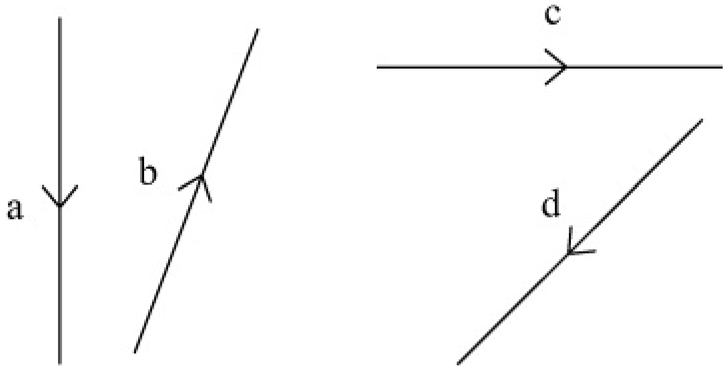
Solutionbank 4
Edexcel AS and A Level Modular Mathematics

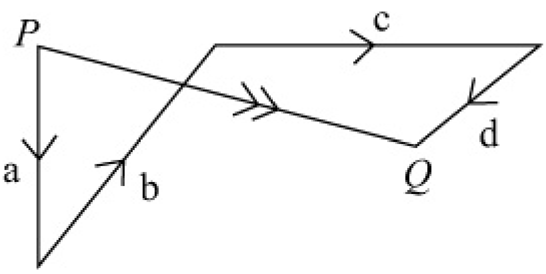
Vectors
Exercise A, Question 1

Question:

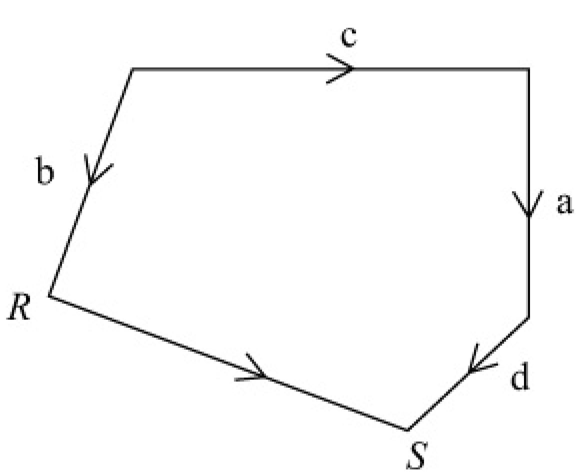
The diagram shows the vectors **a**, **b**, **c** and **d**. Draw a diagram to illustrate the vector addition a + b + c + d.



Solution:



a + b + c + d = PQ
(Vector goes from the start of **a** to the finish of **d**).
The vectors could be added in a different order,
e.g. b + c + a + d:



Here b + c + a + d = RS

(RS = PQ)

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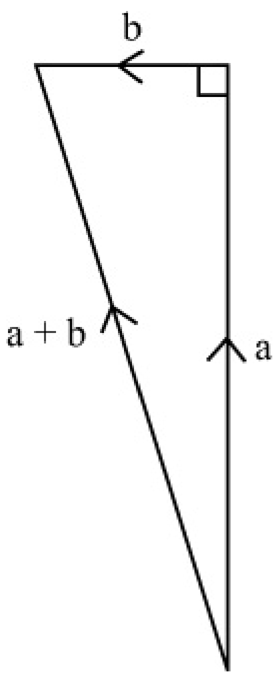
Vectors
Exercise A, Question 2

Question:

The vector **a** is directed due north and $|\mathbf{a}| = 24$. The vector **b** is directed due west and $|\mathbf{b}| = 7$. Find $|\mathbf{a} + \mathbf{b}|$.

Solution:



$|\mathbf{a}| = 24$
$|\mathbf{b}| = 7$
$|\mathbf{a} + \mathbf{b}|^2 = 24^2 + 7^2 = 625$
$\therefore |\mathbf{a} + \mathbf{b}| = 25$

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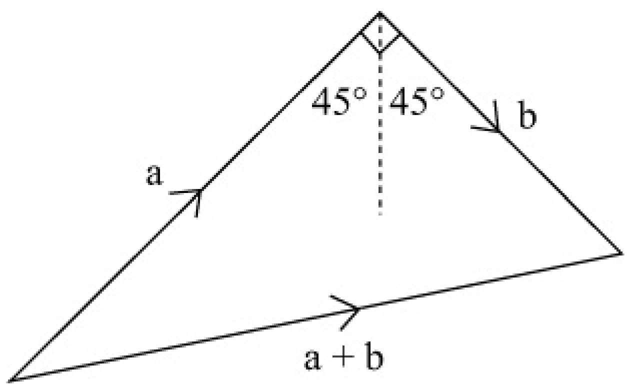
Vectors
Exercise A, Question 3

Question:

The vector **a** is directed north-east and $|\mathbf{a}| = 20$. The vector **b** is directed south-east and $|\mathbf{b}| = 13$. Find $|\mathbf{a} + \mathbf{b}|$.

Solution:



$|\mathbf{a}| = 20$
$|\mathbf{b}| = 13$
$|\mathbf{a} + \mathbf{b}|^2 = 20^2 + 13^2 = 569$
$|\mathbf{a} + \mathbf{b}| = \sqrt{569} = 23.9$ (3 s.f.)

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Vectors
Exercise A, Question 4

Question:

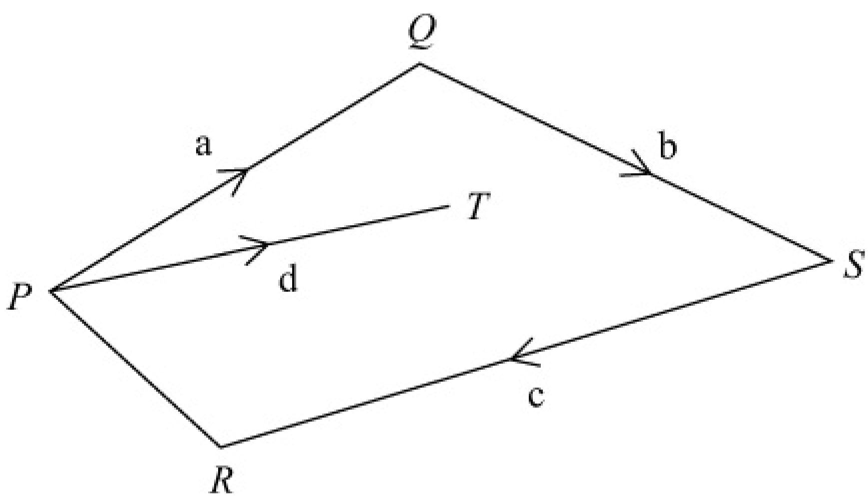
In the diagram, PQ = a, QS = b, SR = c and PT = d. Find in terms of **a**, **b**, **c** and **d**:

(a) QT

(b) PR

(c) TS

(d) TR



Solution:

(a) QT = QP + PT = − a + d

(b) PR = PQ + QS + SR = a + b + c

(c) TS = TP + PQ + QS = − d + a + b = a + b − d

(d) TR = TP + PR = − d + (a + b + c) = a + b + c − d

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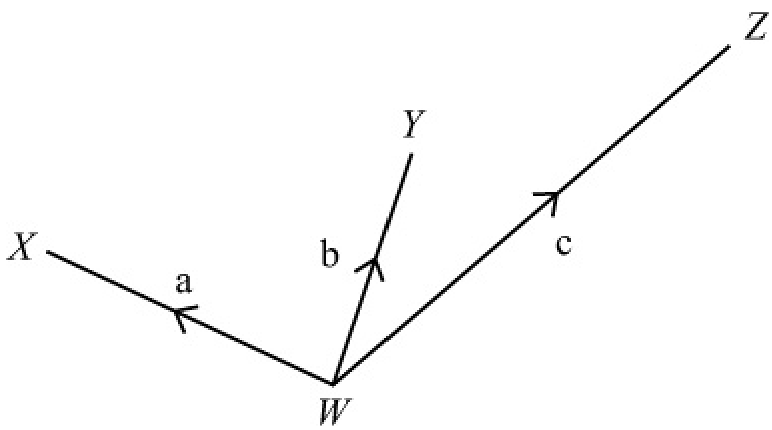
Vectors
Exercise A, Question 5

Question:

In the diagram, WX = a, WY = b and WZ = c. It is given that XY = YZ. Prove that a + c = 2b.
(2**b** is equivalent to b + b).



Solution:

XY = XW + WY = − a + b
YZ = YW + WZ = − b + c
Since XY = YZ,
− a + b = − b + c
b + b = a + c
a + c = 2b

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Vectors
Exercise B, Question 1

Question:

In the triangle PQR, PQ = 2a and QR = 2b. The mid-point of *PR* is *M*.

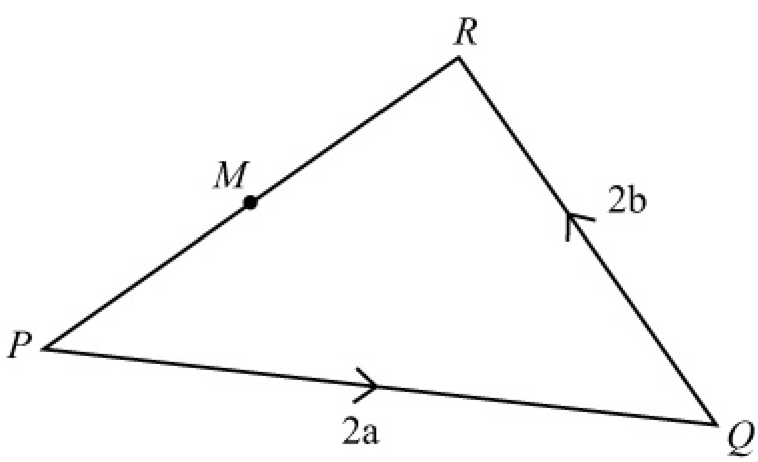
Find, in terms of **a** and **b**:

(a) PR

(b) PM

(c) QM.

Solution:



(a) PR = PQ + QR = 2a + 2b

(b) PM = $\frac{1}{2}$PR = $\frac{1}{2}\left(2a + 2b\right)$ = a + b

(c) QM = QP + PM = − 2a + a + b = − a + b

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Vectors
Exercise B, Question 2

Question:

ABCD is a trapezium with *AB* parallel to *DC* and DC = 3AB. *M* is the mid-point of *DC*, AB = a and BC = b.
Find, in terms of **a** and **b**:

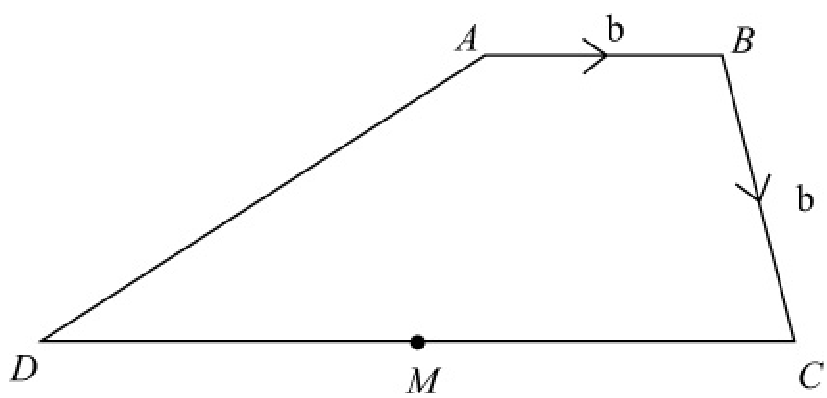
(a) AM

(b) BD

(c) MB

(d) DA.

Solution:



Since DC = 3AB, DC = 3a

Since *M* is the mid-point of *DC*, DM = MC = $\frac{3}{2}$a

(a) AM = AB + BC + CM = a + b − $\frac{3}{2}$a = − $\frac{1}{2}$a + b

(b) BD = BC + CD = b − 3a

(c) MB = MC + CB = $\frac{3}{2}$a − b

(d) DA = DC + CB + BA = 3a − b − a = 2a − b

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Vectors
Exercise B, Question 3

Question:

In each part, find whether the given vector is parallel to a − 3b:

(a) 2a − 6b

(b) 4a − 12b

(c) a + 3b

(d) 3b − a

(e) 9b − 3a

(f) $\frac{1}{2}a - \frac{2}{3}b$

Solution:

(a) 2a − 6b = 2 (a − 3b)
Yes, parallel to a − 3b.

(b) 4a − 12b = 4 (a − 3b)
Yes, parallel to a − 3b.

(c) a + 3b is not parallel to a − 3b

(d) 3b − a = − 1 (a − 3b)
Yes, parallel to a − 3b.

(e) 9b − 3a = − 3 (a − 3b)
Yes, parallel to a − 3b.

(f) $\frac{1}{2}a - \frac{2}{3}b = \frac{1}{2}\left(a - \frac{4}{3}b \right)$
No, not parallel to a − 3b.

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Vectors
Exercise B, Question 4

Question:

The non-zero vectors **a** and **b** are not parallel. In each part, find the value of λ and the value of μ:

(a) $a + 3b = 2\lambda a - \mu b$

(b) $(\lambda + 2)a + (\mu - 1)b = 0$

(c) $4\lambda a - 5b - a + \mu b = 0$

(d) $(1 + \lambda)a + 2\lambda b = \mu a + 4\mu b$

(e) $(3\lambda + 5)a + b = 2\mu a + (\lambda - 3)b$

Solution:

(a) $a + 3b = 2\lambda a - \mu b$
$1 = 2\lambda$ and $3 = -\mu$
$\lambda = \frac{1}{2}$ and $\mu = -3$

(b) $(\lambda + 2)a + (\mu - 1)b = 0$
$\lambda + 2 = 0$ and $\mu - 1 = 0$
$\lambda = -2$ and $\mu = 1$

(c) $4\lambda a - 5b - a + \mu b = 0$
$4\lambda - 1 = 0$ and $-5 + \mu = 0$
$\lambda = \frac{1}{4}$ and $\mu = 5$

(d) $(1 + \lambda)a + 2\lambda b = \mu a + 4\mu b$
$1 + \lambda = \mu$ and $2\lambda = 4\mu$
Since $2\lambda = 4\mu$, $\lambda = 2\mu$
$1 + 2\mu = \mu$
$\mu = -1$ and $\lambda = -2$

(e) $(3\lambda + 5)a + b = 2\mu a + (\lambda - 3)b$
$3\lambda + 5 = 2\mu$ and $1 = \lambda - 3$
$\lambda = 4$ and $2\mu = 12 + 5$

$\lambda = 4$ and $\mu = 8\frac{1}{2}$

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Vectors
Exercise B, Question 5

Question:

In the diagram, OA = a, OB = b and *C* divides *AB* in the ratio 5:1.

