Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Integration
Exercise A, Question 1

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

x^5

Solution:

$$\frac{dy}{dx} = x^5$$

$$y = \frac{x^6}{6} + c$$

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Integration
Exercise A, Question 2

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$10x^4$

Solution:

$\frac{dy}{dx} = 10x^4$

$y = 10 \frac{x^5}{5} + c$

$y = 2x^5 + c$

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Integration
Exercise A, Question 3

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$3x^2$

Solution:

$\frac{dy}{dx} = 3x^2$

$y = 3\frac{x^3}{3} + c$

$y = x^3 + c$

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Integration
Exercise A, Question 4

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-x^{-2}$

Solution:

$\frac{dy}{dx} = -x^{-2}$

$y = -\frac{x^{-1}}{-1} + c$

$y = x^{-1} + c$ or

$y = \frac{1}{x} + c$

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Integration
Exercise A, Question 5

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-4x^{-3}$

Solution:

$\frac{dy}{dx} = -4x^{-3}$

$y = -4\frac{x^{-2}}{-2} + c$

$y = 2x^{-2} + c$ or

$y = \frac{2}{x^2} + c$

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Integration
Exercise A, Question 6

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$x^{\frac{2}{3}}$

Solution:

$$\frac{dy}{dx} = x^{\frac{2}{3}}$$

$$y = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$y = \frac{3}{5}x^{\frac{5}{3}} + c$$

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Integration
Exercise A, Question 7

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$4x^{\frac{1}{2}}$

Solution:

$\frac{dy}{dx} = 4x^{\frac{1}{2}}$

$y = 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$y = \frac{8}{3}x^{\frac{3}{2}} + c$

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Integration
Exercise A, Question 8

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-2x^6$

Solution:

$\frac{dy}{dx} = -2x^6$

$y = -2\frac{x^7}{7} + c$

$y = -\frac{2}{7}x^7 + c$

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Integration
Exercise A, Question 9

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$3x^5$

Solution:

$\frac{dy}{dx} = 3x^5$

$y = 3\frac{x^6}{6} + c$

$y = \frac{1}{2}x^6 + c$

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Integration
Exercise A, Question 10

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$3x^{-4}$

Solution:

$\frac{dy}{dx} = 3x^{-4}$

$y = 3\frac{x^{-3}}{-3} + c$

$y = -x^{-3} + c$ or

$y = -\frac{1}{x^3} + c$

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Integration
Exercise A, Question 11

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$x^{-\frac{1}{2}}$

Solution:

$\frac{dy}{dx} = x^{-\frac{1}{2}}$

$y = \frac{x^{+\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 2x^{\frac{1}{2}} + c$ or

$y = 2\sqrt{x} + c$

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Integration
Exercise A, Question 12

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$5x^{-\frac{3}{2}}$

Solution:

$\frac{dy}{dx} = 5x^{-\frac{3}{2}}$

$y = 5\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$y = -10x^{-\frac{1}{2}} + c$ or

$y = \frac{-10}{\sqrt{x}} + c$

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Integration
Exercise A, Question 13

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-2x^{-\frac{3}{2}}$

Solution:

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}}$$

$$y = -2\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$y = 4x^{-\frac{1}{2}} + c$ or

$$y = \frac{4}{\sqrt{x}} + c$$

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Integration
Exercise A, Question 14

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$6x^{\frac{1}{3}}$

Solution:

$$\frac{dy}{dx} = 6x^{\frac{1}{3}}$$

$$y = 6\,\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$y = \frac{18}{4}x^{\frac{4}{3}} + c$$

$$y = \frac{9}{2}x^{\frac{4}{3}} + c$$

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Integration
Exercise A, Question 15

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$36x^{11}$

Solution:

$\frac{dy}{dx} = 36x^{11}$

$y = 36\frac{x^{12}}{12} + c$

$y = 3x^{12} + c$

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Integration
Exercise A, Question 16

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-14x^{-8}$

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Integration
Exercise A, Question 17

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$-3x^{-\frac{2}{3}}$

Solution:

$$\frac{dy}{dx} = -3x^{-\frac{2}{3}}$$

$$y = -3\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$y = -9x^{\frac{1}{3}} + c$$

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Integration
Exercise A, Question 18

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

-5

Solution:

