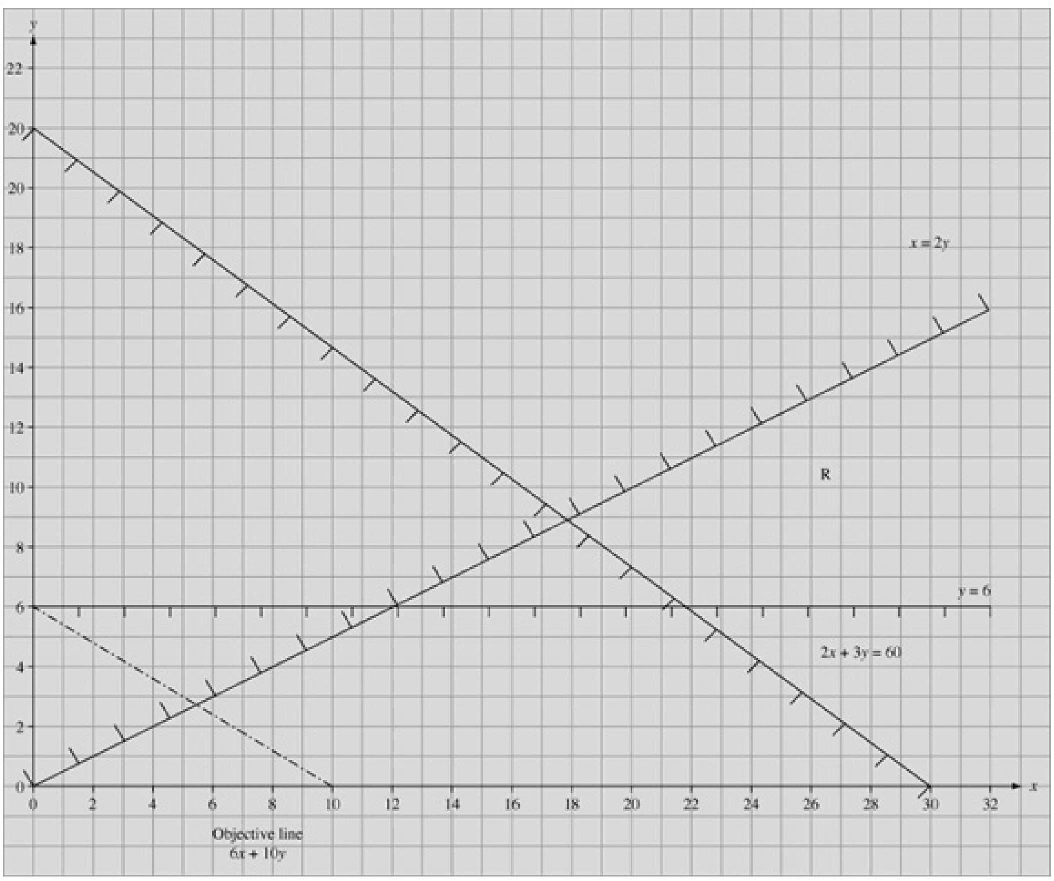
Exercise D, Question 6

Question:

Solve this problem.

A floral display is required for the opening of a new building. The display must be at least 30 m long and is to be made up of two types of planted displays, type A and type B.
Type A is 1 m in length and costs £6
Type B is 1.5 m in length and costs £10
The client wants at least twice as many type A as type B, and at least 6 of type B.
The cost is to be minimised.

Solution:



Intersection of $2x+3y=60$ and $y=6$
(21, 6)
cost $= 6x+10y$ so minimum cost $= 6\times 21+60=$ £186

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Exercise D, Question 7

Question:

Solve this problem.

A toy company makes two types of board game, Cludopoly and Trivscrab. As well as the board each game requires playing pieces and cards.

The company uses two machines, one to produce the pieces and one to produce the cards.

Both machines can only be operated for up to ten hours per day.
The first machine takes 5 minutes to produce a set of pieces for Cludopoly and 8 minutes to produce a set of pieces for Trivscrab.

