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Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C34

Advanced

Tuesday 16 January 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. A curve C has equation

$$3^x + xy = x + y^2, \qquad y > 1$$

The point P with coordinates (4, 11) lies on C.

Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer in the form $a + b\ln 3$, where a and b are rational numbers.

(6)

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Question 1 continued

Q1

(Total 6 marks)

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2. $f(x) = (125 - 5x)^{\frac{2}{3}} \qquad |x| < 25$

(a) Find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2, giving the coefficient of x and the coefficient of x^2 as simplified fractions.

(4)

(b) Use your expansion to find an approximate value for $120^{\frac{2}{3}}$, stating the value of x which you have used and showing your working. Give your answer to 5 decimal places.

(3)

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Question 2 continued

Q2

(Total 7 marks)

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3. $$f(x) = \frac{x^2}{4} + \ln(2x), \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \frac{1}{2}e^{-\frac{1}{4}x^2}$$

(2)

The equation $f(x) = 0$ has a root near 0.5

(b) Starting with $x_1 = 0.5$ use the iterative formula

$$x_{n+1} = \frac{1}{2}e^{-\frac{1}{4}x_n^2}$$

to calculate the values of x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval, show that 0.473 is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

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Question 3 continued

Q3

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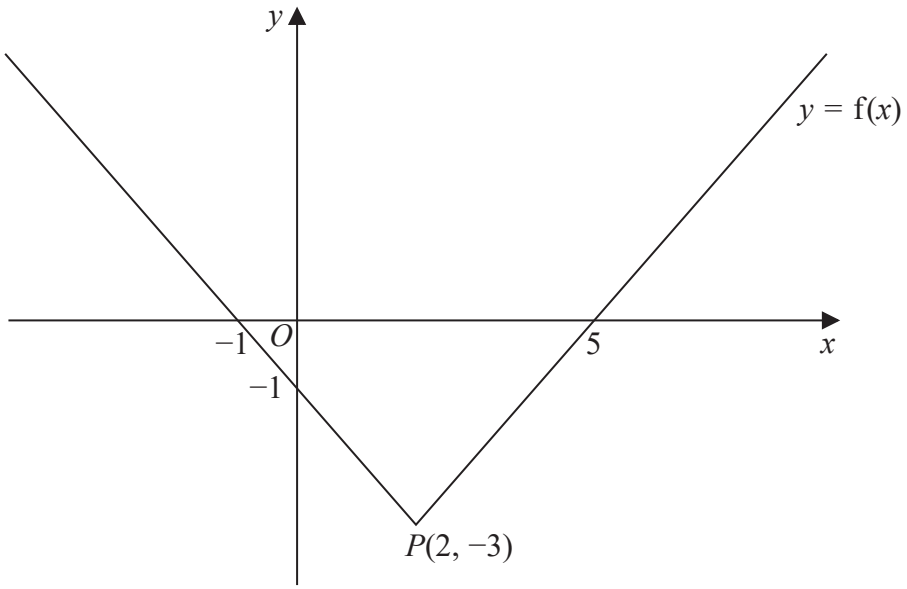

Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x), \quad x \in \mathbb{R}$

The graph consists of two half lines that meet at the point $P\,(2, -3)$, the vertex of the graph.

The graph cuts the y-axis at the point $(0, -1)$ and the x-axis at the points $(-1, 0)$ and $(5, 0)$.

Sketch, on separate diagrams, the graph of

(a) $y = f(|x|)$,

(3)

(b) $y = 2f(x + 5)$.

(3)

In each case, give the coordinates of the points where the graph crosses or meets the coordinate axes.

Also give the coordinates of any vertices corresponding to the point P.

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

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5. (a) Express $\dfrac{9(4+x)}{16-9x^2}$ in partial fractions.

(3)

Given that

$$f(x) = \frac{9(4+x)}{16-9x^2}, \qquad x \in \mathbb{R}, \qquad -\frac{4}{3} < x < \frac{4}{3}$$

(b) express $\int f(x)\,dx$ in the form $\ln(g(x))$, where $g(x)$ is a rational function.

(4)

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Question 5 continued

Q5

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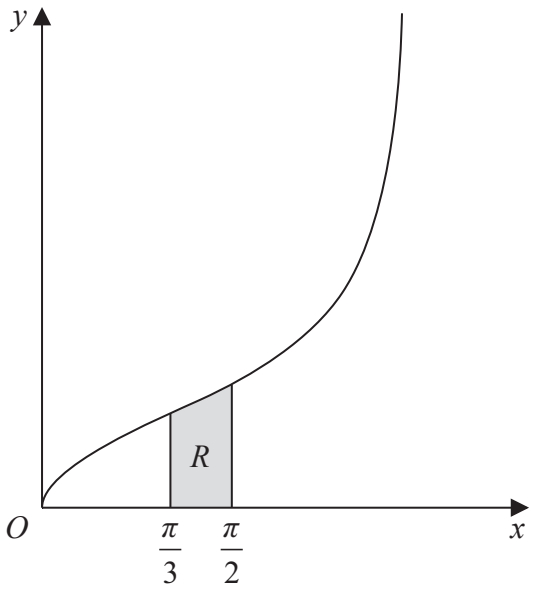

Figure 2

The curve shown in Figure 2 has equation

$$y^2 = 3\tan\left(\frac{x}{2}\right), \qquad 0 < x < \pi, \qquad y > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{\pi}{3}$ the x-axis and the line with equation $x = \frac{\pi}{2}$

The region R is rotated through 360° about the x-axis to generate a solid of revolution.