$\frac{dy}{dx} = -5 = -5x^0$

$y = -5\frac{x^1}{1} + c$

$y = -5x + c$

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Integration
Exercise A, Question 19

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$6x$

Solution:

$\frac{dy}{dx} = 6x$

$y = 6 \frac{x^2}{2} + c$

$y = 3x^2 + c$

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Integration
Exercise A, Question 20

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$2x^{-0.4}$

Solution:

$\frac{dy}{dx} = 2x^{-0.4}$

$y = 2 \frac{x^{0.6}}{0.6} + c$

$y = \frac{20}{6} x^{0.6} + c$

$y = \frac{10}{3} x^{0.6} + c$

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Integration
Exercise B, Question 1

Question:

Find y when $\frac{dy}{dx}$ is given by the following expressions. In each case simplify your answer:

(a) $4x - x^{-2} + 6x^{\frac{1}{2}}$

(b) $15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

(c) $x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

(d) $4x^3 + x^{-\frac{2}{3}} - x^{-2}$

(e) $4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

(f) $5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

(g) $-\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

(h) $5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

Solution:

(a) $\frac{dy}{dx} = 4x - x^{-2} + 6x^{\frac{1}{2}}$

$y = 4\frac{x^2}{2} - \frac{x^{-1}}{-1} + 6\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$y = 2x^2 + x^{-1} + 4x^{\frac{3}{2}} + c$

(b) $\frac{dy}{dx} = 15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

$y = 15\frac{x^3}{3} + 6\frac{x^{-2}}{-2} - 3\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + c$

$y = 5x^3 - 3x^{-2} + 2x^{-\frac{3}{2}} + c$

(c) $\frac{dy}{dx} = x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

$y = \frac{x^4}{4} - \frac{3}{2}\frac{x^{+\frac{1}{2}}}{\frac{1}{2}} - 6\frac{x^{-1}}{-1} + c$

$y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$

(d) $\frac{dy}{dx} = 4x^3 + x^{-\frac{2}{3}} - x^{-2}$

$y = 4\frac{x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c$

$y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$

(e) $\frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

$y = 4x - 12\frac{x^{-3}}{-3} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$

(f) $\frac{dy}{dx} = 5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

$y = 5\frac{x^{\frac{5}{3}}}{\frac{5}{3}} - 10\frac{x^5}{5} + \frac{x^{-2}}{-2} + c$

$y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$

(g) $\frac{dy}{dx} = -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

$y = -\frac{4}{3}\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + 8\frac{x^2}{2} + c$

$y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$

(h) $\frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

$$y = 5\,\frac{x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 12\,\frac{x^{-4}}{-4} + c$$

$$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$$

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Integration
Exercise B, Question 2

Question:

Find f(x) when f′(x) is given by the following expressions. In each case simplify your answer:

(a) $12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

(b) $6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

(c) $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

(d) $10x + 8x^{-3}$

(e) $2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

(f) $9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

(g) $x^2 + x^{-2} + x^{\frac{1}{2}}$

(h) $-2x^{-3} - 2x + 2x^{\frac{1}{2}}$

Solution:

(a) $f'(x) = 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

$f(x) = 12\frac{x^2}{2} + \frac{3}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c$

$f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$

(b) $f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

$f(x) = 6\frac{x^6}{6} + 6\frac{x^{-6}}{-6} - \frac{1}{6}\frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$

$f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$

(c) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$f(x) = \frac{1}{2}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$

(d) $f'(x) = 10x + 8x^{-3}$

$f(x) = 10\frac{x^2}{2} + 8\frac{x^{-2}}{-2} + c$

$f(x) = 5x^2 - 4x^{-2} + c$

(e) $f'(x) = 2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

$f(x) = 2\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 4\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + c$

$f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$

(f) $f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

$f(x) = 9\frac{x^3}{3} + 4\frac{x^{-2}}{-2} + \frac{1}{4}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$

(g) $f'(x) = x^2 + x^{-2} + x^{\frac{1}{2}}$

$f(x) = \frac{x^3}{3} + \frac{x^{-1}}{-1} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$f(x) = \frac{1}{3}x^3 - x^{-1} + \frac{2}{3}x^{\frac{3}{2}} + c$

(h) $f'(x) = -2x^{-3} - 2x + 2x^{\frac{1}{2}}$

$f(x) = -2\frac{x^{-2}}{-2} - 2\frac{x^2}{2} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$f(x) = x^{-2} - x^2 + \frac{4}{3}x^{\frac{3}{2}} + c$