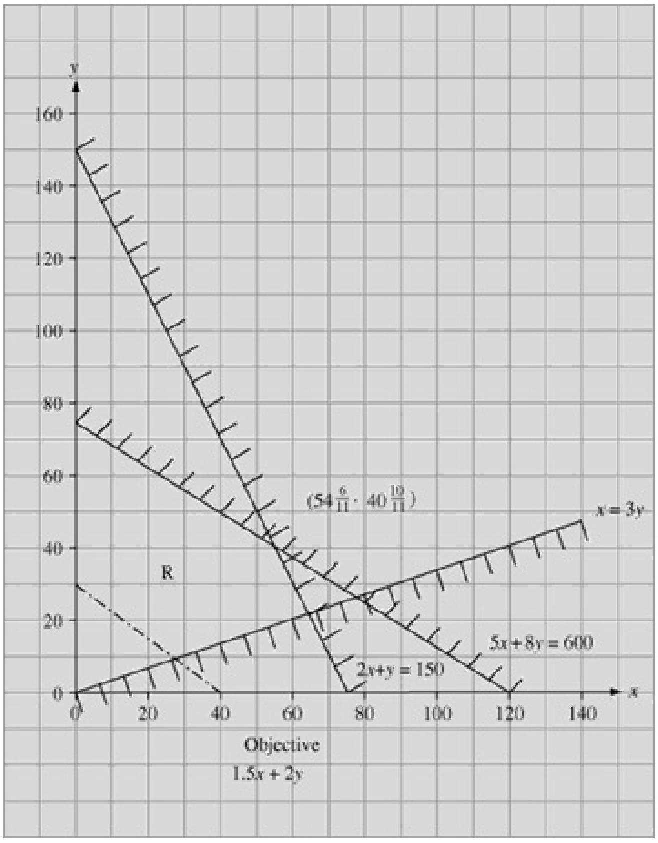
The second machine takes 8 minutes to produce a set of cards for Cludopoly and 4 minutes to produce a set of cards for Trivscrab.

The company knows it will sell at most three times as many games of Cludopoly as Trivscrab.

The profit made on each game of Cludopoly is £1.50 and £2.50 on each game of Trivscrab.

The company wishes to maximise its daily profit.

Solution:



Intersection of $5x+8y=600$

$2x+y=150$ giving $\left(54\frac{6}{11}, 40\frac{10}{11}\right)$

Points	$5x+8y \le 600$	$2x+y \le 150$	$1.5x+2y$
(54, 40)	√	√	161
(54, 41)	√	√	163
(55, 40)	√	√	162.5
(55, 41)	X	X	

so maximum value is 163 at (54, 41).

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Exercise D, Question 8

Question:

Solve this problem.

A librarian needs to purchase bookcases for a new library. She has a budget of £3000 and $240\,\text{m}^2$ of available floor space. There are two types of bookcase, type 1 and type 2, that she is permitted to buy.

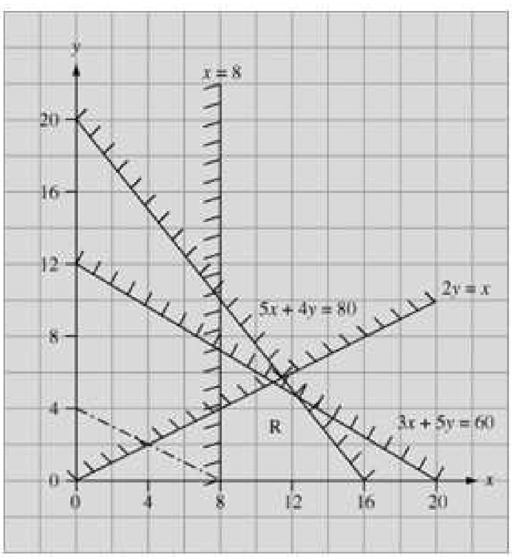
Type 1 costs £150, needs $15\,\text{m}^2$ of floor space and has 40 m of shelving.

Type 2 costs £250, needs $12\,\text{m}^2$ of floor space and has 50 m of shelving.

She must buy at least 8 type 1 bookcases and wants at most $\frac{1}{3}$ of all the bookcases to be type 2.

She wishes to maximise the total amount of shelving.