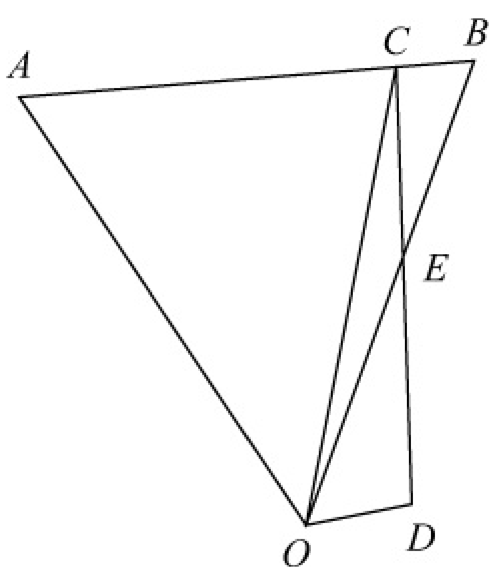
(a) Write down, in terms of **a** and **b**, expressions for AB, AC and OC.
Given that OE = λ b, where λ is a scalar:

(b) Write down, in terms of **a**, **b** and λ, an expression for CE.
Given that OD = μ (b − a) , where μ is a scalar:

(c) Write down, in terms of **a**, **b**, λ and μ, an expression for ED.
Given also that *E* is the mid-point of *CD*:

(d) Deduce the values of λ and μ.

E



Solution:

(a) AB = AO + OB = − a + b

$$AC = \frac{5}{6}AB = \frac{5}{6}\left(-a+b\right)$$

$$OC = OA + AC = a + \frac{5}{6}\left(-a+b\right) = \frac{1}{6}a + \frac{5}{6}b$$

(b) OE = λ b:

$$CE = CO + OE = -\left(\frac{1}{6}a + \frac{5}{6}b\right) + \lambda b = -\frac{1}{6}a + \left(\lambda - \frac{5}{6}\right)b$$

(c) $OD = \mu\,(b - a)$:

$$ED = EO + OD = -\lambda b + \mu\,(b - a) = -\mu a + (\mu - \lambda)\,b$$

(d) If *E* is the mid-point of *CD*, CE = ED:

$$-\frac{1}{6}a + \left(\lambda - \frac{5}{6}\right)b = -\mu a + \left(\mu - \lambda\right)b$$

Since **a** and **b** are not parallel

$$-\frac{1}{6} = -\mu \quad \Rightarrow \quad \mu = \frac{1}{6}$$

and

$$\left(\lambda - \frac{5}{6}\right) = \left(\mu - \lambda\right)$$

$$\Rightarrow \quad 2\lambda = \mu + \frac{5}{6}$$

$$\Rightarrow \quad 2\lambda = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2}$$

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Vectors
Exercise B, Question 6

Question:

In the diagram OA = a, OB = b, 3OC = 2OA and 4OD = 7OB.
The line *DC* meets the line *AB* at *E*.

(a) Write down, in terms of **a** and **b**, expressions for
 (i) AB
 (ii) DC
Given that DE = λ DC and EB = μ AB where λ and μ are constants:

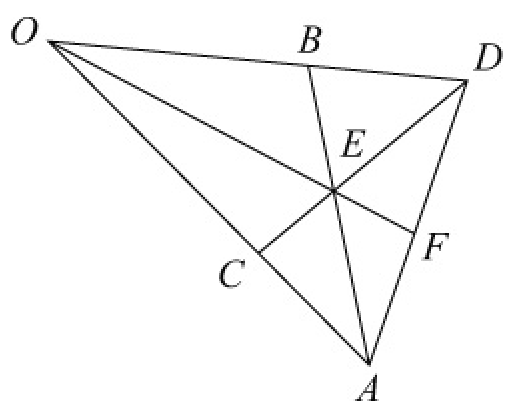
(b) Use $\triangle$ EBD to form an equation relating to **a**, **b**, λ and μ.
Hence:

(c) Show that $\lambda = \frac{9}{13}$.

(d) Find the exact value of μ.

(e) Express OE in terms of **a** and **b**.
The line OE produced meets the line *AD* at *F*.
Given that OF = *k*OE where *k* is a constant and that AF = $\frac{1}{10}\left(7b - 4a \right)$:

(f) Find the value of *k*. E



Solution:

(a) OC = $\frac{2}{3}$OA = $\frac{2}{3}$a, OD = $\frac{7}{4}$OB = $\frac{7}{4}$b
 (i) AB = AO + OB = − a + b

(ii) DC = DO + OC = $\frac{2}{3}$a − $\frac{7}{4}$b

(b) DE = λ DC and EB = μ AB.

From $\triangle$ EBD, DE = DB + BE

Since OD = $\frac{7}{4}$b, BD = OD − OB = $\frac{7}{4}$b − b = $\frac{3}{4}$b

$\therefore$ DB = $-\frac{3}{4}$b

So λ DC = DB − μ AB

$$\lambda\left(\frac{2}{3}a-\frac{7}{4}b\right)=-\frac{3}{4}b-\mu\left(-a+b\right)$$

$$\left(\frac{2}{3}\lambda-\mu\right)a+\left(\frac{3}{4}+\mu-\frac{7}{4}\lambda\right)b=0$$

(c) So $\frac{2}{3}\lambda-\mu=0 \Rightarrow \mu=\frac{2}{3}\lambda$

and $\frac{3}{4}+\mu-\frac{7}{4}\lambda=0$

$$\Rightarrow \frac{3}{4}+\frac{2}{3}\lambda-\frac{7}{4}\lambda=0$$

$$\Rightarrow \frac{13}{12}\lambda=\frac{3}{4}$$

$$\Rightarrow \lambda=\frac{3}{4}\times\frac{12}{13}=\frac{9}{13}$$

(d) $\mu=\frac{2}{3}\lambda=\frac{2}{3}\times\frac{9}{13}=\frac{6}{13}$

(e) OE = OB + BE = OB − μ AB = b − $\frac{6}{13}\left(-a+b\right)=\frac{6}{13}a+\frac{7}{13}b$

(f) OF = kOE and AF = $\frac{7}{10}$b − $\frac{4}{10}$a.

OF = $\frac{6k}{13}a+\frac{7k}{13}b$

From $\triangle$ OFA, OF = OA + AF

$$\frac{6k}{13}a+\frac{7k}{13}b=a+\left(\frac{7}{10}b-\frac{4}{10}a\right)=\frac{6}{10}a+\frac{7}{10}b$$

So $\frac{6k}{13}=\frac{6}{10}$ (and $\frac{7k}{13}=\frac{7}{10}$)

$$\Rightarrow \quad k = \frac{13}{10}$$

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise B, Question 7

Question:

In △ OAB, *P* is the mid-point of *AB* and *Q* is the point on *OP* such that OQ = $\frac{3}{4}$OP. Given that OA = a and OB = b, find, in terms of **a** and **b**:

(a) AB

(b) OP

(c) OQ

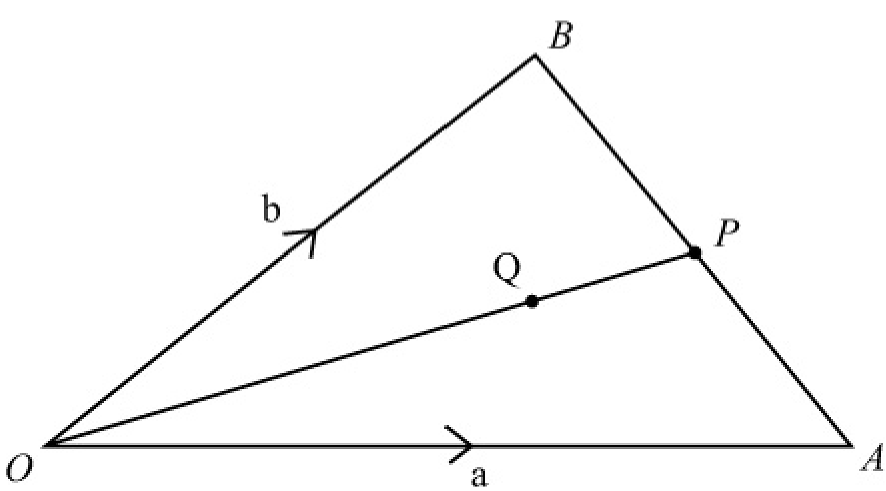
(d) AQ
The point *R* on *OB* is such that OR = kOB, where 0 < *k* < 1.

(e) Find, in terms of **a**, **b** and *k*, the vector AR.
Given that *AQR* is a straight line:

(f) Find the ratio in which *Q* divides *AR* and the value of *k*.

E

Solution:



BP = PA and OQ = $\frac{3}{4}$OP

(a) AB = AO + OB = − a + b

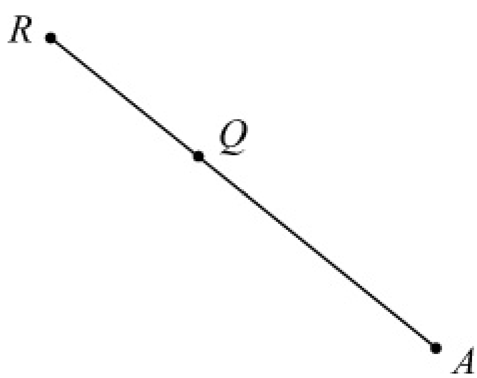
(b) $OP = OA + AP = OA + \frac{1}{2}AB = a + \frac{1}{2}\left(-a + b \right) = \frac{1}{2}a + \frac{1}{2}b$

(c) $OQ = \frac{3}{4}OP = \frac{3}{4}\left(\frac{1}{2}a + \frac{1}{2}b \right) = \frac{3}{8}a + \frac{3}{8}b$

(d) $AQ = AO + OQ = -a + \left(\frac{3}{8}a + \frac{3}{8}b \right) = -\frac{5}{8}a + \frac{3}{8}b$

(e) Given $OR = kOB$ $(0 < k < 1)$
In $\triangle$ OAR, $AR = AO + OR = -a + kb$

(f) Since *AQR* is a straight line, AR and AQ are parallel vectors.



Suppose $AQ = \lambda AR$

$-\frac{5}{8}a + \frac{3}{8}b = \lambda \left(-a + kb \right)$

So $-\frac{5}{8} = -\lambda \Rightarrow \lambda = \frac{5}{8}$

and $\frac{3}{8} = \lambda k$

$\Rightarrow k = \frac{3}{8\lambda}$

$\Rightarrow k = \frac{3}{5}$

Since $AQ = \frac{5}{8}AR$, $RQ = \frac{3}{8}AR$

So *Q* divides *AR* in the ratio 5:3.

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise B, Question 8

Question:

In the figure OE : EA = 1 : 2, AF : FB = 3 : 1 and OG : OB = 3 : 1. The vector OA = a and the vector OB = b.
Find, in terms of **a**, **b** or **a** and **b**, expressions for:

(a) OE

(b) OF

(c) EF

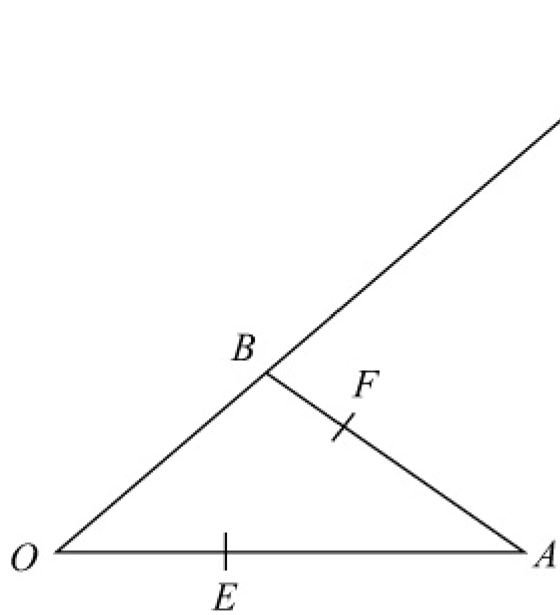
(d) BG

(e) FB

(f) FG

(g) Use your results in (c) and (f) to show that the points *E*, *F* and *G* are collinear and find the ratio EF : FG.

(h) Find EB and AG and hence prove that *EB* is parallel to *AG*.



Solution:

(a) $OE = \frac{1}{3}OA = \frac{1}{3}a$

(b) $OF = OA + AF = OA + \frac{3}{4}AB$

$= a + \frac{3}{4}\left(b - a \right)$

$= a + \frac{3}{4}b - \frac{3}{4}a$

$= \frac{1}{4}a + \frac{3}{4}b$

(c) $EF = EA + AF = \frac{2}{3}OA + \frac{3}{4}AB$

$= \frac{2}{3}a + \frac{3}{4}\left(b - a \right)$

$= \frac{2}{3}a + \frac{3}{4}b - \frac{3}{4}a$

$= -\frac{1}{12}a + \frac{3}{4}b$

(d) $BG = 2OB = 2b$

(e) $FB = \frac{1}{4}AB = \frac{1}{4}\left(b - a \right) = -\frac{1}{4}a + \frac{1}{4}b$

(f) $FG = FB + BG = -\frac{1}{4}a + \frac{1}{4}b + 2b = -\frac{1}{4}a + \frac{9}{4}b$

(g) $FG = -\frac{1}{4}a + \frac{9}{4}b = 3\left(-\frac{1}{12}a + \frac{3}{4}b \right) = 3EF$

So EF and FG are parallel vectors.
So *E*, *F* and *G* are collinear.
EF : FG = 1 : 3

(h) $EB = EO + OB = -\frac{1}{3}a + b$

$AG = AO + OG = -a + 3b = 3\left(-\frac{1}{3}a + b \right) = 3EB$

So *EB* is parallel to *AG*.

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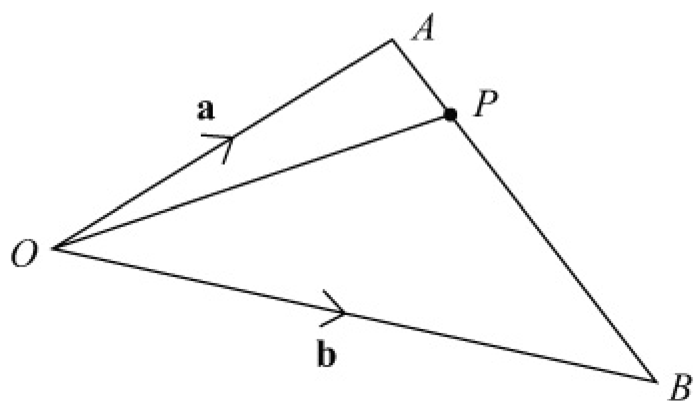
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise C, Question 1

Question:

The points *A* and *B* have position vectors **a** and **b** respectively (referred to the origin *O*).
The point *P* divides *AB* in the ratio 1:5.
Find, in terms of **a** and **b**, the position vector of *P*.