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Integration
Exercise C, Question 1

Question:

Find the following integral:

$\int \left(x^3 + 2x \right) dx$

Solution:

$\int \left(x^3 + 2x \right) dx$

$= \frac{x^4}{4} + 2 \frac{x^2}{2} + c$

$= \frac{1}{4}x^4 + x^2 + c$

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Integration
Exercise C, Question 2

Question:

Find the following integral:

$\int (2x^{-2} + 3) \, dx$

Solution:

$\int (2x^{-2} + 3) \, dx$

$= 2 \frac{x^{-1}}{-1} + 3x + c$

$= -2x^{-1} + 3x + c$

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Integration
Exercise C, Question 3

Question:

Find the following integral:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

Solution:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 3 \frac{x^3}{3} + c$$

$$= 2x^{\frac{5}{2}} - x^3 + c$$

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Integration
Exercise C, Question 4

Question:

Find the following integral:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

Solution:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

$$= 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 4x + c$$

$$= \frac{4}{3} x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$$

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Integration
Exercise C, Question 5

Question:

Find the following integral:

$\int (4x^3 - 3x^{-4} + r) \, dx$

Solution:

$\int (4x^3 - 3x^{-4} + r) \, dx$

$= 4 \frac{x^4}{4} - 3 \frac{x^{-3}}{-3} + rx + c$

$= x^4 + x^{-3} + rx + c$

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Integration
Exercise C, Question 6

Question:

Find the following integral:

$\int \left(3t^2 - t^{-2} \right) \mathrm{d}t$

Solution:

$\int \left(3t^2 - t^{-2} \right) \mathrm{d}t$

$= 3 \frac{t^3}{3} - \frac{t^{-1}}{-1} + c$

$= t^3 + t^{-1} + c$

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Integration
Exercise C, Question 7

Question:

Find the following integral:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

Solution:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

$$= 2\,\frac{t^3}{3} - 3\,\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$$

$$= \frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$$

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Integration
Exercise C, Question 8

Question:

Find the following integral:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

Solution:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

$$= \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$= \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

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Integration
Exercise C, Question 9

Question:

Find the following integral:

$\int (px^4 + 2t + 3x^{-2}) \, dx$

Solution:

$\int (px^4 + 2t + 3x^{-2}) \, dx$

$= p \frac{x^5}{5} + 2tx + 3 \frac{x^{-1}}{-1} + c$

$= \frac{p}{5} x^5 + 2tx - 3x^{-1} + c$

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Integration
Exercise C, Question 10

Question:

Find the following integral:

$\int (pt^3 + q^2 + px^3) \, dt$

Solution:

$\int (pt^3 + q^2 + px^3) \, dt$

$= p \frac{t^4}{4} + q^2 t + px^3 t + c$

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Integration
Exercise D, Question 1

Question:

Find the following integrals:

(a) $\int (2x + 3) x^2 dx$

(b) $\int \frac{(2x^2 + 3)}{x^2} dx$

(c) $\int (2x + 3)^2 dx$

(d) $\int (2x + 3) (x - 1) dx$

(e) $\int (2x + 3) \sqrt{x} dx$

Solution:

(a) $\int (2x + 3) x^2 dx$
$= \int (2x^3 + 3x^2) dx$
$= 2 \frac{x^4}{4} + 3 \frac{x^3}{3} + c$

$= \frac{1}{2} x^4 + x^3 + c$

(b) $\int \frac{(2x^2 + 3)}{x^2} dx$

$= \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2} \right) dx$

$= \int (2 + 3x^{-2}) dx$

$= 2x + 3 \frac{x^{-1}}{-1} + c$

$= 2x - 3x^{-1} + c$

or $= 2x - \frac{3}{x} + c$

(c) $\int (2x + 3)^2 dx$
$= \int (4x^2 + 12x + 9) dx$
$= 4 \frac{x^3}{3} + 12 \frac{x^2}{2} + 9x + c$

$= \frac{4}{3} x^3 + 6x^2 + 9x + c$

(d) $\int (2x + 3) (x - 1) dx$
$= \int (2x^2 + x - 3) dx$
$= 2 \frac{x^3}{3} + \frac{x^2}{2} - 3x + c$

$= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$

(e) $\int (2x + 3) \sqrt{x} \, dx$

$= \int \left(2x + 3 \right) x^{\frac{1}{2}} dx$

$= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx$

$= 2 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

or $= \frac{4}{5}\sqrt{x^5} + 2\sqrt{x^3} + c$

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Integration
Exercise D, Question 2

Question:

Find $\int f(x)dx$ when $f(x)$ is given by the following:

(a) $(x + 2)^2$

(b) $\left(x + \frac{1}{x} \right)^2$

(c) $(\sqrt{x} + 2)^2$

(d) $\sqrt{x} (x + 2)$

(e) $\left(\frac{x + 2}{\sqrt{x}} \right)$

(f) $\left(\frac{1}{\sqrt{x}} + 2 \sqrt{x} \right)$

Solution:

(a) $\int (x + 2)^2 dx$
$= \int (x^2 + 4x + 4) dx$
$= \frac{1}{3}x^3 + \frac{4}{2}x^2 + 4x + c$
$= \frac{1}{3}x^3 + 2x^2 + 4x + c$

(b) $\int \left(x + \frac{1}{x} \right)^2 dx$
$= \int \left(x^2 + 2 + \frac{1}{x^2} \right) dx$
$= \int (x^2 + 2 + x^{-2}) dx$
$= \frac{1}{3}x^3 + 2x + \frac{x^{-1}}{-1} + c$
$= \frac{1}{3}x^3 + 2x - x^{-1} + c$
or $= \frac{1}{3}x^3 + 2x - \frac{1}{x} + c$

(c) $\int (\sqrt{x} + 2)^2 dx$
$= \int (x + 4 \sqrt{x} + 4) dx$

$= \int \left(x + 4x^{\frac{1}{2}} + 4 \right) dx$

$= \frac{1}{2}x^2 + 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$

$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

(d) $\int \sqrt{x}\,(x + 2)\,dx$

$= \int \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

or $= \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + c$

(e) $\int \left(\frac{x + 2}{\sqrt{x}} \right) dx$

$= \int \left(\frac{x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} \right) dx$

$= \int \left(x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$

or $= \frac{2}{3}\sqrt{x^3} + 4\sqrt{x} + c$

(f) $\int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \right) dx$

$= \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

or $= 2\sqrt{x} + \frac{4}{3}\sqrt{x^3} + c$

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Integration
Exercise D, Question 3

Question:

Find the following integrals:

(a) $\int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$

(b) $\int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$

(c) $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$

(d) $\int \left(\frac{2+x}{x^3} + 3 \right) dx$

(e) $\int (x^2 + 3)(x - 1)\, dx$

(f) $\int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$

(g) $\int (x - 3)^2 dx$

(h) $\int \frac{(2x+1)^2}{\sqrt{x}} dx$

(i) $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) dx$

(j) $\int \sqrt{x} (\sqrt{x} + 3)^2 dx$

Solution:

(a) $\int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$

$= \int \left(3x^{\frac{1}{2}} + x^{-2} \right) dx$

$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + c$

$= 2x^{\frac{3}{2}} - x^{-1} + c$

or $= 2\sqrt{x^3} - \frac{1}{x} + c$

(b) $\int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$

$= \int \left(2x^{-\frac{1}{2}} + 3x^2 \right) dx$

$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3}{3}x^3 + c$

$= 4x^{\frac{1}{2}} + x^3 + c$

or $= 4\sqrt{x} + x^3 + c$

(c) $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$

$= \int \left(x^{\frac{2}{3}} + 4x^{-3} \right) dx$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 4\frac{x^{-2}}{-2} + c$

$= \frac{3}{5}x^{\frac{5}{3}} - 2x^{-2} + c$

or $= \frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$

(d) $\int \left(\frac{2+x}{x^3} + 3 \right) dx$

$= \int (2x^{-3} + x^{-2} + 3) dx$

$= 2\frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$

$= -x^{-2} - x^{-1} + 3x + c$

or $= -\frac{1}{x^2} - \frac{1}{x} + 3x + c$

(e) $\int (x^2 + 3) (x - 1) dx$

$= \int (x^3 - x^2 + 3x - 3) dx$

$= \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$

(f) $\int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$

$= \int \left(2x^{-\frac{1}{2}} + 3x^{\frac{3}{2}} \right) dx$

$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 3 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= 4x^{\frac{1}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$

or $= 4 \sqrt{x} + \frac{6}{5}x^2 \sqrt{x} + c$

(g) $\int (x - 3)^2 dx$
$= \int (x^2 - 6x + 9)\, dx$
$= \frac{1}{3}x^3 - \frac{6}{2}x^2 + 9x + c$

