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Principal Examiner Feedback

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General

This paper proved to be a good test of student knowledge and understanding. It discriminated well between the different ability levels. There were many accessible marks available to students who were confident with topics such as arc lengths, hyperbolic functions, conic sections, matrices, vectors and calculus.

Reports on Individual Questions

Question 1

The opening question required students to find the length of an arc of a curve. The method was well known, although a small number of misconceptions were seen such as believing that s was given by $\int y \, \mathrm{d}x$ or the surface area formula.
Most students were able to obtain a correct form for the derivative of arcosh but some mishandled the '2' from the $2x$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times \frac{1}{\sqrt{(2x)^2 - 1}}$, meaning that they were unable to reach an integrable function and could make no further progress. Those who chose to write $\cosh 2y = 2x$ and then differentiate implicitly were usually correct. Most went on to find $\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$ with almost all achieving a common denominator for the sum. A few attempted to deal with the square root by replacing $\sqrt{1+(\mathrm{f}(x))^2}$ with $1 + \mathrm{f}(x)$. However, the root of the numerator was usually taken and the correct simplified integral was seen fairly widely. The integration saw more mixed results with some failing to achieve the correct multiplying constant and many unable to achieve the required form. A significant number of failed attempts by parts were seen. Those who integrated correctly usually proceeded to the correct final answer. A small number attempted to integrate by substitution (usually using $x = \frac{1}{2} \cosh u$). Some successes were seen via this route although many experienced difficulty trying to find exact values for sinh(arcosh k). Others kept their result in u but used the original x limits.

Question 2

This question required a proof of the negative logarithmic form of arcosh and scoring was mixed. Most were able to use the correct exponential definition of cosh and usually proceeded to multiply through by e^y to obtain a quadratic. Some students were unable to make any further progress with this equation but many went on to use the correct quadratic formula (or, less commonly, completing the square) to obtain a correct expression in x for e^y. Since $y < 0$, the negative root needed to be taken and some students either did not attempt to justify this or did so with unsatisfactory reasoning. However, a pleasing number did state that $y < 0$ or $\mathrm{e}^y < 1$ and were able to score all six marks. Attempts using the given answer and working backwards could receive credit although efforts using this approach were mixed and the final mark was unlikely to be available.

Question 3

This question on the normal to an ellipse and an associated locus saw good scoring on the whole although obtaining the equation of the locus was a little discriminating.

In part (a), those who used parametric differentiation generally scored all four marks. Those opting for explicit differentiation were similarly successful by and large although some fell foul of failing to differentiate the "1" on the right-hand side. Direct differentiation was only seen in the weakest attempts. It was rare to see failure to use the perpendicularity condition correctly and most students were able to produce a correct straight line equation, usually via $y - y_1 = m(x - x_1)$. Only a small number then produced the given answer without a required intermediate step.

In part (b), the overwhelming majority were able to obtain correct coordinates or axis intercepts for A and B. Although a small number took these intercepts as the coordinates of M, most were able to achieve the correct midpoint. Those who identified that the Pythagorean trigonometric identity was now required invariably scored full marks, but it was clear that some had little idea about how to change their equation in θ into one in x and y. A very small number of students did not give the equation of the locus of M with integer coefficients.

Question 4

This matrix question was a good source of marks for many students but part (c) proved to be quite difficult with full marks rarely being given.

In part (a), almost all were able to convincingly produce the given expression for the determinant with most multiplying it out conventionally, although a relatively small number used the rule of Sarrus.

Part (b) was answered reasonably well. Most knew the full method and correct final answers were common. A small number of students missed or mis-applied a step in the process – often the application of the sign change matrix to obtain the cofactors from the minors. Occasionally the minors were multiplied by the corresponding matrix element. A very small number forgot the $\frac{1}{\det \mathbf{M}}$ or multiplied the adjugate by det **M**.

Part (c) proved challenging to even the most able students. It was anticipated that most students would determine the volume scale factor by finding the volume of the transformed tetrahedron. Those who attempted this method tended to be the most successful and an appropriate scalar triple product was usually used although numerical slips were not uncommon. The $\frac{1}{6}$ was occasionally missing or $\frac{1}{2}$ was used instead. The next step, to obtain an equation in k by setting $|\det \mathbf{M}|$ equal to the quotient of the volumes, was not carried out by many. Examples of incorrect equations were $\frac{20}{3}\frac{1}{\det(M)} = 50$, $\frac{20}{3}(\det(M))^3 = 50$ and some giving the value of k simply as $\frac{50}{\frac{20}{3}}$. Those with a correct equation in k usually obtained the first solution, but many forgot the modulus sign. Some squared the $5k - 10$ to remove the modulus but did not square the rest of the equation. As a result, only the very best students obtained both solutions.

The alternative was to find the images of the given points and there were often errors seen with the resulting coordinates. Many did not progress any further but those who attempted a scalar triple product often struggled with the algebra that this produced. Some did arrive at $k = \frac{7}{2}$ but as with the other method it was unusual to see both possible values of k correctly found.

Question 5

Question 5 involved two skew lines and a wide range of mark profiles were seen here. This was the most common question to see no response made by students. Part (c) in particular was often omitted.

Marks in part (a) were widely scored although there were the usual slips applying the vector/cross product. The scalar/dot product alternative was seen on occasion.
Part (b) did not see widespread success. Many students seemed unfamiliar with the form of the plane required in part (i) and seemed to largely guess the vectors. Those that did put the line directions with the parameters sometimes chose the given point on l_1 rather than l_2. This wrong point often appeared in part (ii) as well although those who had the correct point invariably found a correct value for p.