Solution:



Intersection of $3x+5y=60$

$5x+4y=80$ giving $\left(12\frac{4}{13}, 4\frac{8}{13}\right)$

Points	$3x+5y \leq 60$	$5x+4y \leq 80$	$40x+60y$
(12, 4)	✓	✓	720
(12, 5)	X	✓	–
(13, 4)	✓	X	–
(13, 5)	X	X	–

Maximum value is 720 at (12, 4).

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Exercise E, Question 1

Question:

Mr Baker is making cakes and fruit loaves for sale at a charity cake stall. Each cake requires 200 g of flour and 125 g of fruit. Each fruit loaf requires 200 g of flour and 50 g of fruit. He has 2800 g of flour and 1000 g of fruit available.

Let the number of cakes that he makes be x and the number of fruit loaves he makes be y.

a Show that these constraints can be modelled by the inequalities $x + y \leq 14$ and $5x + 2y \leq 40$.

Each cake takes 50 minutes to cook and each fruit loaf takes 30 minutes to cook. There are 8 hours of cooking time available.

b Obtain a further inequality, other than $x \geq 0$ and $y \geq 0$, which models this time constraint.

c On graph paper illustrate these three inequalities, indicating clearly the feasible region.

d It is decided to sell the cakes for £3.50 each and the fruit loaves for £1.50 each. Assuming that Mr Baker sells all that he makes, write down an expression for the amount of money P, in pounds, raised by the sale of Mr Baker's products.

e Explaining your method clearly, determine how many cakes and how many fruit loaves Mr Baker should make in order to maximise P. ***E***

Solution:

a *flour*: $200x + 200y \leq 2800$

so $x + y \leq 14$

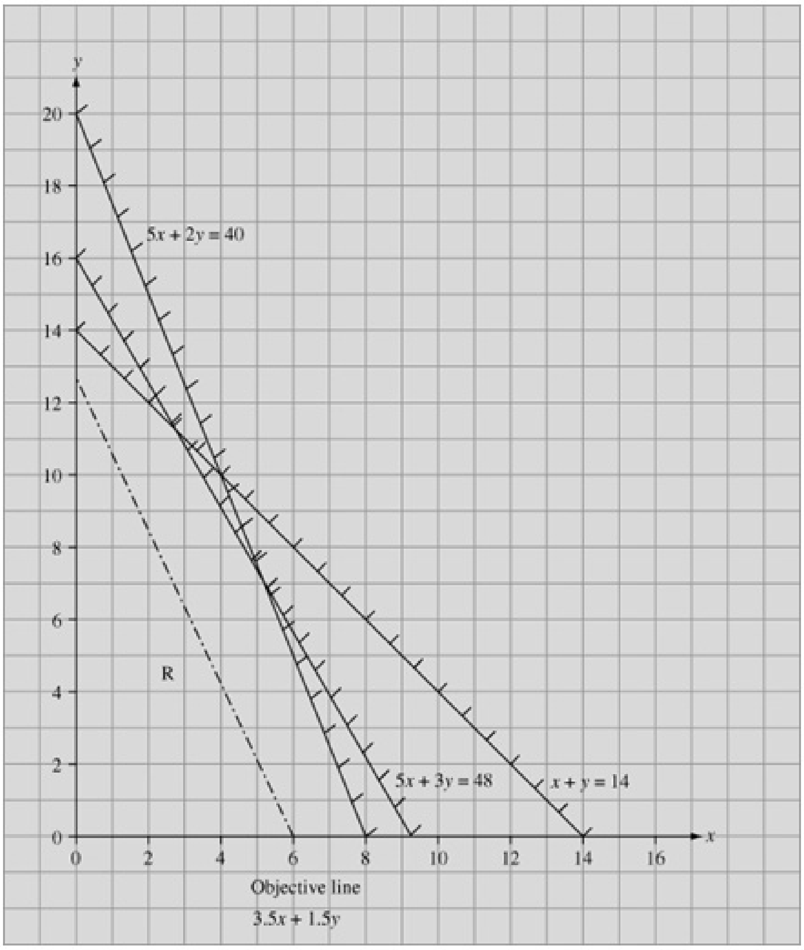
fruit: $125x + 50y \leq 1000$

so $5x + 2y \leq 40$

b Cooking time $50x + 30y \leq 480$

so $5x + 3y \leq 48$

c



d $P = 3.5x + 1.5y$

e Integer solution required (6, 5)

