Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise A, Question 1

Question:

Find the values of x for which $f(x) = x^3 - 3x^2$ is a decreasing function.

Solution:

$f(x) = x^3 - 3x^2$	
$f'(x) = 3x^2 - 6x$	Find $f'(x)$ and put this expression < 0.
$3x^2 - 6x < 0$	
$3x(x - 2) < 0$	Solve the inequality by factorisation, consider the three regions $x < 0$, $0 < x < 2$ and $x > 2$, looking for sign changes.
$f(x)$ is a decreasing function for $0 < x < 2$.	$\frac{dy}{dx} < 0$ for $0 < x < 2$

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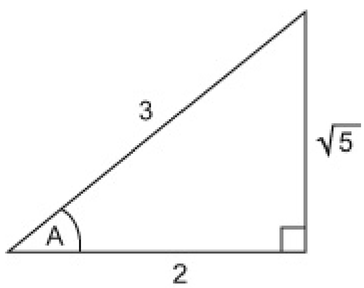
Algebra and functions
Exercise A, Question 2

Question:

Given that A is an acute angle and $\cos A = \frac{2}{3}$, find the exact value of $\tan A$.

Solution:

$\cos A = \frac{2}{3}$



so $\tan A = \frac{\sqrt{5}}{2}$

Draw a diagram and put in the information for cos A.

Use Pythagoras theorem: $a^2 + b^2 = c^2$ with $b = 2$ and $c = 3$.
so

$$a^2 + 2^2 = 3^2$$
$$a^2 + 4 = 9$$
$$a^2 = 5$$
$$a = \sqrt{5}$$

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Algebra and functions
Exercise A, Question 3

Question:

Evaluate $\int_1^3 x^2 - \frac{1}{x^2} dx$.

Solution:

Remember $\int ax^n \, dx = \frac{ax^{n+1}}{n+1}$

$\int_1^3 x^2 - \frac{1}{x^2} \, dx$

Change $\frac{-1}{x^2}$ into index form and integrate:

$\frac{-1}{x^2} = -1x^{-2}$

$\int -1x^{-2} \, dx$

$= \frac{-1x^{-2+1}}{-2+1}$

$= \frac{-1x^{-1}}{-1}$

$= x^{-1}$

$= \frac{1}{x}$

Evaluate the integral: substitute $x = 3$, then $x = 1$, and subtract.

$= \left[\frac{x^3}{3} + \frac{1}{x} \right]_1^3$

$= \left(\frac{(3)^3}{3} + \frac{1}{(3)} \right) - \left(\frac{(1)^3}{3} + \frac{1}{(1)} \right)$

$= \left(9 + \frac{1}{3} \right) - \left(\frac{1}{3} + 1 \right)$

$= 8$

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Algebra and functions
Exercise A, Question 4

Question:

Given that $y = \frac{x^3}{3} + x^2 - 6x + 3$, find the values of x when $\frac{dy}{dx} = 2$.

Solution:

$y = \frac{x^3}{3} + x^2 - 6x + 3$

Remember $\frac{d}{dx}(ax^n) = anx^{n-1}$

$\frac{dy}{dx} = \frac{3x^2}{3} + 2x - 6$

$x^2 + 2x - 6 = 2$

Put $\frac{dy}{dx} = 2$ and solve the equation.

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

Factorise $x^2 + 2x - 8 = 0$:

$(+4) \times (-2) = -8$

$(+4) + (-2) = +2$

so $x^2 + 2x - 8 = (x + 4)(x - 2)$

so $x = -4$, $x = 2$

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Algebra and functions
Exercise A, Question 5

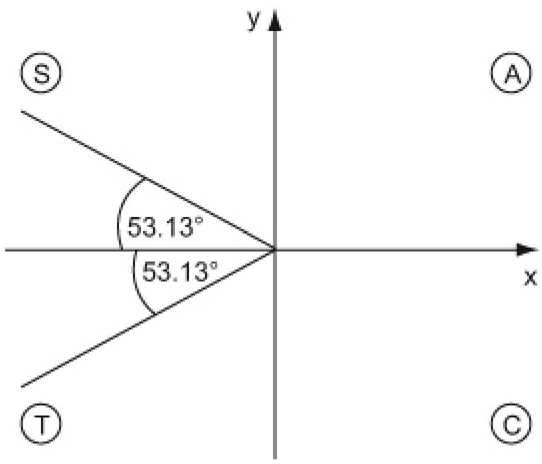
Question:

Solve, for $0 \leq x < 180°$, the equation $\cos 2x = -0.6$, giving your answers to 1 decimal place.

Solution:

$\cos 2x = -0.6$

$2x = 126.87°$



$\cos 2x$ is negative, so your need to look in the 2nd and 3rd quadrants. Here the angle in the 2nd quadrant is $180° - 126.87° = 53.13°$

$2x = 126.87, \quad 233.13$

so $\quad x = 63.4°, \quad 116.6°$

Read off the solutions from your diagram
Find x: divide each value by 2.

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Algebra and functions
Exercise A, Question 6

Question:

Find the area between the curve $y = x^3 - 3x^2$, the x-axis and the lines $x = 2$ and $x = 4$.

Solution:

Area $= \int_2^4 x^3 - 3x^2 \mathrm{d}x$

Remember $\int ax^n \mathrm{d}x = \frac{ax^{n+1}}{n+1}$

$= \left[\frac{x^4}{4} - x^3 \right]_2^4$

$= \left(\frac{(4)^4}{4} - (4)^3 \right) - \left(\frac{(2)^4}{4} - (2)^3 \right)$

Use the limits: Substitute $x = 4$ and $x = 2$ into $\frac{x^4}{4} - x^3$ and subtract.

$= (64 - 64) - \left(\frac{16}{4} - 8 \right)$

$= 0 - (4 - 8)$

$= 0 - (-4)$

$= 4$

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Algebra and functions
Exercise A, Question 7

Question:

Given $f(x) = x^3 - 2x^2 - 4x$,

(a) find (i) $f(2)$, (ii) $f'(2)$, (iii) $f''(2)$

(b) interpret your answer to part (a).

Solution:

$$f(x) = x^3 - 2x^2 - 4x$$
$$f'(x) = 3x^2 - 4x - 4$$
$$f''(x) = 6x - 4$$

Remember $f'(x) = \frac{d}{dx}f(x)$, $f''(x) = \frac{d^2}{dx^2}f(x)$

(a)

(i)

$$f(2) = (2)^3 - 2(2)^2 - 4(2) = 8 - 8 - 8 = -8$$

Find the value of $f(x)$ where $x = 2$; substitute $x = 2$ into $x^3 - 2x^2 - 4x$

(ii)

$$f'(2) = 3(2)^2 - 4(2) - 4 = 12 - 8 - 4 = 0$$

Find the value of $f'(x)$ where $x = 2$; substitute $x = 2$ into $3x^2 - 4x - 4$

(iii)

$$f''(2) = 6(2) - 4 = 12 - 4 = 8$$

Find the value of $f''(x)$ when $x = 2$; Substitute $x = 2$ into $6x - 4$.

(b)

On the graph of $y = f(x)$, the point $(2, -8)$ is a minimum point.