There was plenty of good scoring in part (c), primarily via Way 2. Many students had clearly remembered a correct formula and projected AB onto the normal with very few errors. A smaller number attempted the Way 1 parallel planes method – occasionally the distances of the planes to the origin were not combined correctly. The rather circuitous route of way 3 was quite rare although a few students obtained the correct answer by this method. The question required the answer in its simplest form – a very small number did not achieve this although it was very unusual to see anyone offer a negative value for the distance.

Question 6

This reduction formula question was challenging for many although it was pleasing to see a many fully correct answers here. Part (b) was a good source of marks for students regardless of whether they had been successful with the proof in part (a).

Most who made progress in part (a) did so via Way 1. However many did not perform the "split" and this often resulted in incorrect integrations of $\cos(x^2)$ and $\sin(x^2)$. Those who recognised the need to write $x^n \cos(x^2)$ as $x^{n-1}\, x \cos(x^2)$ usually applied parts correctly and usually went on to repeat the process. There were the usual slips with signs and coefficients along with errors in bracketing, but the first four marks were scored fairly widely. Way 2 was seen far less commonly and attempts via this route were slightly more mixed. Some who opted for this method seemed unaware of where it was leading but were often able to achieve the first four marks with two correct applications of integration by parts. The next mark required explicit evidence of substitution of limits and many just presented the given answer after parts had been completed with no evidence that the limits had been considered.

Those who attempted part (b) were usually able to score something and the full three marks were widely awarded. The correct value for I_1 was commonly seen or used and there were few slips applying the reduction formula to obtain a value for I_5, although 2 x $\frac{1}{2}$ was carelessly calculated as $\frac{1}{4}$ on a few occasions. There was a number of students who were unable to evaluate the integral for I_1 and so were unable to access marks.

Question 7

Question 7 involved a hyperbola and it proved to be quite discriminating. Those who made a reasonable sketch of the situation usually found it to be very beneficial.
The mark in part (a) was widely awarded – most used a correct eccentricity formula and slips were not common. A recurring error was not giving e^2 in terms of a despite having a correct equation. Occasionally the value for b was not substituted or it was replaced with 25 rather than 5 – an error sometimes repeated in (b).

In part (b) most were able to state a correct equation for a directrix although some then proceeded to substitute this into the equation of the hyperbola. Those who used a correct equation for one of the asymptotes were generally successful although some got bogged down with unnecessarily complicated expressions by not working in terms of just a and e. A regular error was to obtain the correct coordinate of A or A' and then to find the distance from O by Pythagoras.

Part (c) was probably the most challenging part of the question. Those who identified that the perpendicular height of the triangle was the sum of the distance to O to the negative focus added to the distance from O to the positive directrix tended to make good progress. Sketches proved particularly useful here. For quite a few students the directrix distance was missing. Working in a and e at the start tended to be a better option than working in a alone but both routes produced a solid number of correct equations in powers of a. Most with a correct equation were generally able to reach the given answer although a small number made some ill-advised choices when tidying up the algebra. A small number of students presented the given answer with insufficient prior working shown.

Part (d) was fully accessible to students who had not made progress with the rest of the question but it tended to be those who had scored well on parts (a) to (c) who picked up marks here. Most attempts involved factorising the cubic and some impressive algebra to achieve this was seen on occasion. Most however opted for long division and more often than not produced a correct quadratic with sufficient working. The factor theorem route was rarer but was usually sufficient when it was seen. To show that there were no further roots, calculating the discriminant was the most popular choice. There needed to be evidence of calculation of
$b^2 – 4ac$ and this was not always present. Some did not follow their discriminant calculation with the reason why no other roots were possible. Completing the square was less common. This approach again needed a reason why no real solutions could result and this was not always provided. A very small number used differentiation followed by the discriminant or completing the square to show that the cubic must be increasing. Several attempts merely solved the cubic on a calculator and the question required algebra to be explicitly used.

Question 8

The final question centred on a method to integrate an arccos function. It proved fairly demanding although it was a good source of marks for a reasonably wide variety of students.

In part (a) most attempts scored well although there were a variety of slips. The chain rule was occasionally absent, $\left(\sqrt{x}\right)^2$ was sometimes replaced with x^2 and the "2" was often incorrectly handled. Most differentiated directly although some took the cosine of both sides first, usually achieving a correct answer. The marks were awarded when any correct derivative was seen so those who went on to lose the minus sign tended to get the marks although this usually led to issues in part (b).

Part (b) saw many correct responses but errors were fairly common. Most provided enough evidence of parts application to secure the M mark but the A mark was not always scored – often due to students merely writing the given answer after very little working. Those with an incorrect answer to (a) often attempted to "fudge" the result. The alternative route was quite rare but usually successful when seen.

Part (c) saw a mixed response. Some did not differentiate the given substitution equation and could not make any creditworthy progress. Most who did differentiate usually had a correct equation involving dx and dθ although the absence of the minus sign was a common slip. Those who reached $-\frac{1}{4}\int \cos^2\theta \, d\theta$ after having worked with the theta limits throughout were often able to spot that all they needed to do was to remove the minus sign and swap the limits around. However, this technique was not that widely understood and there were many instances where the sign was "fudged" in order to obtain the given result and this meant a loss of 2 or 3 of the 4 marks available.

The final part was accessible to many. It was rare to see an incorrect identity used for $\cos^2\theta$ although some attempted an invalid direct integration. Some forgot to integrate after using the identity. The most common error was for students to stop after calculating the definite integral in theta, forgetting that they also needed to incorporate the $x \arccos(2\sqrt{x})$ from part (b).

A wide range of mark profiles resulted for the question as a whole but it was very encouraging to see a significant number of students score all 13 marks with well-presented full solutions.

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