$P_{max} = £28.50$

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Exercise E, Question 2

Question:

A junior librarian is setting up a music recording lending section to loan CDs and cassette tapes. He has a budget of £420 to spend on storage units to display these items.
Let x be the number of CD storage units and y the number of cassette storage units he plans to buy.
Each type of storage unit occupies $0.08\,\mathrm{m}^3$, and there is a total area of $6.4\,\mathrm{m}^3$ available for the display.

a Show that this information can be modelled by the inequality
$x + y \leq 80$

The CD storage units cost £6 each and the cassette storage units cost £4.80 each.

b Write down a second inequality, other than $x \geq 0$ and $y \geq 0$, to model this constraint.

The CD storage unit displays 30 CDs and the cassette storage unit displays 20 cassettes.

The chief librarian advises the junior librarian that he should plan to display at least half as many cassettes as CDs.

c Show that this implies that $3x \leq 4y$.

d On graph paper, display your three inequalities, indicating clearly the feasible region.

The librarian wishes to maximise the total number of items, T, on display. Given that
$T = 30x + 20y$

e determine how many CD storage units and how many cassette storage units he should buy, briefly explaining your method. ***E***

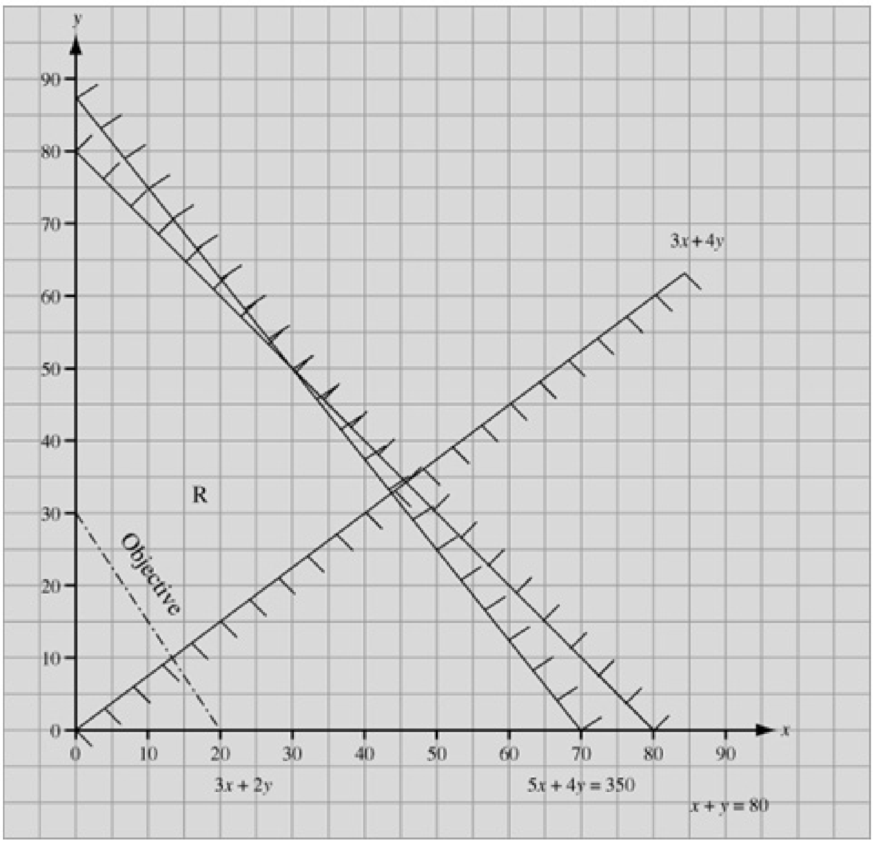
Solution:

a storage : $0.08x + 0.08y \leq 6.4$
so $x + y \leq 80$

b cost : $6x + 4.8y \leq 420$ so $5x + 4y \leq 350$

c Display $30x \leq 2 \times 20y$
$3x \leq 4y$

d



e Integer solution required (43, 33).
He should buy 43 CD storage units and 33 cassette storage units.