$f'(2) = 0$ means there is a stationary point at $x = 2$
$f''(2) = 8 > 0$ means the stationary point is a minimum.
$f(2) = -8$ means the graph of $y = f(u)$ passes through the point $(2, -8)$.

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Algebra and functions
Exercise A, Question 8

Question:

Find all the values of θ in the interval $0 \le \theta < 360°$ for which $2\sin(\theta - 30°) = \sqrt{3}$.

Solution:

$2\sin(\theta - 30°) = \sqrt{3}$

Divide each side by 2.

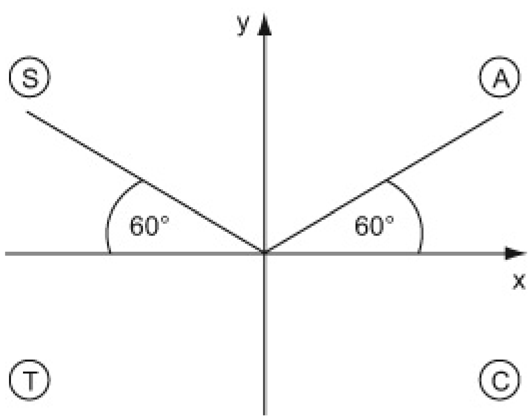
$\sin(\theta - 30°) = \frac{\sqrt{3}}{2}$

$\theta - 30° = 60°$

Solve the equation: let $X = \theta - 30°$ $\sin X = \frac{\sqrt{3}}{2}$, so

$X = 60°$ i.e. $\theta - 30° = 60°$

$\sin(\theta - 30°)$ is positive so you need to look in the 1st and 2nd quadrants.



$\theta - 30° = 60°, 120°$

Read off the solutions from your diagram

so $\theta = 60 + 30°$

$= 90°$

Find θ : add $30°$ to each value.

and $\theta = 120 + 30°$

$= 150°$

$\theta = 90°, 150°$

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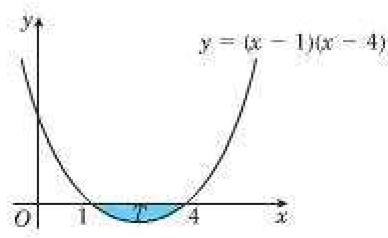
Algebra and functions
Exercise A, Question 9

Question:

The diagram shows the shaded region T which is bounded by the curve $y = (x-1)(x-4)$ and the x-axis. Find the area of the shaded region T.



Solution:

$$\text{Area} = \int_1^4 (x-1)(x-4)\ \mathrm{d}x$$

Expand the brackets so that

$$(x-1)(x-4) = x^2 - 4x - x + 4 = x^2 - 5x + 4$$

$$= \int_1^4 x^2 - 5x + 4\ \mathrm{d}x$$

$$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_1^4$$

Remember $\int ax^n \mathrm{d}x = \frac{ax^{n+1}}{n+1}$

$$= \left(\frac{(4)^3}{3} - \frac{5(4)^2}{2} + 4(4)\right) - \left(\frac{(1)^3}{3} - \frac{5(1)^2}{2} + 4(1)\right)$$

Evaluate the integral. Substitute $x = 4$, then $x = 1$, into $\frac{x^3}{3} - \frac{5x^2}{2} + 4x$ and subtract.

$$= -4\frac{1}{2}$$

The negative value means the area is below the x-axis, as can be seen in the diagram.

$$\text{Area} = 4\frac{1}{2}$$

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Algebra and functions
Exercise A, Question 10

Question:

Find the coordinates of the stationary points on the curve with equation $y = 4x^3 - 3x + 1$.

Solution:

$$y = 4x^3 - 3x + 1$$

Remember $\frac{dy}{dx} = 0$ at a stationary point.

$$\frac{dy}{dx} = 12x^2 - 3$$

$$12x^2 - 3 = 0$$

$$12x^2 = 3$$

$$x^2 = \frac{3}{12}$$

$$= \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}}$$

$$= \pm \frac{1}{2}$$

When $x = \frac{1}{2}$,

Find the coordinates of the stationary points. Substitute $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ into the equation for y.

$$y = 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) + 1$$

$$= 4\left(\frac{1}{8}\right) - \frac{3}{2} + 1$$

$$= 0$$

When $x = \frac{-1}{2}$

Find the coordinates of the stationary points. Substitute $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ into the equation for y.

$$y = 4\left(\frac{-1}{2}\right)^3 - 3\left(-\frac{1}{2}\right) + 1$$

$$= 4\left(-\frac{1}{8}\right) + \frac{3}{2} + 1$$

$$= -\frac{1}{2} + \frac{3}{2} + 1$$

$$= 2$$

So the coordinates of the stationary points are $\left(\frac{1}{2}, 0\right)$ and $\left(-\frac{1}{2}, 2\right)$.

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Algebra and functions
Exercise A, Question 11

Question:

(a) Given that $\sin\ \theta = \cos\ \theta$, find the value of $\tan\ \theta$.

(b) Find the value of θ in the interval $0 \le \theta < 2\pi$ for which $\sin\theta = \cos\ \theta$, giving your answer in terms of π.

Solution:

(a)

$\sin\ \theta = \cos\ \theta$

Remember $\tan\ \theta = \frac{\sin\ \theta}{\cos\ \theta}$

$\frac{\sin\ \theta}{\cos\ \theta} = \frac{\cos\ \theta}{\cos\ \theta}$

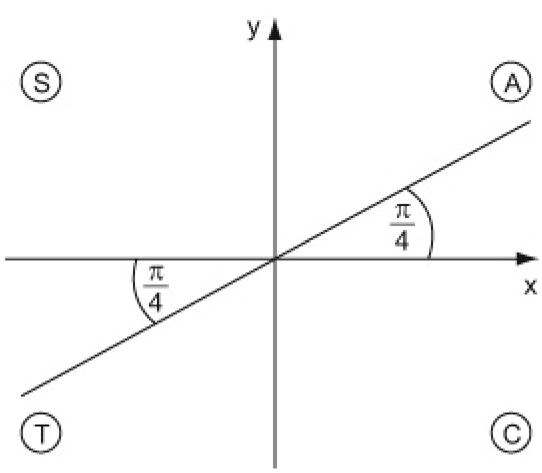
Divide each side by $\cos\ \theta$.

$\tan\ \theta = 1$

(b)

$\theta = \frac{\pi}{4}$

$\tan\ \theta = 1$, so $\theta = 45°$. Remember π (radians) $= 180°$ so $45° = \frac{\pi}{4}$(radians).



$\tan\ \theta$ is positive in the 1st and 3rd quadrants. Read off the solutions, in $0 \le \theta < 2\pi$, from your diagram.

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

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Algebra and functions
Exercise A, Question 12

Question:

(a) Sketch the graph of $y = \frac{1}{x}$, $x > 0$.

(b) Copy and complete the table, giving your values of $\frac{1}{x}$ to 3 decimal places.

