Pure Maths Revision – P1

Indicies

$a^b \times a^c = a^{b+c}$

$a^b \div a^c = a^{b-c}$

$a^0 = 1$

$a^{-m} = 1 / a^m$

$a^{1/m} = {}_m\sqrt{a}$

$a^{n/m} = {}_m\sqrt{a^n}$

$(a^m)^n = a^{mn}$

Quadratic Equations

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Discriminant = $b^2 - 4ac$
If Discriminant > 0 there are 2 solutions
If Discriminant = 0 then the roots are coincident
If Discriminant < 0 there are no real solutions

Remainder theorem (for dividing polynomials)

Function $f(x)$
Divisor (x-2)
Put the value that makes the divisor = 0 (in this case 2) into the function $f(x)$
If a number is produced then this will be the remainder if the function is divided by the divisor. If 0 is returned, then the divisor is a factor of the function $f(x)$.

Co-ordinate Geometry

To find the length of a line AB.
$AB = \sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$

To find the midpoint of the line AB.
$$\text{Mid(AB)} = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

To find the gradient of the line AB

$$\text{Grad(AB)} = \frac{y_2 - y_1}{x_2 - x_1}$$

As the gradient is constant along the length, to find the equation of the line AB. (where y_1, y_2, x_1, and x_2 are the co-ordinates of 2 points on the line. x and y are the general case.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Or if the gradient is already known, then only one point on the line is required (used for finding equations of tangents and normals to a point on a quadratic or polynomial graph. See Differentiation)

$$\frac{y - y_1}{x - x_1} = \text{Gradient}$$

Vertical line gradient = ∞
Parallel lines have the same gradient.
If two perpendicular lines have gradients g_1 and g_2 then:
$g_1 g_2 = -1$

Equations of lines

Straight Line:
$y = mx + c$
$ax + by + c = 0$

Quadratic:
$y = ax^2 + bx + c$
$(ax + b)(cx + d) = y$

Trigonometry

$\text{Sin}\theta = \text{Opp/Hyp}$
$\text{Cos}\theta = \text{Adj/Hyp}$
$\text{Tan}\theta = \text{Opp/Adj}$

$$\frac{\text{SinA}}{a} = \frac{\text{SinB}}{b} = \frac{\text{SinC}}{c}$$

$\text{Area}\Delta = \frac{1}{2}\,\text{abSinC}$

Degrees 360° = Circle
Radians 2π = Circle

$\text{CosX} = \text{Cos}(360\text{-X})$ or $\text{Cos}(2\pi\text{-X})$
$\text{Cos(-X)} = \text{CosX}$

Period = 360 or 2π

SinX = Sin(180-X) or Sin(π-X)
Sin(-X) = -SinX
Period = 360 or 2π

Tan(-X) = -TanX
Period = 180 or π

Graphing

M is a constant in degrees

Sin(X+M) shifts graph M degrees LEFT
Sin(X-M) shifts graph M degrees RIGHT
Sin(MX) decreases period of graph by a factor of M (E.G. if M = 3 then the period is now 1/3 of the original period, 120^o as opposed to 360^o)
Sin(X/M) increases period of graph by a factor of M (E.G. if M = 3 then the period is now 3 times the original period, 1080^o as opposed to 360^o)
MSinX increases amplitude of graph by a factor of M (E.G if M = 3 then the amplitude is now 3 times the original, 3 as opposed to 1)
(SinX)/M decreases the amplitude of the graph by a factor of M (E.G. if M = 3 then the amplitude is now 1/3 of the original, 1/3 as opposed to 1)
SinX + M shifts the graph M places UP
SinX – M shifts the graph M places DOWN
-SinX inverts the graph about the Y-axis. (E.G. when SinX = 1 the graph shows –1)

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

Differentiation

Multiply by the index, and then subtract one from the index. See Indices section at beginning to ensure that the equations are always in the form ax^b and there are no roots or algebraic fractions (eg $1/x^b$).