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Exercise E, Question 3

Question:

The headteacher of a school needs to hire coaches to transport all the year 7, 8 and 9 pupils to take part in the recording of a children's television programme. There are 408 pupils to be taken and 24 adults will accompany them on the coaches. The headteacher can hire either 54 seater (large) or 24 seater (small) coaches. She needs at least two adults per coach. The bus company has only seven large coaches but an ample supply of small coaches.

Let x and y be the number of large and small coaches hired respectively.

a Show that the situation can be modelled by the three inequalities:

i $9x+4y \geq 72$

ii $x+y \leq 12$

iii $x \leq 7$.

b On graph paper display the three inequalities, indicating clearly the feasible region.

A large coach costs £336 and a small coach costs £252 to hire.

c Write down an expression, in terms of x and y, for the total cost of hiring the coaches.

d Explain how you would locate the best option for the headteacher, given that she wishes to minimise the total cost.

e Find the number of large and small coaches that the headteacher should hire in order to minimise the total cost and calculate this minimum total cost.

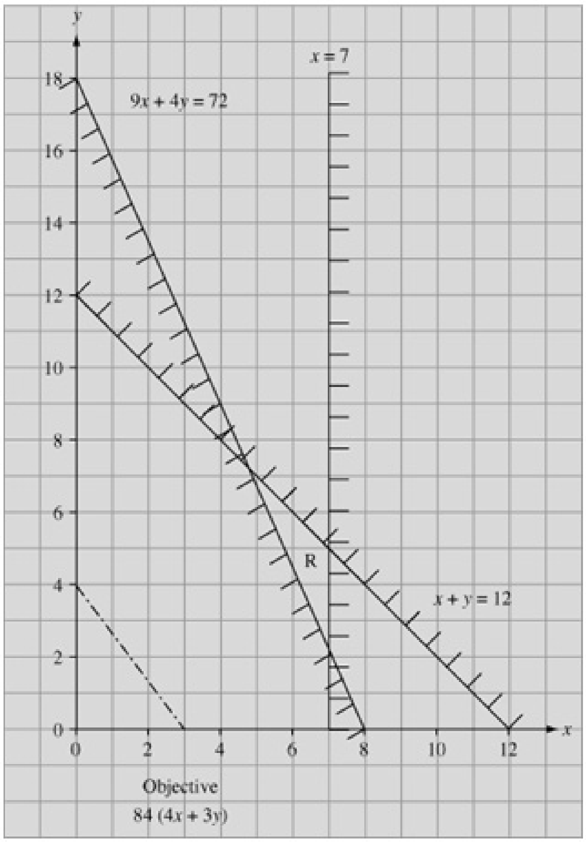
Solution:

a **i** Total number of people $54x+24y \geq 432$ so $9x+4y \geq 72$

ii number of adults $x+y \leq 12$

iii number of large coaches $x \leq 7$

b



c minimise $C = 336x+252y$

$= 84(4x+3y)$

d Objective line passes through (0, 4) (3, 0)