Solution:



AP : PB = 1 : 5

So $AP = \frac{1}{6}AB = \frac{1}{6}\left(b - a \right)$

$OP = OA + AP = a + \frac{1}{6}\left(b - a \right)$

$= a + \frac{1}{6}b - \frac{1}{6}a$

$= \frac{5}{6}a + \frac{1}{6}b$

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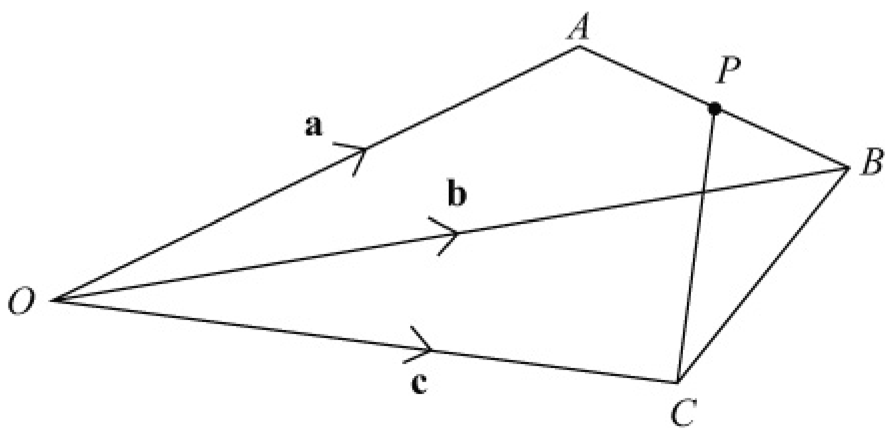
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise C, Question 2

Question:

The points *A*, *B* and *C* have position vectors **a**, **b** and **c** respectively (referred to the origin *O*). The point *P* is the mid-point of *AB*.
Find, in terms of **a**, **b** and **c**, the vector PC.

Solution:



PC = PO + OC = − OP + OC

But OP = OA + AP = OA + $\frac{1}{2}$AB = a + $\frac{1}{2}\left(\text{b} - \text{a}\right) = \frac{1}{2}$a + $\frac{1}{2}$b

So PC = $-\left(\frac{1}{2}\text{a} + \frac{1}{2}\text{b}\right) + \text{c} = -\frac{1}{2}\text{a} - \frac{1}{2}\text{b} + \text{c}$

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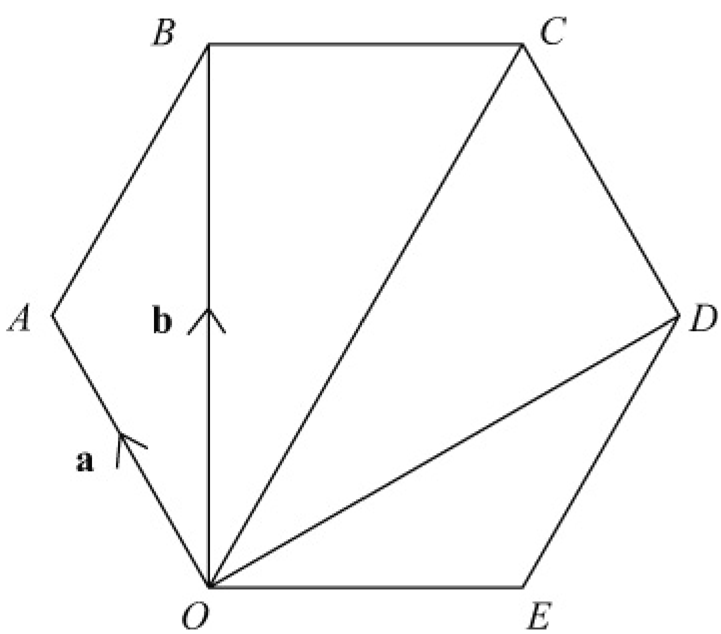
Vectors
Exercise C, Question 3

Question:

OABCDE is a regular hexagon. The points *A* and *B* have position vectors **a** and **b** respectively, referred to the origin *O*.
Find, in terms of **a** and **b**, the position vectors of *C*, *D* and *E*.

Solution:



OC = 2AB = 2 (b − a) = − 2a + 2b
OD = OC + CD = OC + AO = (− 2a + 2b) − a = − 3a + 2b
OE = OD + DE = OD + BA = (− 3a + 2b) + (a − b) = − 2a + b

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Vectors
Exercise D, Question 1

Question:

Given that a = 9i + 7j, b = 11i − 3j and c = − 8i − j, find:

(a) a + b + c

(b) 2a − b + c

(c) 2b + 2c − 3a
(Use column matrix notation in your working.)

Solution:

(a) $a + b + c = \begin{pmatrix} 9 \\ 7 \end{pmatrix} + \begin{pmatrix} 11 \\ -3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$

(b) $2a - b + c = 2\begin{pmatrix} 9 \\ 7 \end{pmatrix} + \begin{pmatrix} -11 \\ 3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 18 \\ 14 \end{pmatrix} + \begin{pmatrix} -11 \\ 3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 16 \end{pmatrix}$

(c) $2b + 2c - 3a = 2\begin{pmatrix} 11 \\ -3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 9 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 22 \\ -6 \end{pmatrix} + \begin{pmatrix} -16 \\ -2 \end{pmatrix} + \begin{pmatrix} -27 \\ -21 \end{pmatrix} = \begin{pmatrix} -21 \\ -29 \end{pmatrix}$

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Vectors
Exercise D, Question 2

Question:

The points *A*, *B* and *C* have coordinates $(3, -1)$, (4, 5) and $(-2, 6)$ respectively, and *O* is the origin.
Find, in terms of **i** and **j**:

(a) the position vectors of *A*, *B* and *C*

(b) AB

(c) AC
Find, in surd form:

(d) $|OC|$

(e) $|AB|$

(f) $|AC|$

Solution:

(a) $a = 3i - j$, $b = 4i + 5j$, $c = -2i + 6j$

(b) $AB = b - a = (4i + 5j) - (3i - j)$
$= 4i + 5j - 3i + j$
$= i + 6j$

(c) $AC = c - a = (-2i + 6j) - (3i - j)$
$= -2i + 6j - 3i + j$
$= -5i + 7j$

(d) $|OC| = |-2i + 6j| = \sqrt{(-2)^2 + 6^2} = \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

(e) $|AB| = |i + 6j| = \sqrt{1^2 + 6^2} = \sqrt{37}$

(f) $|AC| = |-5i + 7j| = \sqrt{(-5)^2 + 7^2} = \sqrt{74}$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise D, Question 3

Question:

Given that a = 4i + 3j, b = 5i − 12j, c = − 7i + 24j and d = i − 3j, find a unit vector in the direction of **a**, **b**, **c** and **d**.

Solution:

$|a| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

Unit vector $= \frac{a}{|a|} = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$|b| = \sqrt{5^2 + (-12)^2} = \sqrt{169} = 13$

Unit vector $= \frac{b}{|b|} = \frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$|c| = \sqrt{(-7)^2 + 24^2} = \sqrt{625} = 25$

Unit vector $= \frac{c}{|c|} = \frac{1}{25}\begin{pmatrix} -7 \\ 24 \end{pmatrix}$

$|d| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

Unit vector $= \frac{d}{|d|} = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise D, Question 4

Question:

Given that a = 5i + j and b = λ i + 3j, and that $|3a + b| = 10$, find the possible values of λ.

Solution:

$$3a + b = 3\begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} \lambda \\ 3 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \end{pmatrix} + \begin{pmatrix} \lambda \\ 3 \end{pmatrix} = \begin{pmatrix} 15 + \lambda \\ 6 \end{pmatrix}$$

$|3a + b| = 10$, so

$\sqrt{(15 + \lambda)^2 + 6^2} = 10$

$(15 + \lambda)^2 + 6^2 = 100$

$225 + 30\lambda + \lambda^2 + 36 = 100$

$\lambda^2 + 30\lambda + 161 = 0$

$(\lambda + 7)(\lambda + 23) = 0$

$\lambda = -7, \lambda = -23$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise E, Question 1

Question:

Find the distance from the origin to the point $P(2, 8, -4)$.

Solution:

Distance $= \sqrt{2^2 + 8^2 + (-4)^2} = \sqrt{4 + 64 + 16} = \sqrt{84} \approx 9.17$ (3 s.f.)

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise E, Question 2

Question:

Find the distance from the origin to the point $P(7, 7, 7)$.

Solution:

Distance $= \sqrt{7^2 + 7^2 + 7^2} = \sqrt{49 + 49 + 49} = \sqrt{147} = 7\sqrt{3} \approx 12.1$ (3 s.f.)

Solutionbank 4
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Vectors
Exercise E, Question 3

Question:

Find the distance between A and B when they have the following coordinates:

(a) $A(3, 0, 5)$ and $B(1, -1, 8)$

(b) $A(8, 11, 8)$ and $B(-3, 1, 6)$

(c) $A(3, 5, -2)$ and $B(3, 10, 3)$

(d) $A(-1, -2, 5)$ and $B(4, -1, 3)$

Solution:

(a) $\text{AB} = \sqrt{(3-1)^2 + [0-(-1)]^2 + (5-8)^2}$
$= \sqrt{2^2 + 1^2 + (-3)^2}$
$= \sqrt{14} \approx 3.74$

(b) $\text{AB} = \sqrt{[8-(-3)]^2 + (11-1)^2 + (8-6)^2}$
$= \sqrt{11^2 + 10^2 + 2^2}$
$= \sqrt{225} = 15$

(c) $\text{AB} = \sqrt{(3-3)^2 + (5-10)^2 + [(-2)-3]^2}$
$= \sqrt{0^2 + (-5)^2 + (-5)^2}$
$= \sqrt{50} = 5\sqrt{2} \approx 7.07$

(d) $\text{AB} = \sqrt{[(-1)-4]^2 + [(-2)-(-1)]^2 + (5-3)^2}$
$= \sqrt{(-5)^2 + (-1)^2 + 2^2}$
$= \sqrt{30} \approx 5.48$

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Vectors
Exercise E, Question 4

Question:

The coordinates of A and B are $(7, -1, 2)$ and $(k, 0, 4)$ respectively. Given that the distance from A to B is 3 units, find the possible values of k.

Solution:

$AB = \sqrt{(7-k)^2 + (-1-0)^2 + (2-4)^2} = 3$

$\sqrt{(49 - 14k + k^2) + 1 + 4} = 3$

$49 - 14k + k^2 + 1 + 4 = 9$

$k^2 - 14k + 45 = 0$

$(k-5)(k-9) = 0$

$k = 5$ or $k = 9$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise E, Question 5

Question:

The coordinates of *A* and *B* are $(5, 3, -8)$ and $(1, k, -3)$ respectively.
Given that the distance from *A* to *B* is $3\sqrt{10}$ units, find the possible values of *k*.

Solution:

$AB = \sqrt{(5-1)^2 + (3-k)^2 + [-8-(-3)]^2} = 3\sqrt{10}$

$\sqrt{16 + (9 - 6k + k^2) + 25} = 3\sqrt{10}$

$16 + 9 - 6k + k^2 + 25 = 9 \times 10$

$k^2 - 6k - 40 = 0$

$(k+4)(k-10) = 0$

$k = -4$ or $k = 10$

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Vectors
Exercise F, Question 1

Question:

Find the modulus of:

(a) 3i + 5j + k

(b) 4i − 2k

(c) i + j − k

(d) 5i − 9j − 8k

(e) i + 5j − 7k

Solution:

(a) $| 3\mathbf{i} + 5\mathbf{j} + \mathbf{k} | = \sqrt{3^2 + 5^2 + 1^2}$
$= \sqrt{9 + 25 + 1} = \sqrt{35}$

(b) $| 4\mathbf{i} - 2\mathbf{k} | = \sqrt{4^2 + 0^2 + (- 2)^2}$
$= \sqrt{16 + 4} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

(c) $| \mathbf{i} + \mathbf{j} - \mathbf{k} | = \sqrt{1^2 + 1^2 + (- 1)^2}$
$= \sqrt{1 + 1 + 1} = \sqrt{3}$

(d) $| 5\mathbf{i} - 9\mathbf{j} - 8\mathbf{k} | = \sqrt{5^2 + (- 9)^2 + (- 8)^2}$
$= \sqrt{25 + 81 + 64} = \sqrt{170}$

(e) $| \mathbf{i} + 5\mathbf{j} - 7\mathbf{k} | = \sqrt{1^2 + 5^2 + (- 7)^2}$
$= \sqrt{1 + 25 + 49} = \sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise F, Question 2

Question:

Given that a = $\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, b = $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and c = $\begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix}$, find in column matrix form:

(a) a + b

(b) b − c

(c) a + b + c

(d) 3a − c

(e) a − 2b + c

(f) | a − 2b + c |

Solution:

(a) a + b = $\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -1 \end{pmatrix}$

(b) b − c = $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ -5 \end{pmatrix}$

(c) a + b + c = $\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ -3 \\ 1 \end{pmatrix}$

(d) 3a − c = $3\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 4 \end{pmatrix}$

(e) $\mathbf{a} - 2\mathbf{b} + \mathbf{c} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix}$

(f) $|\mathbf{a} - 2\mathbf{b} + \mathbf{c}| = \sqrt{8^2 + (-6)^2 + 10^2}$

$= \sqrt{64 + 36 + 100}$

$= \sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

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Vectors
Exercise F, Question 3

Question:

The position vector of the point *A* is 2i − 7j + 3k and AB = 5i + 4j − k. Find the position of the point *B*.

Solution:

AB = b − a, so b = AB + a

$$b = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 2 \end{pmatrix}$$

Position vector of *B* is 7i − 3j + 2k

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Vectors
Exercise F, Question 4

Question:

Given that a = ti + 2j + 3k, and that $|\mathbf{a}| = 7$, find the possible values of t.

Solution:

$|\mathbf{a}| = \sqrt{t^2 + 2^2 + 3^2} = 7$

$\sqrt{t^2 + 4 + 9} = 7$

$t^2 + 4 + 9 = 49$

$t^2 = 36$

$t = 6$ or $t = -6$

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Vectors
Exercise F, Question 5

Question:

Given that a = $5t$i + $2t$j + tk, and that $|\mathbf{a}| = 3\sqrt{10}$, find the possible values of t.

Solution:

$|\mathbf{a}| = \sqrt{(5t)^2 + (2t)^2 + t^2} = 3\sqrt{10}$
$\sqrt{25t^2 + 4t^2 + t^2} = 3\sqrt{10}$
$\sqrt{30t^2} = 3\sqrt{10}$
$30t^2 = 9 \times 10$
$t^2 = 3$
$t = \sqrt{3}$ or $t = -\sqrt{3}$

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Vectors
Exercise F, Question 6

Question:

The points A and B have position vectors $\begin{pmatrix} 2 \\ 9 \\ t \end{pmatrix}$ and $\begin{pmatrix} 2t \\ 5 \\ 3t \end{pmatrix}$ respectively.