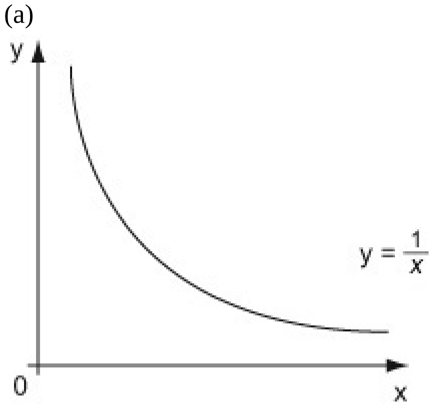
x	1	1.2	1.4	1.6	1.8	2
$\frac{1}{x}$	1					0.5

(c) Use the trapezium rule, with all the values from your table, to find an estimate for the value of $\int_1^2 \frac{1}{x} dx$.

(d) Is this an overestimate or an underestimate for the value of $\int_1^2 \frac{1}{x} dx$? Give a reason for your answer.

Solution:

(a)



Draw the sketch for $x > 0$.

(b)

x	1	1.2	1.4	1.6	1.8	2
$\frac{1}{x}$	1	0.833	0.714	0.625	0.556	0.5

(c)

Area $\doteq \frac{1}{2} \times 0.2 \times [1 + 2 (0.833 + 0.714 + 0.625 + 0.556) + 0.5]$

$\doteq 0.6956$

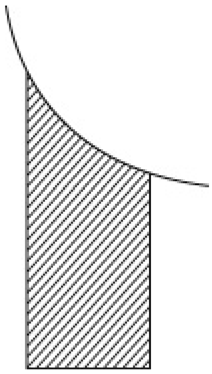
$\doteq 0.70$.

The width of each strip is 0.2

The values of $\frac{1}{x}$ are to 3 decimal places, so give the final answer to 2 decimal places.

(d)

This is an overestimate.



Due to the shape of the curve, each trapezium will give an area slightly larger than the area under the graph.

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Algebra and functions
Exercise A, Question 13

Question:

Show that the stationary point on the curve $y = 4x^3 - 6x^2 + 3x + 2$ is a point of inflexion.

Solution:

$y = 4x^3 - 6x^2 + 3x + 2$

$\frac{dy}{dx} = 12x^2 - 12x + 3$

Find the stationary point. Put $\frac{dy}{dx} = 0$

$12x^2 - 12x + 3 = 0$

$4x^2 - 4x + 1 = 0$

$(2x - 1)(2x - 1) = 0$

$x = \frac{1}{2}$

Simplify. Divide throughout by 4. Factorise
$4x^2 - 4x + 1$
$ac = 4$, and $(-2) + (-2) = -4$
so $4x^2 - 2x - 2x + 1$
$= 2x(2x - 1) - 1(2x - 1)$
$= (2x - 1)(2x - 1)$

When $x = 0$,

$\frac{dy}{dx} = 12(0) - 12(0) + 3$

$= 3 > 0$

Find the gradient of the tangent when $x = 0$ and $x = 1$.

When $x = 1$

$\frac{dy}{dx} = 12(1)^2 - 12(1) + 3$

$= 3 > 0$

x	0	$\frac{1}{2}$	1
$\frac{dy}{dx}$	>0	0	>0
Shape of curve	/	—	/

Look at the gradient of the tangent on either side of the stationary point.

The Stationary point is a point of inflexion.

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Algebra and functions
Exercise A, Question 14

Question:

Find all the values of x in the interval $0 \le x < 360°$ for which $3\tan^2 x = 1$.

Solution:

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$ Rearrange the equation for tan x. Divide each side by 3.

$\tan x = \pm\sqrt{\frac{1}{3}}$ Take the square root of each side.

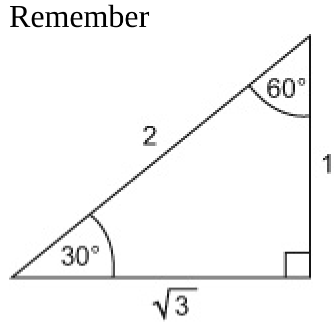
(i)

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = 30°$

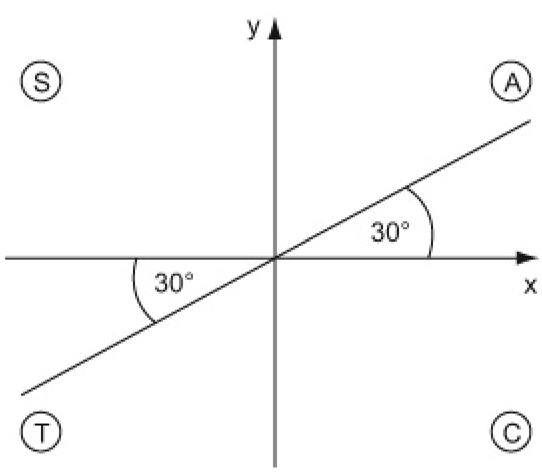
$\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

Remember



so $\tan 30° = \frac{1}{\sqrt{3}}$

Tan x is positive in the 1st and 3rd quadrants.

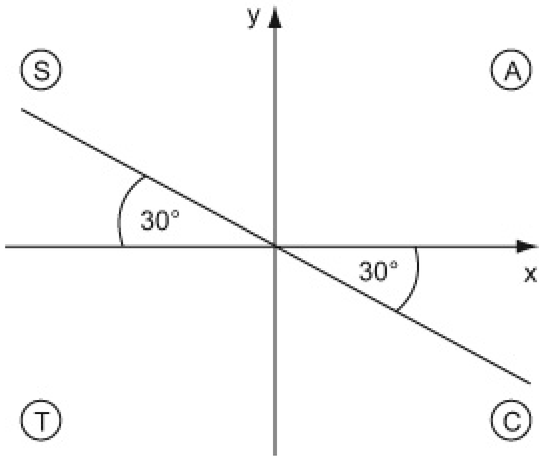


so , $x = 30°, 210°$

(ii)

$\tan x = -\frac{1}{\sqrt{3}}$

$x = 330°$ (i.e. $-30°$)



Tan x is negative in the 2nd and 4th quadrants.

$x = 330°, 150°$

Write down all the solutions in $0 \leq x < 360°$ in order of size.

so, $x = 30°, 150°, 210°, 330°$

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Algebra and functions

Exercise A, Question 15

Question:

Evaluate $\int_1^8 x^{\frac{1}{3}} - x^{-\frac{1}{3}} \, dx$.

Solution:

$\int_1^8 x^{\frac{1}{3}} - x^{\frac{-1}{3}} \, dx$

$= \left[\frac{3}{4} x^{\frac{4}{3}} - \frac{3}{2} x^{\frac{2}{3}} \right]_1^8$

Remember $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

$\int x^{\frac{1}{3}} dx = \frac{x^{\frac{4}{3}}}{\left(\frac{4}{3} \right)} = \frac{3}{4} x^{\frac{4}{3}}$

$\int x^{-\frac{1}{3}} dx = \frac{x^{\frac{2}{3}}}{\left(\frac{2}{3} \right)} = \frac{3}{2} x^{\frac{2}{3}}$

$= \left(\frac{3}{4} (8)^{\frac{4}{3}} - \frac{3}{2} (8)^{\frac{2}{3}} \right) - \left(\frac{3}{4} (1)^{\frac{4}{3}} - \frac{3}{2} (1)^{\frac{2}{3}} \right)$

$(8)^{\frac{4}{3}} = \left(8^{\frac{1}{3}} \right)^4$

$= \left(\sqrt[3]{8} \right)^4$

$= 2^4$

$= 16$

$= \left(\frac{3}{4} (16) - \frac{3}{2} (4) \right) - \left(\frac{3}{4} (1) - \frac{3}{2} (1) \right)$

$= (12 - 6) - \left(\frac{3}{4} - \frac{3}{2} \right)$

$= 6 \frac{3}{4}$

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Algebra and functions
Exercise A, Question 16

Question:

The curve C has equation $y = 2x^3 - 13x^2 + 8x + 1$.