e Integer coordinates needed (7, 3) so hire 7 large coaches and 3 small coaches

cost = £ 3108

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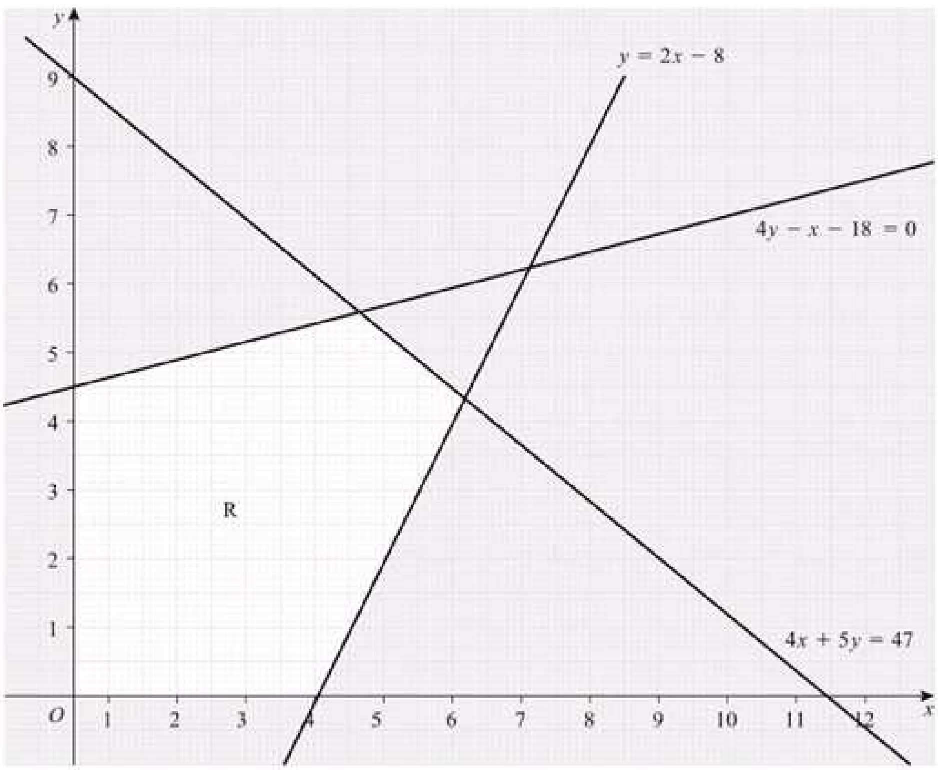
Exercise E, Question 4

Question:

The graph below was drawn to solve a linear programming problem. The feasible region, R, includes the points on its boundary.



a Write down the inequalities that define the region R.
The objective function, P, is given by $P = 3x + 2y$.

b Find the value of x and the value of y that lead to the maximum value of P. Make your method clear.

c **i** Give an example of a practical linear programming problem in which it would be necessary for the variables to have integer values.

ii Given that the solution must have integer values of x and y, find the values that lead to the maximum value of P.

Solution:

a $4x + 5y \le 47$
$y \ge 2x - 8$
$4y - x - 8 \le 0$
$x, y \ge 0$

b Solving simultaneous equations $y = 2x - 8$
$4x + 5y = 47$
$\left(6\frac{3}{14}, 4\frac{3}{7}\right)$

c **i** For example where x and y
- types of car to be hired
- number of people, etc.

ii (6, 4)

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Exercise E, Question 5

Question:

A company produces plates and mugs for local souvenir shops. The plates and mugs are manufactured in a two-stage process. Each day there are 300 minutes available for the completion of the first stage and 400 minutes available for the completion of the second stage. In addition the mugs require some hand painting. There are 150 minutes available each day for hand painting.

Product	Stage 1	Stage 2	Hand painting
Plate	$2\frac{1}{2}$	5	—
Mug	3	2	2

The above table shows the production time, in minutes, required for the plates and the mugs.

All plates and mugs made are sold. The profit on each plate sold is £2 and the profit on each mug sold is £4. The company wishes to determine how many plates and mugs to make so as to maximise its profits each day.

Let x be the number of plates made and y the number of mugs made each day.

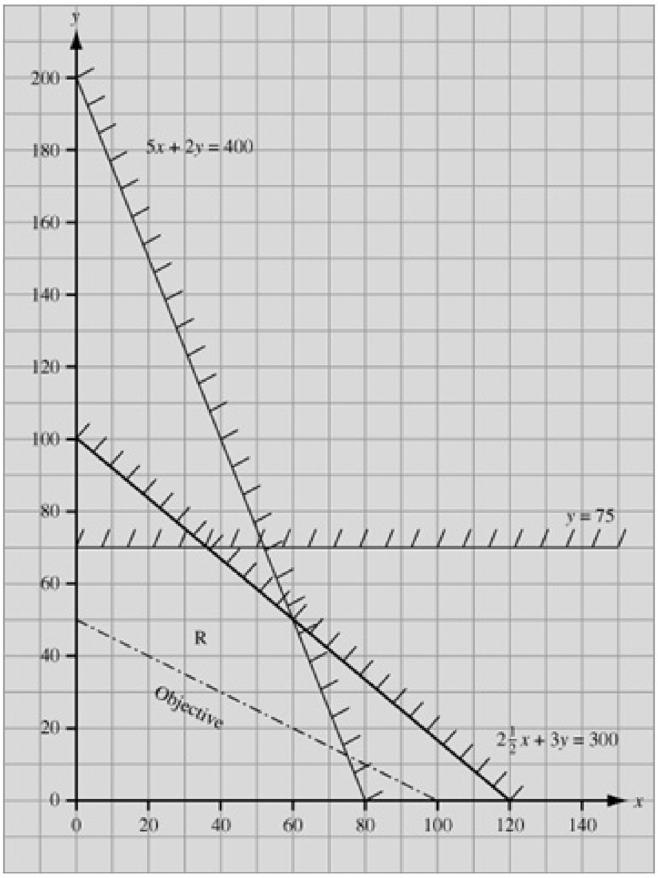
a Write down the three constraints, other than $x \geq 0, y \geq 0$, satisfied by x and y.