(a) Find AB.

(b) Find, in terms of t, $|AB|$.

(c) Find the value of t that makes $|AB|$ a minimum.

(d) Find the minimum value of $|AB|$.

Solution:

(a) $AB = b - a = \begin{pmatrix} 2t \\ 5 \\ 3t \end{pmatrix} - \begin{pmatrix} 2 \\ 9 \\ t \end{pmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2t \end{pmatrix}$

(b) $|AB| = \sqrt{(2t-2)^2 + (-4)^2 + (2t)^2}$
$= \sqrt{4t^2 - 8t + 4 + 16 + 4t^2}$
$= \sqrt{8t^2 - 8t + 20}$

(c) Let $|AB|^2 = p$, then $p = 8t^2 - 8t + 20$

$\frac{dp}{dt} = 16t - 8$

For a minimum, $\frac{dp}{dt} = 0$, so $16t - 8 = 0$, i.e. $t = \frac{1}{2}$

$\frac{d^2P}{dt^2} = 16$, positive, $\therefore$ minimum

(d) When $t = \frac{1}{2}$,

$|AB| = \sqrt{8t^2 - 8t + 20} = \sqrt{2 - 4 + 20} = \sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

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Vectors
Exercise F, Question 7

Question:

The points A and B have position vectors $\begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix}$ respectively.

(a) Find AB.

(b) Find, in terms of t, $|AB|$.

(c) Find the value of t that makes $|AB|$ a minimum.

(d) Find the minimum value of $|AB|$.

Solution:

(a) $AB = b - a = \begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix} = \begin{pmatrix} -t \\ 4-t \\ -1 \end{pmatrix}$

(b) $|AB| = \sqrt{(-t)^2 + (4-t)^2 + (-1)^2}$
$= \sqrt{t^2 + 16 - 8t + t^2 + 1}$
$= \sqrt{2t^2 - 8t + 17}$

(c) Let $|AB|^2 = P$, then $P = 2t^2 - 8t + 17$

$\frac{dP}{dt} = 4t - 8$

For a minimum, $\frac{dP}{dt} = 0$, so $4t - 8 = 0$, i.e. $t = 2$

$\frac{d^2P}{dt^2} = 4$, positive, $\therefore$ minimum

(d) When $t = 2$,
$|AB| = \sqrt{2t^2 - 8t + 17} = \sqrt{8 - 16 + 17} = \sqrt{9} = 3$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 1

Question:

The vectors **a** and **b** each have magnitude 3 units, and the angle between **a** and **b** is 60 ° . Find **a.b**.

Solution:

$$a\,.\,b = |a|\ |b|\ \cos\theta = 3 \times 3 \times \cos 60° = 3 \times 3 \times \frac{1}{2} = \frac{9}{2}$$

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Vectors
Exercise G, Question 2

Question:

In each part, find **a.b**:

(a) a = 5i + 2j + 3k, b = 2i − j − 2k

(b) a = 10i − 7j + 4k, b = 3i − 5j − 12k

(c) a = i + j − k, b = − i − j + 4k

(d) a = 2i − k, b = 6i − 5j − 8k

(e) a = 3j + 9k, b = i + 12j − 4k

Solution:

(a) $a \cdot b = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 10 - 2 - 6 = 2$

(b) $a \cdot b = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \\ -12 \end{pmatrix} = 30 + 35 - 48 = 17$

(c) $a \cdot b = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} = -1 - 1 - 4 = -6$

(d) $a \cdot b = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ -8 \end{pmatrix} = 12 + 0 + 8 = 20$

(e) $a \cdot b = \begin{pmatrix} 0 \\ 3 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 12 \\ -4 \end{pmatrix} = 0 + 36 - 36 = 0$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 3

Question:

In each part, find the angle between **a** and **b**, giving your answer in degrees to 1 decimal place:

(a) a = 3i + 7j, b = 5i + j

(b) a = 2i − 5j, b = 6i + 3j

(c) a = i − 7j + 8k, b = 12i + 2j + k

(d) a = − i − j + 5k, b = 11i − 3j + 4k

(e) a = 6i − 7j + 12k, b = − 2i + j + k

(f) a = 4i + 5k, b = 6i − 2j

(g) a = − 5i + 2j − 3k, b = 2i − 2j + 11k

(h) a = i + j + k, b = i − j + k

Solution:

(a) $a\ .\ b = \begin{pmatrix} 3 \\ 7 \end{pmatrix} . \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 15 + 7 = 22$

$|a| = \sqrt{3^2 + 7^2} = \sqrt{58}$

$|b| = \sqrt{5^2 + 1^2} = \sqrt{26}$

$\sqrt{58}\sqrt{26}\cos\theta = 22$

$\cos\theta = \frac{22}{\sqrt{58}\sqrt{26}}$

$\theta = 55.5\ °$ (1 d.p.)

(b) $a\ .\ b = \begin{pmatrix} 2 \\ -5 \end{pmatrix} . \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 12 - 15 = -3$

$|a| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$

$|b| = \sqrt{6^2 + 3^2} = \sqrt{45}$

$\sqrt{29}\sqrt{45}\cos\theta = -3$

$\cos\theta = \frac{-3}{\sqrt{29}\sqrt{45}}$

$\theta = 94.8°$ (1 d.p.)

(c) $a \cdot b = \begin{pmatrix} 1 \\ -7 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 2 \\ 1 \end{pmatrix} = 12 - 14 + 8 = 6$

$|a| = \sqrt{1^2 + (-7)^2 + 8^2} = \sqrt{114}$

$|b| = \sqrt{12^2 + 2^2 + 1^2} = \sqrt{149}$

$\sqrt{114}\sqrt{149}\cos\theta = 6$

$\cos\theta = \frac{6}{\sqrt{114}\sqrt{149}}$

$\theta = 87.4°$ (1 d.p.)

(d) $a \cdot b = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -3 \\ 4 \end{pmatrix} = -11 + 3 + 20 = 12$

$|a| = \sqrt{(-1)^2 + (-1)^2 + 5^2} = \sqrt{27}$

$|b| = \sqrt{11^2 + (-3)^2 + 4^2} = \sqrt{146}$

$\sqrt{27}\sqrt{146}\cos\theta = 12$

$\cos\theta = \frac{12}{\sqrt{27}\sqrt{146}}$

$\theta = 79.0°$ (1 d.p.)

(e) $a \cdot b = \begin{pmatrix} 6 \\ -7 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -12 - 7 + 12 = -7$

$|a| = \sqrt{6^2 + (-7)^2 + 12^2} = \sqrt{229}$

$|b| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$

$\sqrt{229}\sqrt{6}\cos\theta = -7$

$\cos\theta = \frac{-7}{\sqrt{229}\sqrt{6}}$

$\theta = 100.9°$ (1 d.p.)

(f) $a \cdot b = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} = 24 + 0 + 0 = 24$

$|a| = \sqrt{4^2 + 5^2} = \sqrt{41}$

$|b| = \sqrt{6^2 + (-2)^2} = \sqrt{40}$

$\sqrt{41}\sqrt{40}\cos\theta = 24$

$\cos\theta = \frac{24}{\sqrt{41}\sqrt{40}}$

$\theta = 53.7°$ (1 d.p.)

(g) $\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix} = -10 - 4 - 33 = -47$

$|\mathbf{a}| = \sqrt{(-5)^2 + 2^2 + (-3)^2} = \sqrt{38}$

$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + 11^2} = \sqrt{129}$

$\sqrt{38}\sqrt{129}\cos\theta = -47$

$\cos\theta = \frac{-47}{\sqrt{38}\sqrt{129}}$

$\theta = 132.2°$ (1 d.p.)

(h) $\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 1 - 1 + 1 = 1$

$|\mathbf{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$|\mathbf{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\sqrt{3}\sqrt{3}\cos\theta = 1$

$\cos\theta = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$

$\theta = 70.5°$ (1 d.p.)

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 4

Question:

Find the value, or values, of λ for which the given vectors are perpendicular:

(a) $3\mathbf{i} + 5\mathbf{j}$ and $\lambda\mathbf{i} + 6\mathbf{j}$

(b) $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $\lambda\mathbf{i} - 4\mathbf{j} - 14\mathbf{k}$

(c) $3\mathbf{i} + \lambda\mathbf{j} - 8\mathbf{k}$ and $7\mathbf{i} - 5\mathbf{j} + \mathbf{k}$

(d) $9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$

(e) $\lambda\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 5\mathbf{k}$

Solution:

(a) $\begin{pmatrix} 3 \\ 5 \end{pmatrix} . \begin{pmatrix} \lambda \\ 6 \end{pmatrix} = 3\lambda + 30 = 0$

$\Rightarrow \quad 3\lambda = -30$

$\Rightarrow \quad \lambda = -10$

(b) $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} . \begin{pmatrix} \lambda \\ -4 \\ -14 \end{pmatrix} = 2\lambda - 24 + 14 = 0$

$\Rightarrow \quad 2\lambda = 10$

$\Rightarrow \quad \lambda = 5$

(c) $\begin{pmatrix} 3 \\ \lambda \\ -8 \end{pmatrix} . \begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix} = 21 - 5\lambda - 8 = 0$

$\Rightarrow \quad 5\lambda = 13$

$\Rightarrow \quad \lambda = 2\frac{3}{5}$

(d) $\begin{pmatrix} 9 \\ -3 \\ 5 \end{pmatrix} . \begin{pmatrix} \lambda \\ \lambda \\ 3 \end{pmatrix} = 9\lambda - 3\lambda + 15 = 0$

$\Rightarrow \quad 6\lambda = -15$

$\Rightarrow \quad \lambda = -2\frac{1}{2}$

(e) $\begin{pmatrix} \lambda \\ 3 \\ -2 \end{pmatrix} . \begin{pmatrix} \lambda \\ \lambda \\ 5 \end{pmatrix} = \lambda^2 + 3\lambda - 10 = 0$

$\Rightarrow \quad (\lambda + 5)(\lambda - 2) = 0$

$\Rightarrow \quad \lambda = -5$ or $\lambda = 2$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 5

Question:

Find, to the nearest tenth of a degree, the angle that the vector 9i − 5j + 3k makes with:

(a) the positive *x*-axis

(b) the positive *y*-axis

Solution:

(a) Using a = 9i − 5j + 3k and b = i,

$$a \cdot b = \begin{pmatrix} 9 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 9$$

$$|a| = \sqrt{9^2 + (-5)^2 + 3^2} = \sqrt{115}$$

$$|b| = 1$$

$$\sqrt{115}\cos\theta = 9$$

$$\cos\theta = \frac{9}{\sqrt{115}}$$

$$\theta = 32.9°$$

(b) Using a = 9i − 5j + 3k and b = j,

$$a \cdot b = \begin{pmatrix} 9 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -5$$

$$|a| = \sqrt{115}, \ |b| = 1$$

$$\sqrt{115}\cos\theta = -5$$

$$\cos\theta = \frac{-5}{\sqrt{115}}$$

$$\theta = 117.8°$$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 6

Question:

Find, to the nearest tenth of a degree, the angle that the vector i + 11j − 4k makes with:

(a) the positive *y*-axis

(b) the positive *z*-axis

Solution:

(a) Using a = i + 11j − 4k and b = j,

$$a . b = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 11$$

$$|a| = \sqrt{1^2 + 11^2 + (-4)^2} = \sqrt{138}$$

$$|b| = 1$$

$$\sqrt{138} \cos\theta = 11$$

$$\cos\theta = \frac{11}{\sqrt{138}}$$

$$\theta = 20.5°$$

(b) Using a = i + 11j − 4k and b = k,

$$a . b = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -4$$

$$|a| = \sqrt{138}, \ |b| = 1$$

$$\sqrt{138} \cos\theta = -4$$

$$\cos\theta = \frac{-4}{\sqrt{138}}$$

$$\theta = 109.9°$$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise G, Question 7

Question:

The angle between the vectors i + j + k and 2i + j + k is θ.
Calculate the exact value of $\cos\theta$.

Solution:

Using a = i + j + k and b = 2i + j + k,

$$\mathbf{a}\,.\,\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2 + 1 + 1 = 4$$

$$|\mathbf{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\mathbf{b}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$\sqrt{3}\sqrt{6}\cos\theta = 4$$

$$\cos\theta = \frac{4}{\sqrt{3}\sqrt{6}} = \frac{4}{\sqrt{3}\sqrt{3}\sqrt{2}} = \frac{4}{3\sqrt{2}}$$

$$= \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

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Vectors
Exercise G, Question 8

Question:

The angle between the vectors i + 3j and j + λ k is 60 ° .
Show that $\lambda = \pm \sqrt{\frac{13}{5}}$.

Solution:

Using a = i + 3j and b = j + λ k,

$$a \cdot b = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix} = 0 + 3 + 0 = 3$$

$$|a| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|b| = \sqrt{1^2 + \lambda^2} = \sqrt{1 + \lambda^2}$$

$$\sqrt{10}\sqrt{1 + \lambda^2}\cos 60° = 3$$

$$\sqrt{1 + \lambda^2} = \frac{3}{\sqrt{10}\cos 60°} = \frac{6}{\sqrt{10}}$$

Squaring both sides:

$$1 + \lambda^2 = \frac{36}{10}$$

$$\lambda^2 = \frac{26}{10} = \frac{13}{5}$$

$$\lambda = \pm\sqrt{\frac{13}{5}}$$

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Vectors
Exercise G, Question 9

Question:

Simplify as far as possible:

(a) $a \cdot (b + c) + b \cdot (a - c)$, given that **b** is perpendicular to **c**.

(b) $(a + b) \cdot (a + b)$, given that $|a| = 2$ and $|b| = 3$.

(c) $(a + b) \cdot (2a - b)$, given that **a** is perpendicular to **b**.