(a) Find the coordinates of the turning points of C.

(b) Determine the nature of the turning points of C.

Solution:

$y = 2x^3 - 13x^2 + 8x + 1$

Find the x-coordinate. Solve $\frac{dy}{dx} = 0$.

(a)

$\frac{dy}{dx} = 6x^2 - 26x + 8$

$6x^2 - 26x + 8 = 0$

Divide throughout by 2.

$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

Factorize $3x^2 - 13x + 4 = 0$. $ac = 12$, $(-12) + (-1) = -13$
so $3x^2 - 12x - x + 4$
$= 3x(x - 4) - 1(x - 4)$
$= (3x - 1)(x - 4)$

$x = \frac{1}{3}$, $x = 4$

When $x = \frac{1}{3}$,

Find the y-coordinates. Substitute $x = \frac{1}{3}$ and $x = 4$ into $y = 2x^3 - 13x^2 + 8x + 1$

$$y = 2\left(\frac{1}{3}\right)^3 - 13\left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) + 1$$
$$= 2\frac{8}{27}$$

When $x = 4$

$$y = 2(4)^3 - 13(4)^2 + 8(4) + 1$$
$$= -47$$

so $\left(\frac{1}{3}, 2\frac{8}{27}\right)$, $(4, -47)$.

Give your answer as coordinates

(b)

$\frac{d^2y}{dx^2} = 12x - 26$

Remember $\frac{d^2y}{dx^2} < 0$ is a maximum stationary point, and

When $x = \frac{1}{3}$, $\frac{d^2y}{dx^2} > 0$ is a minimum stationary point.

$$\frac{d^2y}{dx^2} = 12 \left(\frac{1}{3} \right) - 26$$

$$= -22 < 0$$

$\left(\frac{1}{3} , 2 \frac{8}{27} \right)$ is a maximum.

When $x = 4$,

$$\frac{d^2y}{dx^2} = 12 (4) - 26$$

$$= 22 > 0$$

$(4 , - 47)$ is a minimum.

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Algebra and functions
Exercise A, Question 17

Question:

The curve S, for $0 \le x < 360°$, has equation $y = 2\sin\left(\frac{2}{3}x - 30°\right)$.

(a) Find the coordinates of the point where S meets the y-axis.

(b) Find the coordinates of the points where S meets the x-axis.

Solution:

$y = 2\sin\left(\frac{2}{3}x - 30°\right)$

(a)

$x = 0$

$y = 2\sin\left(\frac{2}{3}(0) - 30°\right)$

$= 2\sin(-30°)$

$= -2$

so, $(0, -2)$

The curve $y = 2\sin\left(\frac{2x}{3} - 30°\right)$ meets the y-axis when $x = 0$, so substitute $x = 0$ into $y = 2\sin\left(\frac{2x}{3} - 30°\right)$.

(b)

$y = 0$

$2\sin\left(\frac{2}{3}x - 30°\right) = 0$

$\sin\left(\frac{2}{3}x - 30°\right) = 0$

$\frac{2}{3}x - 30° = 0°, 180°, 360°,$

The curve $y = 2\sin\left(\frac{2x}{3} - 30°\right)$ meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 2\sin\left(\frac{2x}{3} - 30°\right)$.

(i)

$\frac{2}{3}x - 30° = 0$

$\frac{2}{3}x = 30°$

$x = 45°$

Let $\frac{2}{3}x - 30° = X$ so $\sin X = 0$ Now, $X = 0, 180°, 360°$ Solve for x: $X = 0$, so $\frac{2x}{3} - 30° = 0$.

(ii)

$X = 180°$, so $\frac{2x}{3} - 30° = 180°$.

$\frac{2}{3}x - 30° = 180°$

$\frac{2}{3}x = 210°$

$x = 315°$

(iii)

$\frac{2}{3}x - 30° = 360°$

$\frac{2}{3}x = 390°$

$x = 585°$

X = 360°, so $\frac{2x}{3} - 30° = 360°$.

Solution not in $0 \leq x < 360°$.

so $(45°, 0)$, $(315°, 0)$

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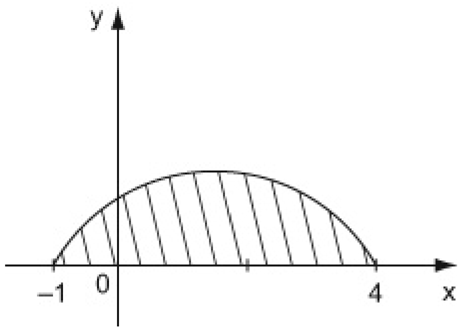
Algebra and functions
Exercise A, Question 18

Question:

Find the area of the finite region bounded by the curve $y = (1 + x)(4 - x)$ and the x-axis.

Solution:



Sketch a graph of the curve $y = (1 + x)(4 - x)$.
Find where the curve meets the x-axis.
The curve meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = (1 + x)(4 - x)$
$(1 + x)(4 - x) = 0$ so $x = -1$ and $x = 4$.

Area $= \int_{-1}^{4} (1 + x)(4 - x) \, dx$

Find the area under the graph and the x-axis. Integrate $y = (1 + x)(4 - x)$ using $x = -1$ and $x = 4$ as the limits of the integration.

$= \int_{-1}^{4} 4 + 3x - x^2 \, dx$

Expand $(1 + x)(4 - x)$.
$(1 + x)(4 - x) = 4 - x + 4x - x^2$
$= 4 + 3x - x^2$

$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^{4}$

Remember $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

$= \left(4(4) + \frac{3}{2}(4)^2 - \frac{(4)^3}{3} \right) - \left(4(-1) + \frac{3}{2}(-1)^2 - \frac{(-1)^3}{3} \right)$

Evaluate the integral. Substitute $x = 4$, then $x = -1$ into $4x + \frac{3x^2}{2} - \frac{x^3}{3}$ and subtract.

$= 18\frac{2}{3} - \left(-2\frac{1}{6} \right)$

$18^{2/3} - \left(-2\frac{1}{6} \right) = 18^{2/3} + 2^{1/6}$
$= 20^{5/6}$

$= 20^{5/6}$

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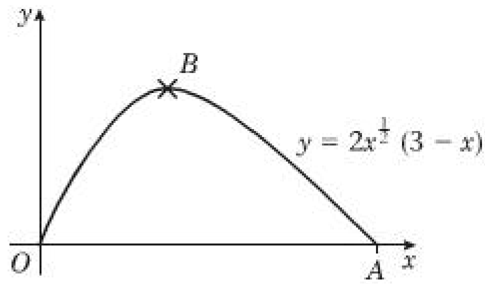
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise A, Question 19

Question:

The diagram shows part of the curve with equation $y = 2x^{\frac{1}{2}}(3 - x)$. The curve meets the x-axis at the points O and A. The point B is the maximum point of the curve.