$= \frac{1}{3}x^3 - 3x^2 + 9x + c$

(h) $\int \frac{(2x + 1)^2}{\sqrt{x}} dx$

$= \int x^{-\frac{1}{2}} \left(4x^2 + 4x + 1 \right) dx$

$= \int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$

$= 4 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

or $= \frac{8}{5}\sqrt{x^5} + \frac{8}{3}\sqrt{x^3} + 2 \sqrt{x} + c$

(i) $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) dx$

$= \int \left(3 + x^{-\frac{1}{2}} + 6x^2 \right) dx$

$= 3x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6}{3}x^3 + c$

$= 3x + 2x^{\frac{1}{2}} + 2x^3 + c$
or $= 3x + 2 \sqrt{x} + 2x^3 + c$

(j) $\int \sqrt{x} (\sqrt{x} + 3)^2 dx$

$= \int x^{\frac{1}{2}} \left(x + 6x^{\frac{1}{2}} + 9 \right) dx$

$= \int \left(x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{6}{2}x^2 + 9\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$

or $= \frac{2}{5}\sqrt{x^5} + 3x^2 + 6\sqrt{x^3} + c$

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Integration
Exercise E, Question 1

Question:

Find the equation of the curve with the given $\frac{dy}{dx}$ that passes through the given point:

(a) $\frac{dy}{dx} = 3x^2 + 2x$; point (2 , 10)

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$; point (1 , 4)

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$; point (4 , 11)

(d) $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$; point (4 , 0)

(e) $\frac{dy}{dx} = (x + 2)^2$; point (1 , 7)

(f) $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$; point (0 , 1)

Solution:

(a) $\frac{dy}{dx} = 3x^2 + 2x$

$\Rightarrow \quad y = \frac{3}{3}x^3 + \frac{2}{2}x^2 + c$

So $y = x^3 + x^2 + c$

$x = 2, y = 10 \quad \Rightarrow \quad 10 = 8 + 4 + c$

So $c = -2$

So equation is $y = x^3 + x^2 - 2$

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$

$\Rightarrow \quad y = \frac{4}{4}x^4 - \frac{2}{2}x^{-2} + 3x + c$

So $y = x^4 - x^{-2} + 3x + c$

$x = 1, y = 4 \quad \Rightarrow \quad 4 = 1 - 1 + 3 + c$

So $c = 1$

So equation is $y = x^4 - x^{-2} + 3x + 1$

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$

$\Rightarrow \quad y = \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{1}{4}\dfrac{x^3}{3} + c$

So $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + c$

$x = 4, y = 11 \quad \Rightarrow \quad 11 = \frac{2}{3} \times 2^3 + \frac{1}{12} \times 4^3 + c$

So $c = \frac{33}{3} - \frac{32}{3} = \frac{1}{3}$

So equation is $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$

(d) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{x}} - x$

$\Rightarrow \quad y = 3\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} - \dfrac{1}{2}x^2 + c$

So $y = 6\sqrt{x} - \frac{1}{2}x^2 + c$

$x = 4, y = 0 \quad \Rightarrow \quad 0 = 6 \times 2 - \frac{1}{2} \times 16 + c$

So $c = -4$

So equation is $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$

(e) $\frac{\mathrm{d}y}{\mathrm{d}x} = (x + 2)^2 = x^2 + 4x + 4$

$\Rightarrow \quad y = \frac{1}{3}x^3 + 2x^2 + 4x + c$

$x = 1, y = 7 \quad \Rightarrow \quad 7 = \frac{1}{3} + 2 + 4 + c$

So $c = \frac{2}{3}$

So equation is $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$

(f) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 + 3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$\Rightarrow \quad y = \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

So $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + c$

$x = 0, y = 1 \quad \Rightarrow \quad 1 = \frac{2}{5} \times 0 + 6 \times 0 + c$

So $c = 1$

So equation of curve is $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

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Integration
Exercise E, Question 2

Question:

The curve C, with equation $y = \text{f}(x)$, passes through the point $(1, 2)$ and $\text{f}'(x) = 2x^3 - \frac{1}{x^2}$. Find the equation of C in the form $y = \text{f}(x)$.