Solution:

(a) $a \cdot (b + c) + b \cdot (a - c)$
$= a \cdot b + a \cdot c + b \cdot a - b \cdot c$
$= 2a \cdot b + a \cdot c$ (because $b \cdot c = 0$)

(b) $(a + b) \cdot (a + b)$
$= a \cdot (a + b) + b \cdot (a + b)$
$= a \cdot a + a \cdot b + b \cdot a + b \cdot b$
$= |a|^2 + 2a \cdot b + |b|^2$
$= 4 + 2a \cdot b + 9$
$= 13 + 2a \cdot b$

(c) $(a + b) \cdot (2a - b)$
$= a \cdot (2a - b) + b \cdot (2a - b)$
$= 2a \cdot a - a \cdot b + 2b \cdot a - b \cdot b$
$= 2|a|^2 - |b|^2$ (because $a \cdot b = 0$)

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Vectors
Exercise G, Question 10

Question:

Find a vector which is perpendicular to both **a** and **b**, where:

(a) a = i + j − 3k, b = 5i − 2j − k

(b) a = 2i + 3j − 4k, b = i − 6j + 3k

(c) a = 4i − 4j − k, b = − 2i − 9j + 6k

Solution:

(a) Let the required vector be xi + yj + zk. Then

$$\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$x + y - 3z = 0$
$5x - 2y - z = 0$
Let $z = 1$:
$x + y = 3 \quad (\times 2)$
$5x - 2y = 1$
$2x + 2y = 6$
$5x - 2y = 1$
Adding, $7x = 7 \quad \Rightarrow \quad x = 1$
$1 + y = 3$, so $y = 2$
So $x = 1$, $y = 2$ and $z = 1$
A possible vector is i + 2j + k.

(b) Let the required vector be xi + yj + zk. Then

$$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$2x + 3y - 4z = 0$
$x - 6y + 3z = 0$
Let $z = 1$:
$2x + 3y = 4$
$x - 6y = -3 \quad (\times 2)$
$2x + 3y = 4$
$2x - 12y = -6$

Subtracting, $15y = 10 \quad \Rightarrow \quad y = \frac{2}{3}$

$2x + 2 = 4$, so $x = 1$

So $x = 1$, $y = \frac{2}{3}$ and $z = 1$

A possible vector is $\mathrm{i} + \frac{2}{3}\mathrm{j} + \mathrm{k}$.

Another possible vector is $3\left(\mathrm{i} + \frac{2}{3}\mathrm{j} + \mathrm{k}\right) = 3\mathrm{i} + 2\mathrm{j} + 3\mathrm{k}$.

(c) Let the required vector be $x\mathrm{i} + y\mathrm{j} + z\mathrm{k}$. Then

$$\begin{pmatrix} 4 \\ -4 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} -2 \\ -9 \\ 6 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$4x - 4y - z = 0$

$-2x - 9y + 6z = 0$

Let $z = 1$:

$4x - 4y = 1$

$-2x - 9y = -6 \qquad (\times 2)$

$4x - 4y = 1$

$-4x - 18y = -12$

Adding, $-22y = -11 \quad \Rightarrow \quad y = \frac{1}{2}$

$4x - 2 = 1$, so $x = \frac{3}{4}$

So $x = \frac{3}{4}$, $y = \frac{1}{2}$ and $z = 1$

A possible vector is $\frac{3}{4}\mathrm{i} + \frac{1}{2}\mathrm{j} + \mathrm{k}$

Another possible vector is $4\left(\frac{3}{4}\mathrm{i} + \frac{1}{2}\mathrm{j} + \mathrm{k}\right) = 3\mathrm{i} + 2\mathrm{j} + 4\mathrm{k}$.

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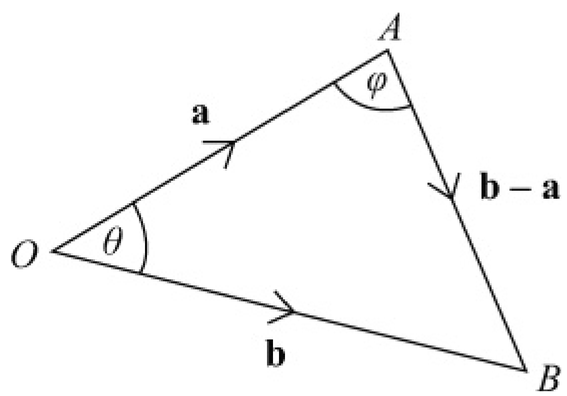
Vectors
Exercise G, Question 11

Question:

The points *A* and *B* have position vectors 2i + 5j + k and 6i + j − 2k respectively, and *O* is the origin.
Calculate each of the angles in △ OAB, giving your answers in degrees to 1 decimal place.

Solution:



Using **a** and **b** to find θ:

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} = 12 + 5 - 2 = 15$$

$$|\mathbf{a}| = \sqrt{2^2 + 5^2 + 1^2} = \sqrt{30}$$

$$|\mathbf{b}| = \sqrt{6^2 + 1^2 + (-2)^2} = \sqrt{41}$$

$$\sqrt{30}\sqrt{41}\cos\theta = 15$$

$$\cos\theta = \frac{15}{\sqrt{30}\sqrt{41}}$$

$$\theta = 64.7°$$

Using AO and AB to find ϕ:

$$AO = -\mathbf{a} = \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix}$$

$$AB = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} \\ -\mathbf{a} \\ \\ \end{pmatrix} . \begin{pmatrix} \\ \mathbf{b}-\mathbf{a} \\ \\ \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix} . \begin{pmatrix} 4 \\ -4 \\ -3 \end{pmatrix} = -8+20+3=15$$

$|-\mathbf{a}| = \sqrt{(-2)^2+(-5)^2+(-1)^2} = \sqrt{30}$

$|\mathbf{b}-\mathbf{a}| = \sqrt{4^2+(-4)^2+(-3)^2} = \sqrt{41}$

$\sqrt{30}\sqrt{41}\cos\phi = 15$

$\cos\phi = \frac{15}{\sqrt{30}\sqrt{41}}$

$\phi = 64.7°$ (1 d.p.)

(Since $|\mathbf{b}-\mathbf{a}| = |\mathbf{b}|$, AB = OB, so the triangle is isosceles).

$\angle$ OBA = 180° − 64.7° − 64.7° = 50.6° (1 d.p.)

Angles are 64.7°, 64.7° and 50.6° (all 1 d.p.)

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Vectors
Exercise G, Question 12

Question:

The points A, B and C have position vectors i + 3j + k, 2i + 7j − 3k and 4i − 5j + 2k respectively.

(a) Find, as surds, the lengths of AB and BC.

(b) Calculate, in degrees to 1 decimal place, the size of $\angle ABC$.

Solution:

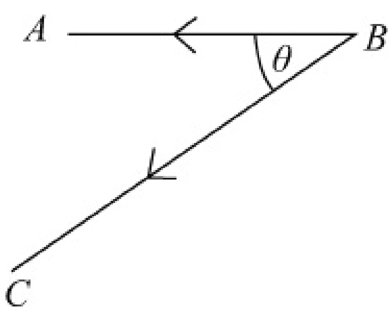
(a) $AB = b - a = \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -4 \end{pmatrix}$

Length of $AB = |AB| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{33}$

$BC = c - b = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -12 \\ 5 \end{pmatrix}$

Length of $BC = |BC| = \sqrt{2^2 + (-12)^2 + 5^2} = \sqrt{173}$

(b)



θ is the angle between BA and BC.

$BA \, . \, BC = \begin{pmatrix} -1 \\ -4 \\ 4 \end{pmatrix} . \begin{pmatrix} 2 \\ -12 \\ 5 \end{pmatrix} = -2 + 48 + 20 = 66$

$\sqrt{33}\sqrt{173}\cos\theta = 66$

$\cos\theta = \frac{66}{\sqrt{33}\sqrt{173}}$

$\theta = 29.1°$ (1 d.p.)

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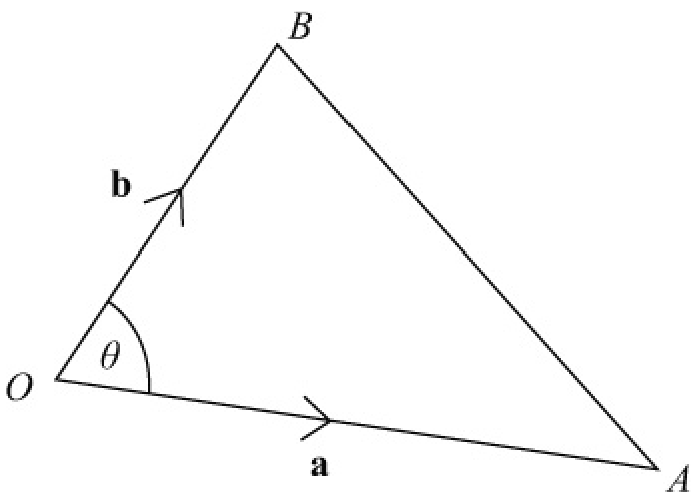
Vectors
Exercise G, Question 13

Question:

Given that the points A and B have coordinates $(7, 4, 4)$ and $(2, -2, -1)$ respectively, use a vector method to find the value of $\cos AOB$, where O is the origin.

Prove that the area of $\triangle AOB$ is $\frac{5\sqrt{29}}{2}$.

Solution:



The position vectors of A and B are

$$\mathbf{a} = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

$$\mathbf{a} . \mathbf{b} = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix} . \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = 14 - 8 - 4 = 2$$

$$|\mathbf{a}| = \sqrt{7^2 + 4^2 + 4^2} = \sqrt{81} = 9$$
$$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$9 \times 3 \times \cos\theta = 2$$

$$\cos\theta = \frac{2}{27}$$

$$\cos \angle AOB = \frac{2}{27}$$

$$\text{Area of } \angle AOB = \frac{1}{2} |\mathbf{a}| \, |\mathbf{b}| \sin \angle AOB$$

Using $\sin^2\theta + \cos^2\theta = 1$:

$\sin^2 \angle AOB = 1 - \left(\frac{2}{27}\right)^2 = \frac{725}{27^2}$

$\sin \angle AOB = \sqrt{\frac{725}{27^2}} = \frac{\sqrt{25}\sqrt{29}}{27} = \frac{5\sqrt{29}}{27}$

Area of $\triangle AOB = \frac{1}{2} \times 9 \times 3 \times \frac{5\sqrt{29}}{27} = \frac{5\sqrt{29}}{2}$

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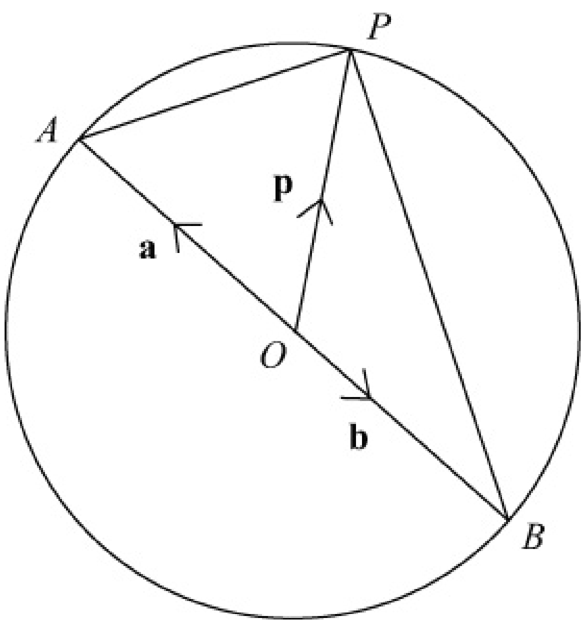
Vectors
Exercise G, Question 14

Question:

AB is a diameter of a circle centred at the origin *O*, and *P* is any point on the circumference of the circle.
Using the position vectors of *A*, *B* and *P*, prove (using a scalar product) that *AP* is perpendicular to *BP* (i.e. the angle in the semicircle is a right angle).

Solution:



Let the position vectors, referred to origin *O*, of *A*, *B* and *P* be **a**, **b** and **p** respectively.
Since $|OA| = |OB|$ and *AB* is a straight line, $b = -a$
$AP = p - a$
$BP = p - b = p - (-a) = p + a$
$AP . BP = (p - a) . (p + a) = p . (p + a) - a . (p + a)$
$= p . p + p . a - a . p - a . a$
$= p . p - a . a$
$p . p = |p|^2$ and $a . a = |a|^2$
Also $|p| = |a|$, since the magnitude of each vector equals the radius of the circle.
So $AP . BP = |p|^2 - |a|^2 = 0$
Since the scalar product is zero, *AP* is perpendicular to *BP*.

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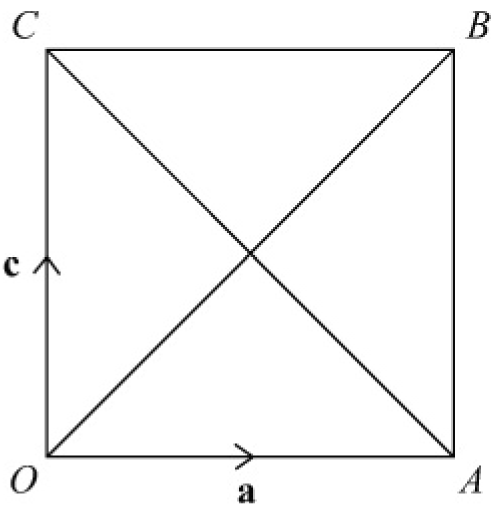
Vectors
Exercise G, Question 15

Question:

Use a vector method to prove that the diagonals of the square *OABC* cross at right angles.

Solution:



Let the position vectors, referred to origin *O*, of *A* and *C* be **a** and **c** respectively.
AB = OC = c
AC = c − a
OB = OA + AB = a + c
AC . OB = (c − a) . (a + c) = c . (a + c) − a . (a + c)
= c . a + c . c − a . a − a . c
= c . c − a . a
= $|c|^2 - |a|^2$
But $|c| = |a|$, since the magnitude of each vector equals the length of the side of the square.
So AC . OB = $|c|^2 - |a|^2 = 0$
Since the scalar product is zero; the diagonals cross at right angles.

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Vectors
Exercise H, Question 1

Question:

Find a vector equation of the straight line which passes through the point A, with position vector **a**, and is parallel to the vector **b**:

(a) a = 6i + 5j − k, b = 2i − 3j − k

(b) a = 2i + 5j, b = i + j + k

(c) a = − 7i + 6j + 2k, b = 3i + j + 2k

(d) $a = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, b = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

(e) $a = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

Solution:

(a) $r = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

(b) $r = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(c) $r = \begin{pmatrix} -7 \\ 6 \\ 2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

(d) $r = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

(e) $$\mathbf{r} = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

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Vectors
Exercise H, Question 2

Question:

Calculate, to 1 decimal place, the distance between the point P, where $t = 1$, and the point Q, where $t = 5$, on the line with equation:

(a) $\mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(3\mathbf{i} - 8\mathbf{j} - \mathbf{k})$

(b) $\mathbf{r} = (\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + t(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$

(c) $\mathbf{r} = (2\mathbf{i} + 5\mathbf{k}) + t(-3\mathbf{i} + 4\mathbf{j} - \mathbf{k})$

Solution:

(a) $t = 1$: $\quad \mathbf{p} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 1\begin{pmatrix} 3 \\ -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 0 \end{pmatrix}$

$t = 5$: $\quad \mathbf{q} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 3 \\ -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 17 \\ -41 \\ -4 \end{pmatrix}$

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 17 \\ -41 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ -9 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ -32 \\ -4 \end{pmatrix}$

Distance $= |\overrightarrow{PQ}| = \sqrt{12^2 + (-32)^2 + (-4)^2}$
$= \sqrt{1184} = 34.4$ (1 d.p.)