(a) Find the coordinates of A.

(b) Show that $\frac{dy}{dx} = 3x^{-\frac{1}{2}}(1 - x)$.

(c) Find the coordinates of B.

Solution:

$y = 2x^{\frac{1}{2}}(3 - x)$

(a)

$2x^{\frac{1}{2}}(3 - x) = 0$

The curve meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 2x^{\frac{1}{2}}(3 - x)$.

(i)

$x^{\frac{1}{2}} = 0$

$x = 0$

(ii)

$3 - x = 0$

$x = 3$

so $A(3, 0)$.

Remember $\frac{d}{dx}(ax^n) = anx^{n-1}$

(b)

Expand the brackets.

$y = 2x^{\frac{1}{2}}(3 - x)$

$= 6x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$

$2x^{\frac{1}{2}} \times 3 = 6x^{\frac{1}{2}}$

$2x^{\frac{1}{2}} \times x = 2x^{\frac{1}{2}} \times x^1$

$= 2x^{\frac{1}{2}+1}$

$= 2x^{\frac{3}{2}}$

$\frac{dy}{dx} = 3x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$

Differentiate.

$\frac{d}{dx}\left(6x^{\frac{1}{2}}\right) = 6 \times \frac{1}{2} \times x^{\frac{1}{2}-1}$

$= 3x^{-\frac{1}{2}}$

$\frac{d}{dx}\left(2x^{\frac{3}{2}}\right) =$ <semantics>$2 \times \frac{3}{2}$</semantics> $\times x^{\frac{3}{2}-1}$

$= 3x^{\frac{1}{2}}$

$= 3x^{-\frac{1}{2}}(1 - x)$ as required

Factorise. Divide each term by $3x^{-\frac{1}{2}}$ so that

$3x^{-\frac{1}{2}} \div 3x^{-\frac{1}{2}} = 1$

$3x^{\frac{1}{2}} \div 3x^{-\frac{1}{2}} =$ <semantics>$\frac{3x^{\frac{1}{2}}}{3x^{-\frac{1}{2}}}$</semantics>

$= x^{\frac{1}{2}-\left(\frac{-1}{2}\right)}$

$= x^{\frac{1}{2}+\frac{1}{2}}$

$= x^1 = x$

(c)

$3x^{-\frac{1}{2}}(1 - x) = 0$

$1 - x = 0$

$x = 1$

When $x = 1$,

$y = 2(1)^{\frac{1}{2}}(3 - (1))$

$= 2 \times 1 \times 2$

$= 4$

so $B(1, 4)$.

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Algebra and functions
Exercise A, Question 20

Question:

(a) Show that the equation $2\cos^2 x = 4 - 5\sin x$ may be written as $2\sin^2 x - 5\sin x + 2 = 0$.

(b) Hence solve, for $0 \leq \theta < 360°$, the equation $2\cos^2 x = 4 - 5\sin x$.

Solution:

(a)

$2\cos^2 x = 4 - 5\sin x$

$2(1 - \sin^2 x) = 4 - 5\sin x$

$2 - 2\sin^2 x = 4 - 5\sin x$

$2\sin^2 x - 5\sin x + 2 = 0$ (as required)

Remember $\cos^2 x + \sin^2 x = 1$ so $\cos^2 x = 1 - \sin^2 x$.

(b)

Let $\sin x = y$

$2y^2 - 5y + 2 = 0$

$(2y - 1)(y - 2) = 0$

Factorise $2y^2 - 5y + 2 = 0$
$ac = 4$, $(-1) + (-4) = -5$
so $2y^2 - 5y + 2 = 2y^2 - y - 4y + 2$
$= y(2y - 1) - 2(2y - 1)$
$= (2y - 1)(y - 2)$

so $y = \frac{1}{2}$, $y = 2$

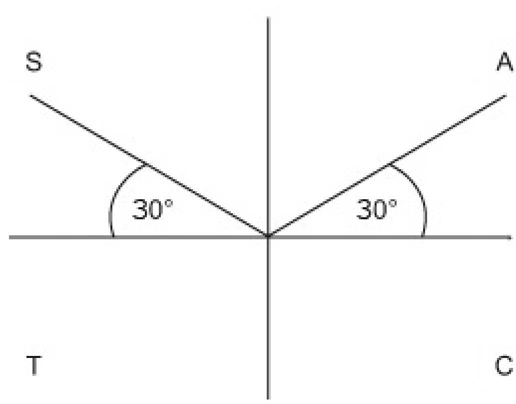
(i)

$\sin x = \frac{1}{2}$

$x = 30°, 150°$

Solve for x.

Substitute (i) $y = \frac{1}{2}$ and (ii) $y = 2$ into $\sin x = y$.



$\sin x$ is positive in the 1st and 2nd quadrants. Read off the solutions in $0 \leq x < 360°$.

(ii)

$\sin x = 2$ (Impossible) No solutions exist as $-1 \le \sin x \le 1$.

so $x = 30°, 150°$

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Algebra and functions
Exercise A, Question 21

Question:

Use the trapezium rule with 5 equal strips to find an estimate for $\int_0^1 x\sqrt{(1+x)}\,dx$.

Solution:

x	0	0.2	0.4	0.6	0.8	1
$x\sqrt{(1+x)}$	0	0.219	0.473	0.759	1.073	1.414

Divide the interval into 5 equal strips.
Use $h = \frac{b-a}{n}$. Here $b = 1$, $a = 0$ and $n = 5$. So that $h = \frac{1-0}{5} = 0.2$

The trapezium rule gives an approximation to the area of the graph. Here we work to an accuracy of 3 decimal places.

$$\int_0^1 x\sqrt{(1+x)}\,dx \simeq \frac{1}{2} \times 0.2 \times [0 + 2(0.219 + 0.473 + 0.759 + 1.073) + 1.414]$$

$$\simeq 0.6462 \text{ or } 0.65$$

Remember $A \simeq \frac{1}{2}h[y_0 + 2(y_1 + y_2 + \ldots) + y_n]$

The values of $x\sqrt{1+x}$ are to 3 decimal places, so give your final answer to 2 decimal places.

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Algebra and functions
Exercise A, Question 22

Question:

A sector of a circle, radius r cm, has a perimeter of 20 cm.

(a) Show that the area of the sector is given by $A = 10r - r^2$.

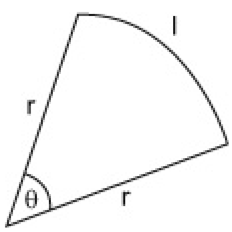
(b) Find the maximum value for the area of the sector.

Solution:

(a)

Remember: The length of an arc of a circle is $l = r\theta$. The area of a sector of a circle is $A = \frac{1}{2}r^2\theta$.

Draw a diagram. Let θ be the center angle and l be the arc length.