Solution:

$\text{f}'(x) = 2x^3 - \frac{1}{x^2} = 2x^3 - x^{-2}$

So $\text{f}(x) = \frac{2}{4}x^4 - \frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 + \frac{1}{x} + c$

But $\text{f}(1) = 2$

So $2 = \frac{1}{2} + 1 + c$

$\Rightarrow \quad c = \frac{1}{2}$

So $\text{f}(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

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Integration
Exercise E, Question 3

Question:

The gradient of a particular curve is given by $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$. Given that the curve passes through the point $(9, 0)$, find an equation of the curve.

Solution:

$$\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2} = x^{-\frac{3}{2}} + 3x^{-2}$$

$$\Rightarrow \quad y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 3\frac{x^{-1}}{-1} + c$$

So $y = -2x^{-\frac{1}{2}} - 3x^{-1} + c = -\frac{2}{\sqrt{x}} - \frac{3}{x} + c$

$x = 9, y = 0 \quad \Rightarrow \quad 0 = -\frac{2}{3} - \frac{3}{9} + c$

So $c = \frac{2}{3} + \frac{1}{3} = 1$

So equation is $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$

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Integration
Exercise E, Question 4

Question:

A set of curves, that each pass through the origin, have equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$... where $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = x^2$.

(a) Find $f_2(x)$, $f_3(x)$.

(b) Suggest an expression for $f_n(x)$.

Solution:

(a) $f_2'(x) = f_1(x) = x^2$

So $f_2(x) = \frac{1}{3}x^3 + c$

The curve passes through (0 , 0) so $f_2(0) = 0 \Rightarrow c = 0$.

So $f_2(x) = \frac{1}{3}x^3$

$f_3'(x) = \frac{1}{3}x^3$

$f_3(x) = \frac{1}{12}x^4 + c$, but $c = 0$ since $f_3(0) = 0$.

So $f_3(x) = \frac{1}{12}x^4$

(b) $f_2(x) = \frac{x^3}{3}$, $f_3(x) = \frac{x^4}{3 \times 4}$

So power of x is $n + 1$ for $f_n(x)$, denominator is $3 \times 4 \times$ ... up to $n + 1$:

$f_n(x) = \frac{x^{n+1}}{3 \times 4 \times 5 \times \ldots \times (n+1)}$

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Integration
Exercise E, Question 5

Question:

A set of curves, with equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$ ... all pass through the point $(0, 1)$ and they are related by the property $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = 1$.

Find $f_2(x)$, $f_3(x)$, $f_4(x)$.

Solution:

$f_2'(x) = f_1(x) = 1$

$\Rightarrow \quad f_2(x) = x + c$

But $f_2(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_2(x) = x + 1$

$f_3'(x) = f_2(x) = x + 1$

$\Rightarrow \quad f_3(x) = \frac{1}{2}x^2 + x + c$

But $f_3(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_3(x) = \frac{1}{2}x^2 + x + 1$

$f_4'(x) = f_3(x) = \frac{1}{2}x^2 + x + 1$

$\Rightarrow \quad f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + c$

But $f_4(0) = 1 \quad \Rightarrow \quad 1 = 0 + c \quad \Rightarrow \quad c = 1$

So $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

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Integration
Exercise F, Question 1

Question:

Find:

(a) $\int (x + 1) (2x - 5) dx$

(b) $\int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx$.

Solution:

(a) $\int (x + 1) (2x - 5) dx$
$= \int (2x^2 - 3x - 5) dx$
$= 2 \frac{x^3}{3} - 3 \frac{x^2}{2} - 5x + c$
$= \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x + c$

(b) $\int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c$

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Integration
Exercise F, Question 2

Question:

The gradient of a curve is given by $f'(x) = x^2 - 3x - \frac{2}{x^2}$. Given that the curve passes through the point $(1, 1)$, find the equation of the curve in the form $y = f(x)$.