b Write down the objective function to be maximised.

c Using the graphical method, solve the resulting linear programming problem. Determine the optimal number of plates and mugs to be made each day and the resulting profit.

d When the optimal solution is adopted determine which, if any, of the stages has available time which is unused. State the amount of unused time. ***E***

Solution:



a $2\frac{1}{2}x+3y \le 300$ $[5x+6y \le 600]$

$5x+2y \le 400$

$2y \le 150$ $[y \le 75]$

b Maximise $P = 2x+4y$

c (30, 75) $P = 360$

d The optimal point is at the intersection of $y = 75$ and $2\frac{1}{2}x+3y = 300$.

So the constraint $5x+2y \le 400$ is not at its limit.

At (30, 75) $5x+2y = 300$ so 100 minutes are unused.

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Exercise E, Question 6

Question:

Two fertilizers are available, a liquid X and a powder Y. A bottle of X contains 5 units of chemical A, 2 units of chemical B and $\frac{1}{2}$ unit of chemical C. A packet of Y contains 1 unit of A, 2 units of B and 2 units of C. A professional gardener makes her own fertilizer. She requires at least 10 units of A, at least 12 units of B and at least 6 units of C. She buys x bottles of X and y packets of Y.

a Write down the inequalities which model this situation.

b Construct and label the feasible region.

A bottle of X costs £2 and a packet of Y costs £3.

c Write down an expression, in terms of x and y, for the total cost £T.

d Using your graph, obtain the values of x and y that give the minimum value of T. Make your method clear and calculate the minimum value of T.

e Suggest how the situation might be changed so that it could no longer be represented graphically.

Solution:

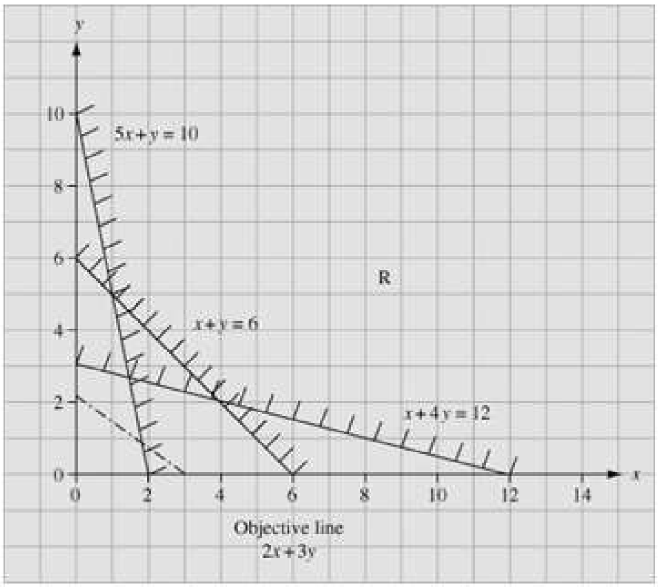
a Chemical A $5x+y\geq 10$

Chemical B $2x+2y\geq 12 \quad [x+y\geq 6]$

Chemical C $\frac{1}{2}x+2y\geq 6 \quad [x+4y\geq 12]$

$x, y\geq 6$

b



c $T=2x+3y$

d (4, 2) $T=14$

e If there were 3 or more variables the problem could not be solved graphically.
So adding a third fertiliser z, would mean a graphical method could not be used.