(b) $t = 1$: $\quad \mathbf{p} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + 1\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$

$t = 5$: $\quad \mathbf{q} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 31 \\ -6 \\ 16 \end{pmatrix}$

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 31 \\ -6 \\ 16 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 24 \\ -8 \\ 12 \end{pmatrix}$

Distance $= |\overrightarrow{PQ}| = \sqrt{24^2 + (-8)^2 + 12^2}$
$= \sqrt{784} = 28$ (exact)

(c) $t = 1$: $\mathbf{p} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + 1 \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 4 \end{pmatrix}$

$t = 5$: $\mathbf{q} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + 5 \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -13 \\ 20 \\ 0 \end{pmatrix}$

$\mathrm{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} -13 \\ 20 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ 16 \\ -4 \end{pmatrix}$

$\text{Distance} = |\mathrm{PQ}| = \sqrt{(-12)^2 + 16^2 + (-4)^2}$
$= \sqrt{416} = 20.4 \text{ (1 d.p.)}$

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Vectors
Exercise H, Question 3

Question:

Find a vector equation for the line which is parallel to the *z*-axis and passes through the point $(4, -3, 8)$.

Solution:

Vector k = $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is in the direction of the *z*-axis.

The point $(4, -3, 8)$ has position vector $\begin{pmatrix} 4 \\ -3 \\ 8 \end{pmatrix}$.

The equation of the line is

$$\mathrm{r} = \begin{pmatrix} 4 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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Vectors
Exercise H, Question 4

Question:

Find a vector equation for the line which passes through the points:

(a) (2, 1, 9) and $(4 , - 1 , 8)$

(b) $(- 3 , 5 , 0)$ and (7, 2, 2)

(c) $(1 , 11 , - 4)$ and (5, 9, 2)

(d) $(- 2 , - 3 , - 7)$ and $(12 , 4 , - 3)$

Solution:

(a) $a = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$

$$b - a = \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

Equation is

$$r = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

(b) $a = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix}$

$$b - a = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$$

Equation is

$$r = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$$

(c) $\mathbf{a} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 9 \\ 2 \end{pmatrix}$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 5 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$$

(d) $\mathbf{a} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 12 \\ 4 \\ -3 \end{pmatrix}$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 12 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \\ 4 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 14 \\ 7 \\ 4 \end{pmatrix}$$

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Vectors
Exercise H, Question 5

Question:

The point $(1, p, q)$ lies on the line l. Find the values of p and q, given that the equation is l is:

(a) $\mathrm{r} = (2\mathrm{i} - 3\mathrm{j} + \mathrm{k}) + t(\mathrm{i} - 4\mathrm{j} - 9\mathrm{k})$

(b) $\mathrm{r} = (-4\mathrm{i} + 6\mathrm{j} - \mathrm{k}) + t(2\mathrm{i} - 5\mathrm{j} - 8\mathrm{k})$

(c) $\mathrm{r} = (16\mathrm{i} - 9\mathrm{j} - 10\mathrm{k}) + t(3\mathrm{i} + 2\mathrm{j} + \mathrm{k})$

Solution:

(a) $\mathrm{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \\ -9 \end{pmatrix}$

$x = 1: \quad 2 + t = 1 \quad \Rightarrow \quad t = -1$

$\mathrm{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} \\ -1 \\ \end{pmatrix}\begin{pmatrix} 1 \\ -4 \\ -9 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$

So $p = 1$ and $q = 10$.

(b) $\mathrm{r} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix}$

$x = 1: \quad -4 + 2t = 1 \quad \Rightarrow \quad 2t = 5 \quad \Rightarrow \quad t = \frac{5}{2}$

$\mathrm{r} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} + \frac{5}{2}\begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix} = \begin{pmatrix} 1 \\ -6\frac{1}{2} \\ -21 \end{pmatrix}$

So $p = -6\frac{1}{2}$ and $q = -21$.

(c) $\mathrm{r} = \begin{pmatrix} 16 \\ -9 \\ -10 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$x = 1: \quad 16 + 3t = 1 \quad \Rightarrow \quad 3t = -15 \quad \Rightarrow \quad t = -5$

$$r = \begin{pmatrix} 16 \\ -9 \\ -10 \end{pmatrix} + \begin{pmatrix} \\ -5 \\ \\ \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -19 \\ -15 \end{pmatrix}$$

So $p = -19$ and $q = -15$.

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Vectors
Exercise I, Question 1

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 1 \\ 14 \\ 16 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 2+2t \\ 4+t \\ -7+3t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 1+s \\ 14-s \\ 16-2s \end{pmatrix}.$$

At an intersection point: $\begin{pmatrix} 2+2t \\ 4+t \\ -7+3t \end{pmatrix} = \begin{pmatrix} 1+s \\ 14-s \\ 16-2s \end{pmatrix}$

$2+2t = 1+s$
$4+t = 14-s$
Adding: $6+3t = 15$
$\Rightarrow \quad 3t = 9$
$\Rightarrow \quad t = 3$
$2+6 = 1+s$
$\Rightarrow \quad s = 7$
If the lines intersect, $-7+3t = 16-2s$ must be true.
$-7+3t = -7+9 = 2$
$16-2s = 16-14 = 2$
The z components are equal, so the lines do intersect.
Intersection point:

$$\begin{pmatrix} 2+2t \\ 4+t \\ -7+3t \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ 2 \end{pmatrix}$$

Coordinates (8, 7, 2)

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Vectors
Exercise I, Question 2

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 9 \\ -2 \\ -1 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 2 + 9t \\ 2 - 2t \\ -3 - t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 3 + 2s \\ -1 - s \\ 2 + 3s \end{pmatrix}$$

At an intersection point: $\begin{pmatrix} 2 + 9t \\ 2 - 2t \\ -3 - t \end{pmatrix} = \begin{pmatrix} 3 + 2s \\ -1 - s \\ 2 + 3s \end{pmatrix}$

$2 + 9t = 3 + 2s$
$2 - 2t = -1 - s \qquad (\times 2)$
$2 + 9t = 3 + 2s$
$4 - 4t = -2 - 2s$
Adding: $6 + 5t = 1$
$\Rightarrow \quad 5t = -5$
$\Rightarrow \quad t = -1$
$2 - 9 = 3 + 2s$
$\Rightarrow \quad 2s = -10$
$\Rightarrow \quad s = -5$
If the lines intersect, $-3 - t = 2 + 3s$ must be true.
$-3 - t = -3 + 1 = -2$
$2 + 3s = 2 - 15 = -13$
The z components are not equal, so the lines do not intersect.

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise I, Question 3

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 12 \\ 4 \\ -6 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ 6 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 12-2t \\ 4+t \\ -6+4t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 8+2s \\ -2+s \\ 6-5s \end{pmatrix}$$

At an intersection point: $\begin{pmatrix} 12-2t \\ 4+t \\ -6+4t \end{pmatrix} = \begin{pmatrix} 8+2s \\ -2+s \\ 6-5s \end{pmatrix}$

$12-2t=8+2s$
$4+t=-2+s \quad (\times 2)$
$12-2t=8+2s$
$8+2t=-4+2s$
Adding: $20=4+4s$
$\Rightarrow \quad 4s=16$
$\Rightarrow \quad s=4$
$12-2t=8+8$
$\Rightarrow \quad 2t=-4$
$\Rightarrow \quad t=-2$
If the lines intersect, $-6+4t=6-5s$ must be true.
$-6+4t=-6-8=-14$
$6-5s=6-20=-14$
The z components are equal, so the lines do intersect. Intersection point:

$$\begin{pmatrix} 12-2t \\ 4+t \\ -6+4t \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ -14 \end{pmatrix}.$$

Coordinates $(16, 2, -14)$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise I, Question 4

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} -2 \\ -9 \\ 12 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 1+4t \\ 2t \\ 4+6t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} -2+s \\ -9+2s \\ 12-s \end{pmatrix}$$

At an intersection point: $\begin{pmatrix} 1+4t \\ 2t \\ 4+6t \end{pmatrix} = \begin{pmatrix} -2+s \\ -9+2s \\ 12-s \end{pmatrix}$

$1 + 4t = -2 + s$
$2t = -9 + 2s \quad (\times 2)$
$1 + 4t = -2 + s$
$4t = -18 + 4s$
Subtracting: $1 = 16 - 3s$
$\Rightarrow \quad 3s = 15$
$\Rightarrow \quad s = 5$
$1 + 4t = -2 + 5$
$\Rightarrow \quad 4t = 2$
$\Rightarrow \quad t = \frac{1}{2}$

If the lines intersect, $4 + 6t = 12 - s$ must be true.
$4 + 6t = 4 + 3 = 7$
$12 - s = 12 - 5 = 7$
The z components are equal, so the lines do intersect. Intersection point:

$$\begin{pmatrix} 1+4t \\ 2t \\ 4+6t \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}.$$

Coordinates $(3, 1, 7)$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise I, Question 5

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 6 \\ -4 \\ 1 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 3+2t \\ -3+t \\ 1-4t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 3+6s \\ 4-4s \\ 2+s \end{pmatrix}$$

At an intersection point: $\begin{pmatrix} 3+2t \\ -3+t \\ 1-4t \end{pmatrix} = \begin{pmatrix} 3+6s \\ 4-4s \\ 2+s \end{pmatrix}$

$3 + 2t = 3 + 6s$
$-3 + t = 4 - 4s \quad (\times 2)$
$3 + 2t = 3 + 6s$
$-6 + 2t = 8 - 8s$
Subtracting: $9 = -5 + 14s$
$\Rightarrow \quad 14s = 14$
$\Rightarrow \quad s = 1$
$3 + 2t = 3 + 6$
$\Rightarrow \quad 2t = 6$
$\Rightarrow \quad t = 3$
If the lines intersect, $1 - 4t = 2 + s$ must be true.
$1 - 4t = 1 - 12 = -11$
$2 + s = 2 + 1 = 3$
The z components are not equal, so the lines do not intersect.

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise J, Question 1

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:
r = (2i + j + k) + t (3i − 5j − k)
and r = (7i + 4j + k) + s (2i + j − 9k)

Solution:

Direction vectors are $\mathbf{a} = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -9 \end{pmatrix}$

$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\ |\mathbf{b}|}$

$\mathbf{a}\cdot\mathbf{b} = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -9 \end{pmatrix} = 6 - 5 + 9 = 10$

$|\mathbf{a}| = \sqrt{3^2 + (-5)^2 + (-1)^2} = \sqrt{35}$
$|\mathbf{b}| = \sqrt{2^2 + 1^2 + (-9)^2} = \sqrt{86}$

$\cos\theta = \dfrac{10}{\sqrt{35}\sqrt{86}}$

$\theta = 79.5°$ (1 d.p.)
The acute angle between the lines is 79.5 ° (1 d.p.)

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise J, Question 2

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:
$\mathbf{r} = (\mathbf{i} - \mathbf{j} + 7\mathbf{k}) + t(-2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$
and $\mathbf{r} = (8\mathbf{i} + 5\mathbf{j} - \mathbf{k}) + s(-4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

Solution:

Direction vectors are $\mathbf{a} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$

$\cos\theta = \dfrac{\mathbf{a} . \mathbf{b}}{|\mathbf{a}|\ |\mathbf{b}|}$

$\mathbf{a} . \mathbf{b} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix} = 8 + 2 + 3 = 13$

$|\mathbf{a}| = \sqrt{(-2)^2 + (-1)^2 + 3^2} = \sqrt{14}$
$|\mathbf{b}| = \sqrt{(-4)^2 + (-2)^2 + 1^2} = \sqrt{21}$

$\cos\theta = \dfrac{13}{\sqrt{14}\sqrt{21}}$

$\theta = 40.7°$ (1 d.p.)
The acute angle between the lines is $40.7°$ (1 d.p.)

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise J, Question 3

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:
$\mathbf{r} = (3\mathbf{i} + 5\mathbf{j} - \mathbf{k}) + t(\mathbf{i} + \mathbf{j} + \mathbf{k})$
and $\mathbf{r} = (-\mathbf{i} + 11\mathbf{j} + 5\mathbf{k}) + s(2\mathbf{i} - 7\mathbf{j} + 3\mathbf{k})$

Solution:

Direction vectors are $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}$

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix} = 2 - 7 + 3 = -2$

$|\mathbf{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$
$|\mathbf{b}| = \sqrt{2^2 + (-7)^2 + 3^2} = \sqrt{62}$

$\cos\theta = \dfrac{-2}{\sqrt{3}\sqrt{62}}$

$\theta = 98.4°$ (1 d.p.)
This is the angle between the two vectors.
The acute angle between the lines is $180° - 98.4° = 81.6°$ (1 d.p.).

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise J, Question 4

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:
r = (i + 6j − k) + t (8i − j − 2k)
and r = (6i + 9j) + s (i + 3j − 7k)

Solution:

Direction vectors are a = $\begin{pmatrix} 8 \\ -1 \\ -2 \end{pmatrix}$ and b = $\begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix}$

$\cos\theta = \dfrac{a \, . \, b}{|\,a\,| \; |\,b\,|}$

a . b = $\begin{pmatrix} 8 \\ -1 \\ -2 \end{pmatrix} . \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} = 8 - 3 + 14 = 19$

$|\,a\,| = \sqrt{8^2 + (-1)^2 + (-2)^2} = \sqrt{69}$
$|\,b\,| = \sqrt{1^2 + 3^2 + (-7)^2} = \sqrt{59}$

$\cos\theta = \dfrac{19}{\sqrt{69}\sqrt{59}}$

$\theta = 72.7\,°$ (1 d.p.)
The acute angle between the lines is 72.7 ° (1 d.p.)

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise J, Question 5

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:
$\mathbf{r} = (2\mathbf{i} + \mathbf{k}) + t(11\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$
and $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + s(-3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$

Solution:

Direction vectors are $\mathbf{a} = \begin{pmatrix} 11 \\ 5 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix}$

$\cos\theta = \dfrac{\mathbf{a}\,.\,\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

$\mathbf{a}\,.\,\mathbf{b} = \begin{pmatrix} 11 \\ 5 \\ -3 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix} = -33 + 25 - 12 = -20$

$|\mathbf{a}| = \sqrt{11^2 + 5^2 + (-3)^2} = \sqrt{155}$
$|\mathbf{b}| = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50}$

$\cos\theta = \dfrac{-20}{\sqrt{155}\sqrt{50}}$

$\theta = 103.1°$ (1 d.p.)
This is the angle between the two vectors.
The acute angle between the lines is $180° - 103.1° = 76.9°$ (1 d.p.).