Solution:

$f'(x) = x^2 - 3x - \frac{2}{x^2} = x^2 - 3x - 2x^{-2}$

So $f(x) = \frac{x^3}{3} - 3\frac{x^2}{2} - 2\frac{x^{-1}}{-1} + c$

So $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + c$

But $f\left(1\right) = 1 \quad \Rightarrow \quad \frac{1}{3} - \frac{3}{2} + 2 + c = 1$

So $c = \frac{1}{6}$

So the equation is $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$

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Integration
Exercise F, Question 3

Question:

Find

(a) $\int (8x^3 - 6x^2 + 5) \, dx$

(b) $\int \left(5x + 2 \right) x^{\frac{1}{2}} \, dx$.

Solution:

(a) $\int (8x^3 - 6x^2 + 5) \, dx$

$= 8 \frac{x^4}{4} - 6 \frac{x^3}{3} + 5x + c$

$= 2x^4 - 2x^3 + 5x + c$

(b) $\int \left(5x + 2 \right) x^{\frac{1}{2}} \, dx$

$= \int \left(5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$

$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

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Integration
Exercise F, Question 4

Question:

Given $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$, find $\int y dx$.

Solution:

$y = \frac{(x+1)(2x-3)}{\sqrt{x}}$

$y = \left(2x^2 - x - 3\right) x^{-\frac{1}{2}}$

$y = 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$

$\int y dx = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\right) dx$

$= 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$

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Integration
Exercise F, Question 5

Question:

Given that $\frac{dx}{dt} = 3t^2 - 2t + 1$ and that $x = 2$ when $t = 1$, find the value of x when $t = 2$.

Solution:

$\frac{dx}{dt} = 3t^2 - 2t + 1$

$\Rightarrow \quad x = 3\frac{t^3}{3} - 2\frac{t^2}{2} + t + c$

So $x = t^3 - t^2 + t + c$
But when $t = 1$, $x = 2$.
So $2 = 1 - 1 + 1 + c$
$\Rightarrow \quad c = 1$
So $x = t^3 - t^2 + t + 1$
When $t = 2$, $x = 8 - 4 + 2 + 1$
So $x = 7$

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Integration
Exercise F, Question 6

Question:

Given $y = 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$, $x > 0$, find $\int y\,\mathrm{d}x$.

Solution:

$$\int y\,\mathrm{d}x = \int \left(3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) \mathrm{d}x$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

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Integration
Exercise F, Question 7

Question:

Given that $\frac{dx}{dt} = (t+1)^2$ and that $x = 0$ when $t = 2$, find the value of x when $t = 3$.

Solution:

$\frac{dx}{dt} = (t+1)^2 = t^2 + 2t + 1$

$\Rightarrow \quad x = \frac{t^3}{3} + 2\frac{t^2}{2} + t + c$

But $x = 0$ when $t = 2$.

So $0 = \frac{8}{3} + 4 + 2 + c$

$\Rightarrow \quad c = -\frac{26}{3}$

So $x = \frac{1}{3}t^3 + t^2 + t - \frac{26}{3}$

When $t = 3$, $x = \frac{27}{3} + 9 + 3 - \frac{26}{3}$

So $x = 12\frac{1}{3}$ or $\frac{37}{3}$

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Integration
Exercise F, Question 8

Question:

Given that $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$:

(a) Show that $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$, where A and B are constants to be found.

(b) Hence find $\int y\mathrm{d}x$. **[E]**

Solution:

(a) $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$

So $y = \left(x^{\frac{1}{3}} + 3 \right)^2$

So $y = \left(x^{\frac{1}{3}} \right)^2 + 6x^{\frac{1}{3}} + 9$

So $y = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9$
($A = 6$, $B = 9$)

(b) $\int y\mathrm{d}x = \int \left(x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \right) \mathrm{d}x$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c$

$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$

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Integration
Exercise F, Question 9

Question:

Given that $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \left(x > 0 \right)$:

(a) Find $\frac{dy}{dx}$.

(b) Find $\int y dx$. **[E]**

Solution:

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

(a) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 4 \times \left(- \frac{1}{2} \right) x^{-\frac{3}{2}}$

So $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

(b) $\int y dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \right) dx$

$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$

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Integration

Exercise F, Question 10

Question:

Find $\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) \mathrm{d}x$.**[E]**

Solution:

$$\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) \mathrm{d}x$$

$$= \int \left(1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 \right) \mathrm{d}x$$

$$= \int \left(5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) \mathrm{d}x$$

$$= 5x - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$