$y = ax^b$
$^{dy}/_{dx} = abx^{b-1}$

Notation

Function	Derived/Gradient Function	Second Derivative
f(x)	f'(x)	f''(x)
y	$^{dy}/_{dx}$	$^{d2y}/_{dx2}$

Use differentiation on an equation for a graph to get an equation for the gradient at any point, then substitute in the x value for the point to get the gradient at that point. You can then use Co-ordinate geometry to find equations for tangents and normals (the normal being perpendicular to the tangent). When finding the equation for the normal, first calculate the gradient of the normal, and then use the equation.

To find stationary values (maximum/minimum/turning points on the graph) differentiate the equation of the graph to get the gradient function and where the gradient = 0 will be where the max/min/turning points are. Set gradient function = 0 and solve for X. (gives x co-ordinate of stationary points, then you can put this x co-ordinate into the original equation to get the y co-ordinate).

A point of inflection is where the graph has two turning points in the same place, both a maximum and a minimum (E.G $y = x^3$ at $x = 0$).

There are 3 methods to determine the type of stationary point.

1. Look at the y-values at the point and adjacent to the point. If both adjacent points are lower then it is a maximum, if the adjacent are both higher then it is a minimum, if one is higher and one is lower then it is a point of inflection.
2. Find the gradient ($^{dy}/_{dx}$) close to the point (One before, one after).
   Gradient Before = Gradient After ⇒ Point of Inflection
   Gradient Before –ve, Gradient After +ve ⇒ Minimum
   Gradient Before +ve, Gradient After –ve ⇒ Maximum
3. Find the second derivative ($^{d2y}/_{dx2}$) (Differentiate the gradient function). Substitute in the x-value for which $^{dy}/_{dx} = 0$ (the x value for the stationary point).
   $^{d2y}/_{dx2}$ Positive ⇒ Minimum
   $^{d2y}/_{dx2}$ Negative ⇒ Maximum
   $^{d2y}/_{dx2} = 0$ ⇒ Method Fails ☹

When sketching graphs work out where they cross the x-axis and where the stationary points are, and what kind they are.

If asked to find the angle between the x-axis and the tangent/normal remember the gradient is rise over run. From this you can produce a right angled triangle with the x-axis as the adjacent with a length of 1, the tangent as the hypotenuse, and the opposite with a height equal to the gradient of the tangent. As the adjacent is the run and its value is 1, so the gradient of the tangent will be equal to the length of the opposite. Tan^{-1} of the gradient can then be used to calculate the angle.

Sequences

Convergent, Periodic (where is cycles through a set of results), Oscillating (where it is periodic but the difference between the values increases), and Divergent.
Sequence – commas separate the terms.
Series – the terms are added together.

Arithmetic Progression
a = First Term
d = Common Difference
l = Last Term
n = nth Term

Sequence Definition – $U_n = a + (n - 1)d$
Series Definition – $S_n = \frac{1}{2}n(2a + (n - 1)d)$

Geometric Progression
r = Common Ratio

Sequence Definition – $U_n = ar^{n-1}$
Series Definition – $S_n = \dfrac{a(1 - r^n)}{1 - r}$

Integration

Integration is the reverse of differentiation. Add one to the index and divide by the new index. See Indices section at beginning to ensure that the equations are always in the form ax^b and there are no roots or algebraic fractions (eg $1/x^b$). However an integration constant must be added because differentiating a constant removes it, so an unknown must be added to every intergral.

$$\int(x^n)dx = \frac{x^{n+1}}{n+1} + C$$

UNLESS N = –1

Use integrals to find the area under a curve, between two points. If two definite points are specified (E.G. x = 0 and x = 2) then NO constant is required.

$$\int_0^2(x^n)dx = \left[\frac{x^{n+1}}{n+1}\right]_0^2$$

If asked to find the area enclosed between two curves or a curve and a line.

1. Find the points of intersection. (E.G. $y=2x$ and $y=x^2$ Therefore $2x=x^2$ will solve to find the x co-ordinates of the intersection. So $x^2 - 2x = 0$, then $x(x-2) = 0$, meaning the points of intersection are $x = 0$ and $x = 2$).
2. Sketch the curves.
3. Integrate both functions between +ve intersection x-values to get the areas under the curve, (if one is negative then integrate that separately between +ve and 0 and between –ve and 0) and subtract one from the other, using the sketch to work out which.