$2r + l = 20$

$l = r\theta$

so $2r + r\theta = 20$

$r\theta = 20 - 2r$

$\theta = \frac{20}{r} - 2$

The perimeter of the sector is $r + r + l = 2r + l$ so $2r + l = 20$

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}r^2\left(\frac{20}{r} - 2\right)$$

$$= \frac{1}{2}r^2 \times \frac{20}{r} - \frac{1}{2}r^2 \times 2$$

$$= 10r - r^2 \text{ (as required)}$$

Expand the brackets and simplify.

$$\frac{1}{2}r^2 \times \frac{20}{r} = \frac{1}{2} \times 20 \times \frac{r^2}{r}$$

$$= 10r^{2-1}$$

$$= 10r^1 = 10r$$

$$\frac{1}{2}r^2 \times 2 = 2 \times \frac{1}{2}r^2$$

$$= r^2$$

(b)

Find the value of r for the area to have a maximum. Solve $\frac{dA}{dr} = 0$.

$\frac{dA}{dr} = 10 - 2r$

$10 - 2r = 0$

$2r = 10$

$r = 5$

when $r = 5$

Area $= 10(5) - 5^2$ Find the maximum area. Substitute $r = 5$ into $A = 10r - r^2$

$= 50 - 25$

$= 25 \text{ cm}^2$

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Algebra and functions

Exercise A, Question 23

Question:

Show that, for all values of x:

(a) $\cos^2 x \left(\tan^2 x + 1 \right) = 1$

(b) $\sin^4 x - \cos^4 x = \left(\sin x - \cos x \right) \left(\sin x + \cos x \right)$

Solution:

(a)

$\cos^2 x \left(\tan^2 x + 1 \right)$

$= \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1 \right)$

Remember $\tan x = \frac{\sin x}{\cos x}$ so $\tan^2 x = \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos^2 x}$

$= \cos^2 x \times \frac{\sin^2 x}{\cos^2 x} + \cos^2 x \times 1$

Expand the brackets and simplify.

$\cos^2 x \times \frac{\sin^2 x}{\cos^2 x}$ = <semantics>$\cancel{\cos^2 x} \times \frac{\sin^2 x}{\cancel{\cos^2 x}}$</semantics>

$= \sin^2 x$

$= \sin^2 x + \cos^2 x$

Remember $\sin^2 x + \cos^2 x = 1$

$= 1$ (as required)

(b)

$\sin^4 x - \cos^4 x$

$= \left(\sin^2 x - \cos^2 x \right) \left(\sin^2 x + \cos^2 x \right)$

Remember $a^2 - b^2 = \left(a - b \right) \left(a + b \right)$. Here $a = \sin^2 x$ and $b = \cos^2 x$

$= \left(\sin^2 x - \cos^2 x \right) \times 1$

Remember $\sin^2 x + \cos^2 x = 1$

$= \sin^2 x - \cos^2 x$

$= \left(\sin x - \cos x \right) \left(\sin x + \cos x \right)$ (as required)

Use $a^2 - b^2 = \left(a - b \right) \left(a + b \right)$ again. Here $a = \sin x$ and $b = \cos x$.

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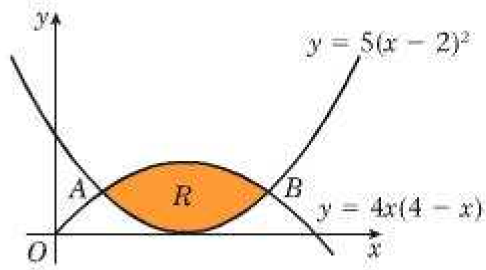
Algebra and functions
Exercise A, Question 24

Question:

The diagram shows the shaded region R which is bounded by the curves $y = 4x(4 - x)$ and $y = 5(x - 2)^2$.



The curves intersect at the points A and B.

(a) Find the coordinates of the points A and B.

(b) Find the area of the shaded region R.

Solution:

$y = 4x(4 - x)$, $y = 5(x - 2)^2$

$4x(4 - x) = 5(x - 2)^2$

Solve the equations $y = 4x(4 - x)$ and $y = 5(x - 2)^2$ simultaneously. Eliminate y so that $4x(4 - x) = 5(x - 2)^2$.

$16x - 4x^2 = 5(x^2 - 4x + 4)$

$16x - 4x^2 = 5x^2 - 20x + 20$

Expand the brackets and simplify.

$9x^2 - 36x + 20 = 0$

Rearrange the equation into the form $ax^2 + bx + c = 0$

$(3x - 10)(3x - 2) = 0$

$x = \frac{10}{3}$, $x = \frac{2}{3}$

Factorise $9x^2 - 36x + 20 = 0$
$ac = 180$, $(-6) + (-30) = -36$
$9x^2 - 6x - 30x + 20$
$= 3x(3x - 2) - 10(3x - 2)$
$= (3x - 2)(3x - 10)$.

(i)

When $x = \frac{10}{3}$,

$y = 4\left(\frac{10}{3}\right)\left(4 - \frac{10}{3}\right)$

$= 4 \times \frac{10}{3} \times \frac{2}{3}$

$= \frac{80}{3}$

Find the coordinator of A and B. Substitute (i) $x = \frac{10}{3}$ and (ii) $x = \frac{2}{3}$, into $y = 4x(4 - x)$.

(ii)

When $x = \frac{2}{3}$

Find the coordinator of A and B. Substitute (i) $x =$

$\frac{10}{3}$ and (ii) $x = \frac{2}{3}$, into $y = 4x(4 - x)$.

$y = 4\left(\frac{2}{3}\right)\left(4 - \frac{2}{3}\right)$

$= 4 \times \frac{2}{3} \times \frac{10}{3}$

$= \frac{80}{3}$

so $A\left(\frac{2}{3}, \frac{80}{3}\right)$, $B\left(\frac{10}{3}, \frac{80}{3}\right)$

(b)

$$\text{Area} = \int_{\frac{2}{3}}^{\frac{10}{3}} 4x(4 - x) - 5(x - 2)^2 dx$$

Remember Area $= \int_a^b (y_1 - y_2)\,dx$. Here $y_1 = 4x(4 - x)$, $y_2 = 5(x - 2)^2$, $a = \frac{2}{3}$ and $b = \frac{10}{3}$.

$$= \int_{\frac{2}{3}}^{\frac{10}{3}} 16x - 4x^2 - 5(x^2 - 4x + 4)\,dx$$

Expand the brackets and simplify.

$$= \int_{\frac{2}{3}}^{\frac{10}{3}} 16x - 4x^2 - 5x^2 + 20x - 20\,dx$$

Remember $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

$$= \int_{\frac{2}{3}}^{\frac{10}{3}} 36x - 9x^2 - 20\,dx$$

$$= \left[18x^2 - 3x^3 - 20x\right]_{\frac{2}{3}}^{\frac{10}{3}}$$

Evaluate the integral. Substitute $x = \frac{10}{3}$, then $x = \frac{2}{3}$, into $18x^2 - 3x^3 - 20x$ and subtract.

$$= \left(18\left(\frac{10}{3}\right)^2 - 3\left(\frac{10}{3}\right)^3 - 20\left(\frac{10}{3}\right)\right) - \left(18\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right)^3 - 20\left(\frac{2}{3}\right)\right)$$

$$= \left(18\left(\frac{100}{9}\right) - 3\left(\frac{1000}{27}\right) - \frac{200}{3}\right) - \left(18\left(\frac{4}{9}\right) - 3\left(\frac{8}{27}\right) - \frac{40}{3}\right)$$

$$= \left(22\frac{2}{9}\right) - \left(-6\frac{2}{9}\right)$$

$$= 28\frac{4}{9}$$

$22\frac{2}{9} - \left(-6\frac{2}{9}\right) = 22\frac{2}{9} + 6\frac{2}{9}$

$= 28\frac{4}{9}$

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Algebra and functions
Exercise A, Question 25

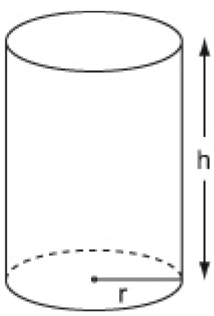
Question:

The volume of a solid cylinder, radius r cm, is 128π.