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Vectors
Exercise J, Question 6

Question:

The straight lines l_1 and l_2 have vector equations
r = (i + 4j + 2k) + t (8i + 5j + k) and r = (i + 4j + 2k) + s (3i + j)
respectively, and P is the point with coordinates (1 , 4 , 2) .

(a) Show that the point Q (9 , 9 , 3) lies on l_1.

(b) Find the cosine of the acute angle between l_1 and l_2.

(c) Find the possible coordinates of the point R, such that R lies on l_2 and PQ = PR.

Solution:

(a) Line l_1: $\mathrm{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}$

When $t = 1$, $\mathrm{r} = \begin{pmatrix} 9 \\ 9 \\ 3 \end{pmatrix}$

So the point (9, 9, 3) lies on l_1.

(b) Direction vectors are $\mathrm{a} = \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}$ and $\mathrm{b} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$$\mathrm{a} \,.\, \mathrm{b} = \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix} . \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = 24 + 5 + 0 = 29$$

$$|\,\mathrm{a}\,| = \sqrt{8^2 + 5^2 + 1^2} = \sqrt{90}$$
$$|\,\mathrm{b}\,| = \sqrt{3^2 + 1^2 + 0^2} = \sqrt{10}$$
$$\cos\theta = \frac{29}{\sqrt{90}\sqrt{10}} = \frac{29}{\sqrt{900}} = \frac{29}{30}$$

(c) $\mathrm{PQ} = \sqrt{(9-1)^2 + (9-4)^2 + (3-2)^2}$
$= \sqrt{8^2 + 5^2 + 1^2} = \sqrt{90}$

Line l_2: $\quad \mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 + 3s \\ 4 + s \\ 2 \end{pmatrix}$

Let the coordinates of R be $(1 + 3s, 4 + s, 2)$

$$PR = \sqrt{(1 + 3s - 1)^2 + (4 + s - 4)^2 + (2 - 2)^2}$$

$$= \sqrt{9s^2 + s^2} = \sqrt{10s^2}$$

$PQ^2 = PR^2$: $\quad 90 = 10s^2$

$\Rightarrow \quad s^2 = 9$

$\Rightarrow \quad s = \pm 3$

When $s = 3$, $\mathbf{r} = \begin{pmatrix} 10 \\ 7 \\ 2 \end{pmatrix}$ $\quad R : \left(10, 7, 2 \right)$

When $s = -3$, $\mathbf{r} = \begin{pmatrix} -8 \\ 1 \\ 2 \end{pmatrix}$ $\quad R : \left(-8, 1, 2 \right)$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise K, Question 1

Question:

With respect to an origin O, the position vectors of the points L, M and N are $(4\mathbf{i} + 7\mathbf{j} + 7\mathbf{k})$, $(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ and $(2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$ respectively.

(a) Find the vectors ML and MN.

(b) Prove that $\cos \angle LMN = \frac{9}{10}$.

E

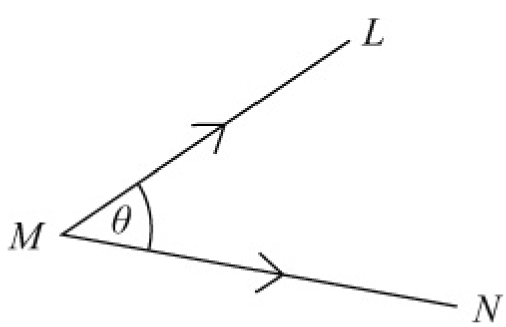
Solution:

$$\mathbf{l} = \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

(a) $$\mathrm{ML} = \mathbf{l} - \mathbf{m} = \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

$$\mathrm{MN} = \mathbf{n} - \mathbf{m} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

(b)



$$\cos\theta = \frac{\mathrm{ML} \cdot \mathrm{MN}}{|\mathrm{ML}| \ |\mathrm{MN}|}$$

$$\mathrm{ML} \cdot \mathrm{MN} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = 3 + 4 + 20 = 27$$

$$| ML | = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$$

$$| MN | = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18}$$

$$\cos\theta = \frac{27}{\sqrt{50}\sqrt{18}} = \frac{27}{\sqrt{25}\sqrt{2}\sqrt{9}\sqrt{2}} = \frac{27}{5 \times 3 \times 2} = \frac{9}{10}.$$

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

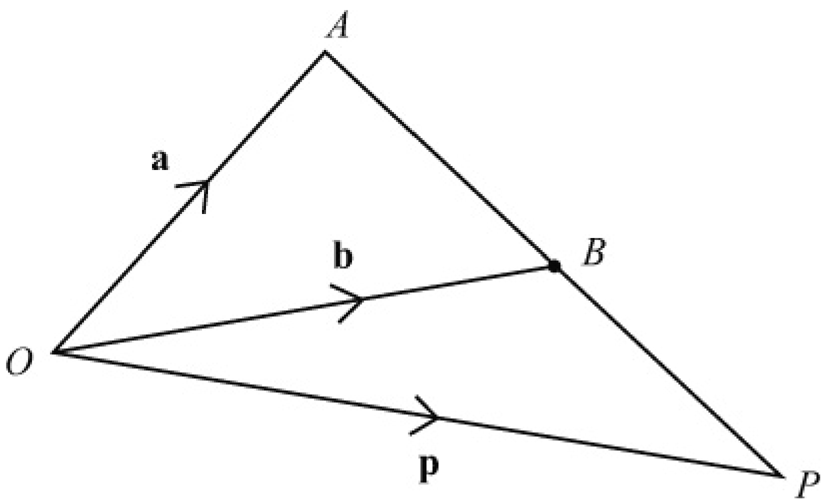
Vectors
Exercise K, Question 2

Question:

The position vectors of the points *A* and *B* relative to an origin *O* are 5i + 4j + k, − i + j − 2k respectively. Find the position vector of the point *P* which lies on *AB* produced such that AP = 2BP.

E

Solution:



$$a = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

OP = OA + AP = OA + 2AB

p = a + 2 (b − a) = 2b − a

$$p = 2\begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \\ -5 \end{pmatrix}$$

The position vector of *P* is − 7i − 2j − 5k.

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Vectors
Exercise K, Question 3

Question:

Points A, B, C, D in a plane have position vectors a = 6i + 8j, b = $\frac{3}{2}$a, c = 6i + 3j, d = $\frac{5}{3}$c respectively. Write down vector equations of the lines AD and BC and find the position vector of their point of intersection.

E

Solution:

$a = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$, $b = \frac{3}{2}a = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

$c = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$, $d = \frac{5}{3}c = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

Line AD: $AD = d - a = \begin{pmatrix} 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$r = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Line BC: $BC = c - b = \begin{pmatrix} 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 9 \\ 12 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$

$r = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + s\begin{pmatrix} -3 \\ -9 \end{pmatrix}$

or

$r = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Where AD and BC intersect, $\begin{pmatrix} 6 + 4t \\ 8 - 3t \end{pmatrix} = \begin{pmatrix} 9 + s \\ 12 + 3s \end{pmatrix}$ (Using the last version of BC)

$6 + 4t = 9 + s \quad (\times 3)$

$8 - 3t = 12 + 3s$

$18 + 12t = 27 + 3s$

$8 - 3t = 12 + 3s$

Subtracting: $10 + 15t = 15$

$\Rightarrow \quad 15t = 5$

$\Rightarrow \quad t = \frac{1}{3}$

Intersection: $\quad \mathbf{r} = \begin{pmatrix} 6 + 4t \\ 8 - 3t \end{pmatrix} = \begin{pmatrix} \frac{22}{3} \\ 7 \end{pmatrix}$

$\mathbf{r} = \frac{22}{3}\mathbf{i} + 7\mathbf{j}$

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Vectors
Exercise K, Question 4

Question:

Find the point of intersection of the line through the points (2, 0, 1) and $(-1, 3, 4)$ and the line through the points $(-1, 3, 0)$ and $(4, -2, 5)$.
Calculate the acute angle between the two lines.

E

Solution:

Line through (2, 0, 1) and $(-1, 3, 4)$.

Let $\mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

Equation: $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$

or $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

Line through $(-1, 3, 0)$ and $(4, -2, 5)$.

Let $\mathbf{c} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$

$$\mathbf{d} - \mathbf{c} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$$

Equation: $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$

or $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

At the intersection point: $\begin{pmatrix} 2-t \\ t \\ 1+t \end{pmatrix} = \begin{pmatrix} -1+s \\ 3-s \\ s \end{pmatrix}$

$2 - t = -1 + s$
$t = 3 - s$
$1 + t = s$
Adding the second and third equations:
$1 + 2t = 3$
$2t = 2$
$t = 1$
$s = 2$
Intersection point:

$\mathrm{r} = \begin{pmatrix} 2-t \\ t \\ 1+t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ Coordinates (1, 1, 2)

Direction vectors of the lines are $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Calling these **m** and **n**:

$\cos\theta = \frac{\mathrm{m}\,.\,\mathrm{n}}{|\mathrm{m}|\ |\mathrm{n}|}$

$\mathrm{m}\,.\,\mathrm{n} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = -1 - 1 + 1 = -1$

$|\mathrm{m}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$
$|\mathrm{n}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\cos\theta = \frac{-1}{\sqrt{3}\sqrt{3}} = \frac{-1}{3}$

$\theta = 109.5°$ (1 d.p.)
This is the angle between the two vectors.
The acute angle between the lines is $180° - 109.5° = 70.5°$ (1 d.p.).

Solutionbank 4
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise K, Question 5

Question:

Show that the lines
$\mathbf{r} = (-2\mathbf{i} + 5\mathbf{j} - 11\mathbf{k}) + \lambda (3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$
$\mathbf{r} = 8\mathbf{i} + 9\mathbf{j} + \mu (4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$
intersect. Find the position vector of their common point.

E

Solution:

$$\mathbf{r} = \begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 8 + 4\mu \\ 9 + 2\mu \\ 5\mu \end{pmatrix}$$

At an intersection point: $\begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix} = \begin{pmatrix} 8 + 4\mu \\ 9 + 2\mu \\ 5\mu \end{pmatrix}$

$-2 + 3\lambda = 8 + 4\mu$
$5 + \lambda = 9 + 2\mu \quad (\times 2)$
$-2 + 3\lambda = 8 + 4\mu$
$10 + 2\lambda = 18 + 4\mu$
Subtracting: $-12 + \lambda = -10$
$\Rightarrow \quad \lambda = 12 - 10$
$\Rightarrow \quad \lambda = 2$
$-2 + 6 = 8 + 4\mu$
$\Rightarrow \quad 4\mu = -4$
$\Rightarrow \quad \lambda = -1$
If the lines intersect, $-11 + 3\lambda = 5\mu$:
$-11 + 3\lambda = -11 + 6 = -5$
$5\mu = -5$
The z components are equal, so the lines do intersect. Intersection point:

$$\mathbf{r} = \begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}.$$

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Vectors
Exercise K, Question 6

Question:

Find a vector that is perpendicular to both 2i + j − k and i + j − 2k.

E.

Solution:

Let the required vector be xi + yj + zk.

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$2x + y - z = 0$
$x + y - 2z = 0$
Let $z = 1$:
$2x + y = 1$
$x + y = 2$
Subtracting: $x = -1, y = 3$
So $x = -1, y = 3$ and $z = 1$
A possible vector is − i + 3j + k.

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise K, Question 7

Question:

State a vector equation of the line passing through the points A and B whose position vectors are $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ respectively. Determine the position vector of the point C which divides the line segment AB internally such that $AC = 2CB$.

E

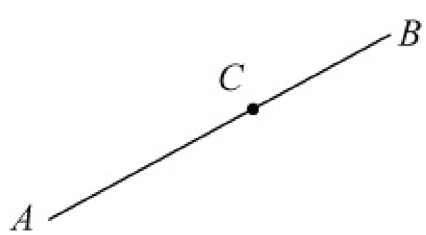
Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Equation of line:

$\mathbf{r} = \mathbf{a} + t (\mathbf{b} - \mathbf{a})$

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$$



but $AC = 2CB$

Position vector of C:

$$\mathbf{c} = \mathbf{a} + \frac{2}{3} \left(\mathbf{b} - \mathbf{a} \right)$$

$$= \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \frac{7}{3} \end{pmatrix}$$

$$= \mathbf{i} + \mathbf{j} + \frac{7}{3}\mathbf{k}$$

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Vectors
Exercise K, Question 8

Question:

Vectors **r** and **s** are given by
$r = \lambda i + (2\lambda - 1)j - k$
$s = (1 - \lambda)i + 3\lambda j + (4\lambda - 1)k$
where λ is a scalar.

(a) Find the values of λ for which **r** and **s** are perpendicular.
When $\lambda = 2$, **r** and **s** are the position vectors of the points *A* and *B* respectively, referred to an origin *0*.

(b) Find AB.

(c) Use a scalar product to find the size of $\angle$ BAO, giving your answer to the nearest degree.

E

Solution:

$r = \begin{pmatrix} \lambda \\ 2\lambda - 1 \\ -1 \end{pmatrix}$, and $s = \begin{pmatrix} 1 - \lambda \\ 3\lambda \\ 4\lambda - 1 \end{pmatrix}$

(a) If **r** and **s** are perpendicular, r . s = 0

$$r \cdot s = \begin{pmatrix} \lambda \\ 2\lambda - 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \lambda \\ 3\lambda \\ 4\lambda - 1 \end{pmatrix}$$
$$= \lambda(1 - \lambda) + 3\lambda(2\lambda - 1) - 1(4\lambda - 1)$$
$$= \lambda - \lambda^2 + 6\lambda^2 - 3\lambda - 4\lambda + 1$$
$$= 5\lambda^2 - 6\lambda + 1$$
$$\therefore 5\lambda^2 - 6\lambda + 1 = 0$$
$$(5\lambda - 1)(\lambda - 1) = 0$$
$$\lambda = \frac{1}{5} \text{ or } \lambda = 1$$

(b) $\lambda = 2$: $\quad a = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 6 \\ 7 \end{pmatrix}$

$$AB = b - a = \begin{pmatrix} -1 \\ 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix}$$

$$= -3i + 3j + 8k$$

(c) Using vectors AB and AO:

$$AB = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix}, AO = -a = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$$

$$\cos \angle BAO = \frac{AB \,.\, AO}{|AB|\ |AO|}$$

$$AB \,.\, AO = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix} . \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = 6 - 9 + 8 = 5$$

$$|AB| = \sqrt{(-3)^2 + 3^2 + 8^2} = \sqrt{82}$$

$$|AO| = \sqrt{(-2)^2 + (-3)^2 + 1^2} = \sqrt{14}$$

$$\cos \angle BAO = \frac{5}{\sqrt{82}\sqrt{14}}$$

$$\angle BAO = 82° \text{ (nearest degree)}$$

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Vectors
Exercise K, Question 9

Question:

With respect to an origin O, the position vectors of the points L and M are $2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and $5\mathbf{i} + \mathbf{j} + c\mathbf{k}$ respectively, where c is a constant. The point N is such that $OLMN$ is a rectangle.

