Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Friday 22 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. Relative to a fixed origin O, the points A, B, C and D have coordinates (0, 4, 1), (4, 0, 0), (3, 5, 2) and (2, 2, k) respectively, where k is a constant.

(a) Determine the exact area of triangle ABC.

(3)

(b) Determine in terms of k, the volume of the tetrahedron $ABCD$, simplifying your answer.

(3)

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Question 1 continued

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Question 1 continued

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Question 1 continued

(Total 6 marks)

Q1

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2.

$$y = \ln(\tanh 2x) \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = p\,\text{cosech}\,4x$$

where p is a constant to be determined.

(4)

(b) Hence determine, in simplest form, the exact value of x for which $\dfrac{dy}{dx} = 1$

(2)

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Question 2 continued

Q2

(Total 6 marks)

3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k.

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 6 marks)

Q3

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4. Using the substitution $x = 4\cosh\theta$ show that

$$\int \frac{1}{\left(x^2 - 16\right)^{\frac{3}{2}}}\,\mathrm{d}x = \frac{ax}{\sqrt{x^2 - 16}} + c \qquad |x| > 4$$

where a is a constant to be determined and c is an arbitrary constant.

(6)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 6 marks)

P 6 6 6 4 2 A 0 1 5 3 6

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5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of **M**

(a) determine an eigenvector corresponding to the eigenvalue 8

(2)

(b) Determine the other two eigenvalues of **M**.

(3)

(c) Hence find an orthogonal matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6.

$$I_n = \int \frac{x^n}{\sqrt{x^2 + 3}}\,\mathrm{d}x \qquad n \in \mathbb{N}$$

(a) Show that

$$I_n = \frac{x^{n-1}}{n}\left(x^2 + 3\right)^{\frac{1}{2}} - \frac{3(n-1)}{n} I_{n-2} \qquad n \geqslant 3$$

(6)

(b) Hence show that

$$\int \frac{x^5}{\sqrt{x^2 + 3}}\,\mathrm{d}x = \frac{1}{5}\left(x^2 + 3\right)^{\frac{1}{2}}\left(x^4 + px^2 + q\right) + k$$

where p and q are integers to be determined and k is an arbitrary constant.

(4)

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Question 6 continued

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Q6

(Total 10 marks)

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7. The point P has coordinates (1, 2, 1)

The line l has Cartesian equation

$$\frac{x-3}{5} = \frac{y+1}{3} = \frac{z+5}{-8}$$

The plane Π_1 contains the point P and the line l.

(a) Show that a Cartesian equation for Π_1 is

$$6x - 2y + 3z = 5$$

(5)

The point Q has coordinates $(2, k, -7)$, where k is a constant.

(b) Show that the shortest distance between Π_1 and Q is

$$\frac{2}{7}|k + 7|$$

(2)

The plane Π_2 has Cartesian equation $8x - 4y + z = -3$

Given that the shortest distance between Π_1 and Q is the same as the shortest distance between Π_2 and Q,

(c) determine the possible values of k.

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 11 marks)

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8. The curve C has equation

$$y = 2 + \ln(1 - x^2) \qquad \frac{1}{2} \leqslant x \leqslant \frac{3}{4}$$

(a) Show that the length of the curve C is given by

$$\int_{\frac{1}{2}}^{\frac{3}{4}} \left(\frac{1 + x^2}{1 - x^2}\right) \mathrm{d}x$$

(4)

(b) Hence, using algebraic integration, show that the length of the curve C is $p + \ln q$ where p and q are rational numbers to be determined.

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 9 marks)

Q8

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9. The ellipse E has equation

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 4\sin\theta)$ where $0<\theta<\frac{\pi}{2}$

The line l is the normal to the ellipse at the point P.

(a) Show that an equation for l is

$$5x\sin\theta - 4y\cos\theta = 9\sin\theta\cos\theta$$

(5)

The point F is the focus of E that lies on the positive x-axis.

(b) Determine the coordinates of F.

(2)

The line l crosses the x-axis at the point Q.

(c) Show that

$$\frac{|QF|}{|PF|}=e$$

where e is the eccentricity of E.

(5)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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