(a) Show that the surface area of the cylinder is given by $S = \frac{256\pi}{r} + 2\pi r^2$.

(b) Find the minimum value for the surface area of the cylinder.

Solution:



Draw a diagram. Let h be the height of cylinder.

(a)

Surface area, $S = 2\pi rh + 2\pi r^2$

(volume =)$128\pi = \pi r^2 h$

$$h = \frac{128\pi}{\pi r^2} = \frac{128}{r^2}$$

so $S = 2\pi r \times \frac{128}{r^2} + 2\pi r^2$

$= \frac{256\pi}{r} + 2\pi r^2$ (as required)

Find expressions for the surface area and volume of the cylinder in terms of π, r and h.

Eliminate h between the expressions $S = 2\pi rh + 2\pi r^2$ and $128\pi = \pi r^2 h$. Rearrange $128\pi = \pi r^2 h$ for h so that

$$\pi r^2 h = 128\pi$$
$$h = \frac{128\pi}{\pi r^2} = \frac{128}{r^2}$$

Substitute $h = \frac{128}{r^2}$ into $S = 2\pi rh + 2\pi r^2$ and simplify the expression.

(b)

$$\frac{\mathrm{d}s}{\mathrm{d}r} = 4\pi r - \frac{256\pi}{r^2}$$

Find the volume of r for which $S = \frac{256\pi}{r} + 2\pi r^2$ has a stationary value. Solve $\frac{\mathrm{d}s}{\mathrm{d}r} = 0$. Differentiate $\frac{256\pi}{r} + 2\pi r^2$ with respect to r, so that

$$4\pi r - \frac{256\pi}{r^2} = 0$$

$$4\pi r = \frac{256\pi}{r^2}$$

$$r^3 = 64$$

$$r = 4 \text{ cm}$$

$$\frac{d}{dr}\left(\frac{256\pi}{r}\right) = \frac{d}{dr}256\pi r^{-1}$$

$$= -256\pi r^{-1-1}$$

$$= -256\pi r^{-2}$$

$$= \frac{-256\pi}{r^2}$$

$$\frac{d}{dr}\left(2\pi r^2\right) = 2 \times 2\pi r^{2-1}$$

$$= 4\pi r^1$$

$$= 4\pi r$$

When $r = 4$,

$$S = \frac{256\pi}{(4)} + 2\pi(4)^2$$

$$= 64\pi + 32\pi$$

$$= 96\pi \text{ cm}^2$$

Find the value of S when $r = 4$. Substitute $r = 4$ into $S = \frac{256\pi}{r} + 2\pi r^2$.

Give the exact answer. Leave your answer in terms of π.

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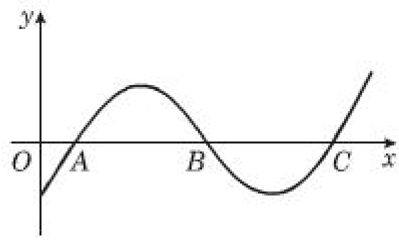
Algebra and functions
Exercise A, Question 26

Question:

The diagram shows part of the curve $y = \sin \left(ax - b \right)$, where a and b are constants and $b < \frac{\pi}{2}$.



Given that the coordinates of A and B are $\left(\frac{\pi}{6}, 0 \right)$ and $\left(\frac{5\pi}{6}, 0 \right)$ respectively,

(a) write down the coordinates of C,

(b) find the value of a and the value of b.

Solution:

(a)

AB = BC

AB is half the period, so AB = BC

$$AB = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

Find the coordinates of C. Work out the length of AB, AB = OB − OA. Work with exact values. Leave your answer in terms of π.

$$\text{so, OC} = \frac{5\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6} + \frac{4\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

OC = OB + BC and AB = BC. So, OC = OB + AB.

so, $\text{C} \left(\frac{3\pi}{2}, 0 \right)$

(b)

(i)

$$\sin \left(a \left(\frac{\pi}{6} \right) - b \right) = 0$$

so $a \left(\frac{\pi}{6} \right) - b = 0$

$\sin \left(0 \right) = 0$ and $\sin \left(\pi \right) = 0$. So, at A, $x = \frac{\pi}{6}$ and a (

$\frac{\pi}{6}) - b = 0$ and at B, $x = \frac{5\pi}{6}$ and $a(\frac{5\pi}{6}) - b = \pi$.

(ii)

$\sin(a(\frac{5\pi}{6}) - b) = 0$

so $a(\frac{5\pi}{6}) - b = \pi$

Solving Simultaneously

$a(\frac{5\pi}{6}) - b = \pi$

$-a(\frac{\pi}{6}) - b = 0$

$a(\frac{4\pi}{6}) = \pi$

$a = \frac{\pi}{(\frac{4\pi}{6})}$

$= \frac{6}{4}$

$= \frac{3}{2}$

Solve the equations $a(\frac{\pi}{6}) - b = 0$and $a(\frac{5\pi}{6}) - b = \pi$ simultaneously. Subtract the equations.

When $a = \frac{3}{2}$

$(\frac{3}{2})(\frac{\pi}{6}) - b = 0$

$b = \frac{\pi}{4}$

Find b. Substitute $a = \frac{3}{2}$ into $a(\frac{\pi}{6}) - b = 0$.

check

sub $a = \frac{3}{2}$, $b = \frac{\pi}{4}$ into

$a(5\frac{\pi}{6}) - b$

$(\frac{3}{2})(\frac{5\pi}{6}) - \frac{\pi}{4} = \frac{5\pi}{4} - \frac{\pi}{4}$

$= \pi$ (as required)

so $a = \frac{3}{2}$ and $b = \frac{\pi}{4}$.

Check answer by substituting $a = \frac{3}{2}$ and $b = \frac{\pi}{6}$ into a $(\frac{5\pi}{6}) - b$.

$(\frac{3}{2})(\frac{5\pi}{6}) = \frac{1}{2} \times \frac{5\pi}{2} = \frac{5\pi}{4}$

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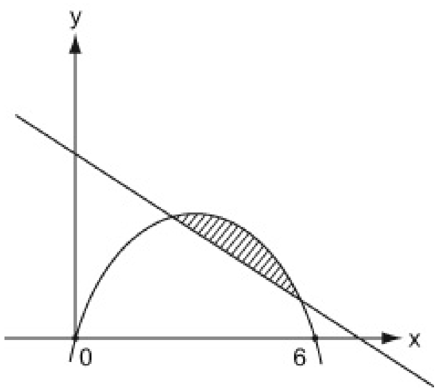
Algebra and functions
Exercise A, Question 27

Question:

Find the area of the finite region bounded by the curve with equation $y = x(6 - x)$ and the line $y = 10 - x$.