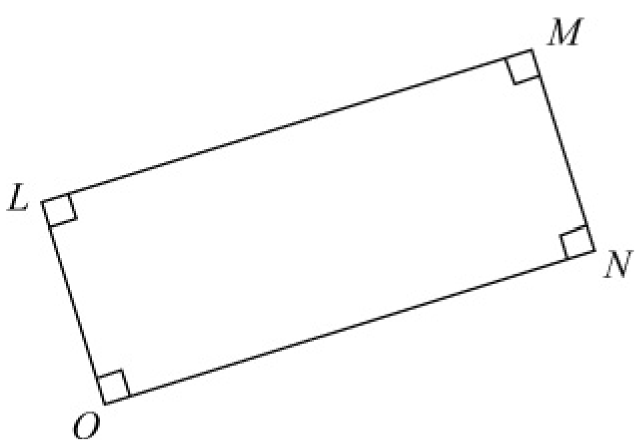
(a) Find the value of c.

(b) Write down the position vector of N.

(c) Find, in the form $\mathbf{r} = \mathbf{p} + t\mathbf{q}$, an equation of the line MN.

E

Solution:



(a) $\mathbf{l} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$ and $\mathbf{m} = \begin{pmatrix} 5 \\ 1 \\ c \end{pmatrix}$

$$\mathbf{LM} = \mathbf{m} - \mathbf{l} = \begin{pmatrix} 5 \\ 1 \\ c \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix}$$

Since OL and LM are perpendicular, OL . LM = O

$$\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} . \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix} = 0$$

$6 - 12 + 3 (c - 3) = 0$
$6 - 12 + 3c - 9 = 0$
$3c = 15$

$c = 5$

(b) $\mathbf{n} = \overrightarrow{ON} = \overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

$\mathbf{n} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

(c) The line *MN* is parallel to *OL*.
Using the point *M* and the direction vector **l**:

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$$

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Vectors
Exercise K, Question 10

Question:

The point A has coordinates $(7, -1, 3)$ and the point B has coordinates $(10, -2, 2)$. The line l has vector equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + \mathbf{k})$, where λ is a real parameter.

(a) Show that the point A lies on the line l.

(b) Find the length of AB.

(c) Find the size of the acute angle between the line l and the line segment AB, giving your answer to the nearest degree.

(d) Hence, or otherwise, calculate the perpendicular distance from B to the line l, giving your answer to two significant figures.

E

Solution:

(a) Line l: $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

Point A is $(7, -1, 3)$

Using $\lambda = 2$, $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix}$

So A lies on the line l.

(b) $AB = \sqrt{(10-7)^2 + [-2-(-1)]^2 + (2-3)^2}$
$= \sqrt{3^2 + (-1)^2 + (-1)^2} = \sqrt{11}$

(c) $AB = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 10 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$

Angle between the vectors $\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$:

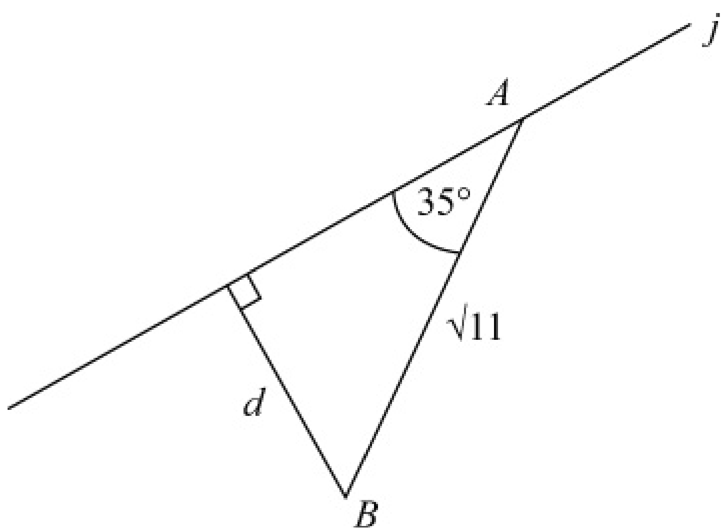
$$\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} . \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 9 + 1 - 1 = 9$$

The magnitude of each of the vectors is $\sqrt{11}$

So $\cos\theta = \dfrac{9}{\sqrt{11}\sqrt{11}} = \dfrac{9}{11}$

$\Rightarrow \quad \theta = 35°$ (nearest degree)

(d)



$\sin 35° = \dfrac{d}{\sqrt{11}}$

$d = \sqrt{11}\sin 35° = 1.9$ (2 s.f.)

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Vectors
Exercise K, Question 11

Question:

Referred to a fixed origin O, the points A and B have position vectors $(5\mathrm{i} - \mathrm{j} - \mathrm{k})$ and $(\mathrm{i} - 5\mathrm{j} + 7\mathrm{k})$ respectively.

(a) Find an equation of the line AB.

(b) Show that the point C with position vector $4\mathrm{i} - 2\mathrm{j} + \mathrm{k}$ lies on AB.

(c) Show that OC is perpendicular to AB.

(d) Find the position vector of the point D, where $D \not\equiv A$, on AB such that $| \mathrm{OD} | = | \mathrm{OA} |$.

E

Solution:

(a) $\mathrm{a} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix}$, $\mathrm{b} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix}$

$$\mathrm{b} - \mathrm{a} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$$

Equation of AB:

$$\mathrm{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$$

or

$$\mathrm{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

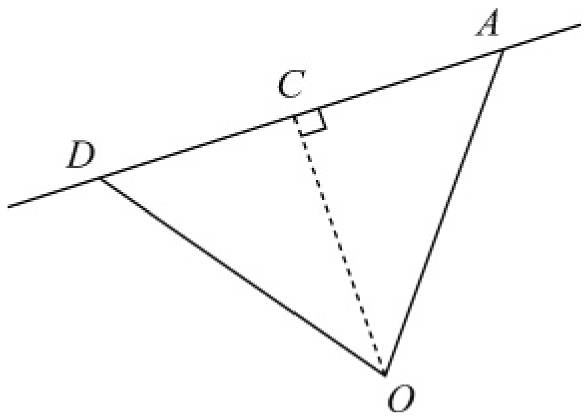
(b) Using $t = 1$: $\mathrm{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

So the point with position vector $4\mathrm{i} - 2\mathrm{j} + \mathrm{k}$ lies on AB.

(c) $OC \cdot AB = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix} = -16 + 8 + 8 = 0$

Since the scalar product is zero, *OC* is perpendicular to *AB*.

(d)



Since OD = OA, DC = CA, so DC = CA.

$CA = a - c = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$DC = c - d = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

So $d = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$d = 3i - 3j + 3k$

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Vectors
Exercise K, Question 12

Question:

Referred to a fixed origin O, the points A, B and C have position vectors $(9\mathrm{i} - 2\mathrm{j} + \mathrm{k})$, $(6\mathrm{i} + 2\mathrm{j} + 6\mathrm{k})$ and $(3\mathrm{i} + p\mathrm{j} + q\mathrm{k})$ respectively, where p and q are constants.

(a) Find, in vector form, an equation of the line l which passes through A and B.
Given that C lies on l:

(b) Find the value of p and the value of q.

(c) Calculate, in degrees, the acute angle between OC and AB,
The point D lies on AB and is such that OD is perpendicular to AB.

(d) Find the position vector of D.

E

Solution:

$$\mathrm{a} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix}, \mathrm{b} = \begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix}, \mathrm{c} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}$$

(a) $$\mathrm{b} - \mathrm{a} = \begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

Equation of l:

$$\mathrm{r} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

(b) Since C lies on l,

$$\begin{pmatrix} 3 \\ p \\ q \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

$3 = 9 - 3t$
$3t = 6$
$t = 2$

So $p = -2 + 4t = 6$
and $q = 1 + 5t = 11$

(c) $\cos\theta = \dfrac{\text{OC . AB}}{|\text{OC}|\ |\text{AB}|}$

$$\text{OC . AB} = \begin{pmatrix} 3 \\ 6 \\ 11 \end{pmatrix} . \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = -9 + 24 + 55 = 70$$

$|\text{OC}| = \sqrt{3^2 + 6^2 + 11^2} = \sqrt{166}$
$|\text{AB}| = \sqrt{(-3)^2 + 4^2 + 5^2} = \sqrt{50}$

$\cos\theta = \dfrac{70}{\sqrt{166}\sqrt{50}}$

$\theta = 39.8°$ (1 d.p.)

(d) If *OD* and *AB* are perpendicular, $\text{d} . (\text{b} - \text{a}) = 0$

Since **d** lies on *AB*, use $\text{d} = \begin{pmatrix} 9 - 3t \\ -2 + 4t \\ 1 + 5t \end{pmatrix}$

$$\begin{pmatrix} 9 - 3t \\ -2 + 4t \\ 1 + 5t \end{pmatrix} . \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = 0$$

$-3(9 - 3t) + 4(-2 + 4t) + 5(1 + 5t) = 0$
$-27 + 9t - 8 + 16t + 5 + 25t = 0$
$50t = 30$

$t = \dfrac{3}{5}$

$$\text{d} = \begin{pmatrix} 9 - \frac{9}{5} \\ -2 + \frac{12}{5} \\ 1 + 3 \end{pmatrix} = \frac{36}{5}\text{i} + \frac{2}{5}\text{j} + 4\text{k}$$

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Vectors
Exercise K, Question 13

Question:

Referred to a fixed origin O, the points A and B have position vectors $(\mathrm{i} + 2\mathrm{j} - 3\mathrm{k})$ and $(5\mathrm{i} - 3\mathrm{j})$ respectively.

(a) Find, in vector form, an equation of the line l_1 which passes through A and B.
The line l_2 has equation $\mathrm{r} = (4\mathrm{i} - 4\mathrm{j} + 3\mathrm{k}) + \lambda (\mathrm{i} - 2\mathrm{j} + 2\mathrm{k})$, where λ is a scalar parameter.

(b) Show that A lies on l_2.

(c) Find, in degrees, the acute angle between the lines l_1 and l_2.
The point C with position vector $(2\mathrm{i} - \mathrm{k})$ lies on l_2.

(d) Find the shortest distance from C to the line l_1.

E

Solution:

(a) $\mathrm{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \mathrm{b} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix}$

$$\mathrm{b} - \mathrm{a} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$$

Equation of l_1:

$$\mathrm{r} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$$

(b) Equation of l_2:

$$\mathrm{r} = \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

Using $\lambda = -3$, $\mathbf{r} = \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

So A lies on the line l_2.

(c) Direction vectors of l_1 and l_2 are $\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

Calling these **m** and **n**:

$\cos\theta = \dfrac{\mathbf{m}\,.\,\mathbf{n}}{|\mathbf{m}|\;|\mathbf{n}|}$

$\mathbf{m}\,.\,\mathbf{n} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 4 + 10 + 6 = 20$

$|\mathbf{m}| = \sqrt{4^2 + (-5)^2 + 3^2} = \sqrt{50}$

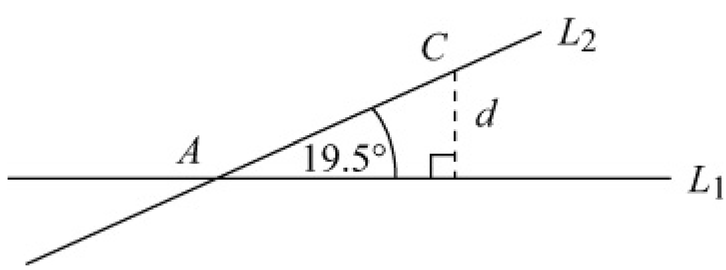
$|\mathbf{n}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$

$\cos\theta = \dfrac{20}{3\sqrt{50}}$

$\theta = 19.5°$ (1 d.p.)

The angle between l_1 and l_2 is $19.5°$ (1 d.p.).

(d) $\mathbf{c} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$



$|AC| = \sqrt{(2-1)^2 + (0-2)^2 + [-1-(-3)]^2}$
$= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$

$\sin\theta = \dfrac{d}{AC}$

$d = AC\sin\theta = 3 \times \dfrac{1}{3} = 1$

The shortest distance from C to l_1 is 1 unit.

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Vectors
Exercise K, Question 14

Question:

Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines, l_1 and l_2, along which they travel are
r = 3i + 4j − 5k + λ (i − 2j + 2k)
and r = 9i + j − 2k + μ (4i + j − k)
where λ and μ are scalars.

(a) Show that the submarines are moving in perpendicular directions.

(b) Given that l_1 and l_2 intersect at the point *A*, find the position vector of *A*.
The point *B* has position vector 10j − 11k.

(c) Show that only one of the submarines passes through the point *B*.

(d) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance *AB*.

E

Solution:

(a) Line l_1: $r = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

Line l_2: $r = \begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$

Using the direction vectors:

$$\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} . \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = 4 - 2 - 2 = 0$$

Since the scalar product is zero, the directions are perpendicular.

(b) At an intersection point: $\begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \\ -5 + 2\lambda \end{pmatrix} = \begin{pmatrix} 9 + 4\mu \\ 1 + \mu \\ -2 - \mu \end{pmatrix}$

$$3 + \lambda = 9 + 4\mu \qquad (\times 2)$$
$$4 - 2\lambda = 1 + \mu$$
$$6 + 2\lambda = 18 + 8\mu$$
$$4 - 2\lambda = 1 + \mu$$

Adding: $10 = 19 + 9\mu$

$\Rightarrow \quad 9\mu = -9$

$\Rightarrow \quad \mu = -1$

$3 + \lambda = 9 - 4$

$\Rightarrow \quad \lambda = 2$

Intersection point: $\begin{pmatrix} 3+\lambda \\ 4-2\lambda \\ -5+2\lambda \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$

Position vector of A is a $= 5\mathbf{i} - \mathbf{k}$.

(c) Position vector of B: b $= 10\mathbf{j} - 11\mathbf{k} = \begin{pmatrix} 0 \\ 10 \\ -11 \end{pmatrix}$

For l_1, to give zero as the x component, $\lambda = -3$.

$$\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -11 \end{pmatrix}$$

So B lies on l_1.

For l_2, to give -11 as the z component, $\mu = 9$.

$$\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + 9\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 45 \\ 10 \\ -11 \end{pmatrix}$$

So B does not lie on l_2.

So only one of the submarines passes through B.

(d) $|AB| = \sqrt{(0-5)^2 + (10-0)^2 + [-11-(-1)]^2}$

$= \sqrt{(-5)^2 + 10^2 + (-10)^2}$

$= \sqrt{225} = 15$

Since 1 unit represents 100 m, the distance AB is $15 \times 100 = 1500\,\text{m} = 1.5\,\text{km}$.