Solution:

$y = x(6 - x)$, $y = 10 - x$



$x(6 - x) = 10 - x$

$6x - x^2 = 10 - x$

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

so $x = 2$ and $x = 5$.

Find the x-coordinates of the points where the line $y = 10 - x$ meets the curve $y = x(6 - x)$. Solve the equations simultaneously, eliminate y.

Factorise $x^2 - 7x + 10$. $(-5) \times (-2) = 10$ and $(-5) + (-2) = -7$ so $x^2 - 7x + 10 = (x - 5)(x - 2)$.

Finite area $= \int_2^5 x(6 - x) - (10 - x)\,dx$

Use area $= \int_a^b (y_1 - y_2)\,dx$. Have $y_1 = x(6 - x)$, $y_2 = 10 - x$, $a = 2$ and $b = 5$.

$= \int_2^5 6x - x^2 - 10 + x\,dx$

Remember $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

$= \int_2^5 7x - x^2 - 10\,dx$

$= \left[\frac{7x^2}{2} - \frac{x^3}{3} - 10x \right]_2^5$

Evaluate the integral. Substitute $x = 5$, then $x = 2$, into $\frac{7x^2}{2} - \frac{x^3}{3} - 10x$ and subtract.

$= \left(\frac{7(5)^2}{2} - \frac{(5)^3}{3} - 10(5) \right) - \left(\frac{7(2)^2}{2} - \frac{(2)^3}{3} - 10(2) \right)$

$= \left(-4\frac{1}{6} \right) - \left(-8\frac{2}{3} \right)$

$= 4\frac{1}{2}$

$-4\frac{1}{6} - \left(-8\frac{2}{3} \right) = -4\frac{1}{6} + 8\frac{2}{3}$

$= 4\frac{1}{2}$

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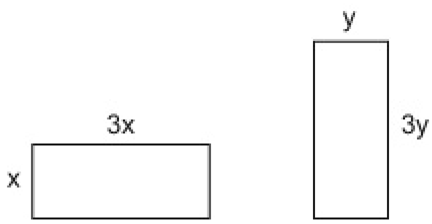
Algebra and functions
Exercise A, Question 28

Question:

A piece of wire of length 80 cm is cut into two pieces. Each piece is bent to form the perimeter of a rectangle which is three times as long as it is wide. Find the lengths of the two pieces of wire if the sum of the areas of the rectangles is to be a maximum.

Solution:



Draw a diagram. Let the width of each rectangle be x and y respectively.

Total length of wire $= 80$

so $80 = 8x + 8y$

Write down an equation in terms of x and y for the total length of the wire.

$x + y = 10$

Divide throughout by 8.

Total Area $= A$

$A = 3x^2 + 3y^2$

Write down an equation in terms of x and y for the total area enclosed by the two pieces of wire.

$$A = 3x^2 + 3(10 - x)^2$$
$$= 3x^2 + 3(100 - 20x + x^2)$$
$$= 3x^2 + 300 - 60x + 3x^2$$
$$= 6x^2 - 60x + 300$$

Solve the equations $x + y = 10$ and $A = 3x^2 + 3y^2$simultaneously. Eliminate y. Rearrange $x + y = 10$, so that $y = 10 - x$, and substitute into $A = 3x^2 + 3y^2$

$$\frac{dA}{dx} = 12x - 60$$

$12x - 60 = 0$

$12x = 6$

$x = 5$ cm

Find the value of x for which A is a maximum. Solve $\frac{dA}{dx} = 0$.

The length of each piece of wire is $(8x =)\ 40$ cm .

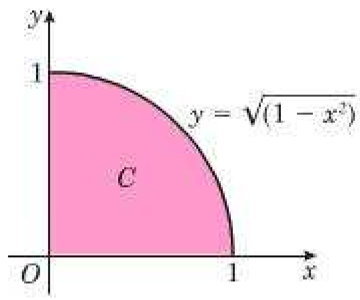
Total length is 80 cm, so $40 + 40 = 80$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise A, Question 29

Question:

The diagram shows the shaded region C which is bounded by the circle $y = \sqrt{(1 - x^2)}$ and the coordinate axes.



(a) Use the trapezium rule with 10 strips to find an estimate, to 3 decimal places, for the area of the shaded region C.

The actual area of C is $\frac{\pi}{4}$.

(b) Calculate the percentage error in your estimate for the area of C.

Solution:

Remember $A \simeq \frac{1}{2}h \left[y_0 + 2 (y_1 + y_2 + \cdots) + y_n \right]$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\sqrt{(1-x^2)}$	1	0.9950	0.9798	0.9539	0.9165	0.8660	0.8	0.7141	0.6	0.4359	0

Area $\simeq \frac{1}{2} \times 0.1 \times \left[1 + 2 (0.9950 + 0.9798 + \ldots + 0.4359) + 0 \right]$

Divide the interval into 10 equal stufs. Use $h = \frac{b-a}{n}$. Here $a = 0$, $b = 1$ and $n = 10$, so that $\frac{1-0}{10} = \frac{1}{10} = 0.1$

The trapezium rule gives an approximation to the area under the graph. Here we round to 4 decimal places.

$\simeq 0.77612$ or 0.776

The values of $\sqrt{1 - x^2}$are rounded to 4 decimal place. Give your final another to 3 decimal places.

(b)

$\frac{\pi}{4} =$

$$\% \text{ error } = \frac{\frac{\pi}{4} - 0.776}{\left(\frac{\pi}{4}\right)} \times 100$$

$$= 1.2\ \%$$

Use percentage error $=$ True value $\times 100$

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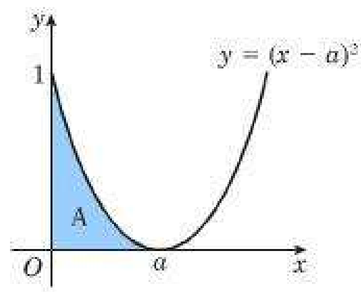
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise A, Question 30

Question:

The area of the shaded region A in the diagram is 9 cm^2. Find the value of the constant a.



Solution:

$\int_0^a (x-a)^2 dx = 9$

Write down an equation in terms of a for the area of region A.

$\int_0^a x^2 - 2ax + a^2 dx = 9$

Expand $(x-a)^2$ so that

$(x-a)(x-a) = x^2 - ax - ax + a^2$
$= x^2 - 2ax + a^2$

$\left[\frac{x^3}{3} - ax^2 + a^2 x \right]_0^a = 9$

Remember $\int ax^n dx = \frac{ax^{n+1}}{n+1}$.

Here

$\int x^2 dx = \frac{x^3}{3}$

$\int 2ax \, dx = \frac{2ax^2}{2}$

$= ax^2$

$\int a^2 dx = a^2 x$

$\left(\frac{(a)^3}{3} - a(a)^2 + a^2(a) \right) - \left(\frac{(0)^3}{3} - a(0)^2 + a^2(0) \right) = 9$

Evaluate the integral. Substitute $x = a$, then $x = 0$, into $\frac{x^3}{3} - ax^2 + a^2 x$, and subtract.

$\left(\frac{a^3}{3} - a^3 + a^3 \right) - 0 = 9$

$\frac{a^3}{3} = 9$

$a^3 = 27$

$a = 3$