Show that the exact value of the volume of the solid generated may be written as $A\ln\left(\frac{3}{2}\right)$, where A is a constant to be found.

(5)

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Question 6 continued

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Question 6 continued

Q6

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7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (13\mathbf{i} + 15\mathbf{j} - 8\mathbf{k}) + \lambda(3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (7\mathbf{i} - 6\mathbf{j} + 14\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection, B.

(6)

(b) Find the acute angle between the lines l_1 and l_2

(3)

The point A has position vector $-5\mathbf{i} - 3\mathbf{j} + 16\mathbf{k}$

(c) Show that A lies on l_1

(1)

The point C lies on the line l_1 where $\overrightarrow{AB} = \overrightarrow{BC}$

(d) Find the position vector of C.

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total 13 marks)

Q7

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8. Given that

$$y = 8\tan(2x), \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{A}{B + y^2}$$

where A and B are integers to be found.

(4)

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Question 8 continued

Q8

(Total 4 marks)

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9. (a) Show that

$$\frac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\frac{\cot^2 x}{1+\cot^2 x} = 8\cos 2x + 2\cos x$$

Give each solution in degrees to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

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10. It is given that

$$f(x) = e^{-2x} \qquad x \in \mathbb{R}$$

$$g(x) = \frac{x}{x-3} \qquad x > 3$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of any points where the graph crosses the axes.

(2)

(b) Find the range of g

(2)

(c) Find $g^{-1}(x)$, stating the domain of g^{-1}

(4)

(d) Using algebra, find the exact value of x for which $fg(x) = 3$

(4)

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Question 10 continued

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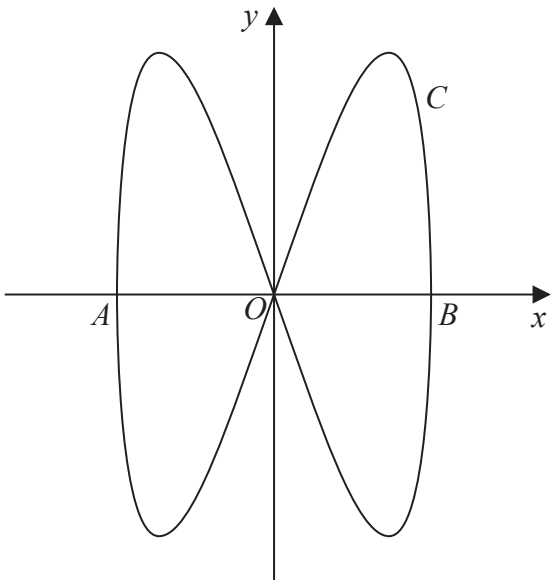

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = 3\cos t, \qquad y = 9\sin 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C meets the x-axis at the origin and at the points A and B, as shown in Figure 3.

(a) Write down the coordinates of A and B.

(2)

(b) Find the values of t at which the curve passes through the origin.

(2)

(c) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, and hence find the gradient of the curve when $t = \dfrac{\pi}{6}$

(4)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined.

(4)

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Question 11 continued

Q11

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12. (a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 significant figures.

(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leqslant t \leqslant 365$$

where t is the number of days since he began recording.

(b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.

(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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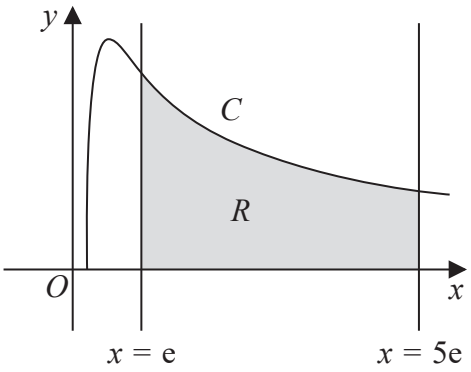

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2x}\ln 2x, \qquad x > \frac{1}{2}$$

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = \text{e}$ and $x = 5\text{e}$.

The table below shows corresponding values of x and y for $y = \frac{1}{2x}\ln 2x$. The values for y are given to 4 significant figures.

x	e	2e	3e	4e	5e
y	0.3114	0.2195	0.1712	0.1416	0.1215

(a) Use the trapezium rule with all the y values in the table to find an approximate value for the area of R, giving your answer to 3 significant figures.

(3)

(b) Using the substitution $u = \ln 2x$, or otherwise, find $\int \frac{1}{2x}\ln 2x \, \mathrm{d}x$

(3)

(c) Use your answer to part (b) to find the true area of R, giving your answer to 3 significant figures.

(2)

(d) Using calculus, find an equation for the tangent to the curve at the point where $x = \frac{\text{e}^2}{2}$, giving your answer in the form $y = mx + c$ where m and c are exact multiples of powers of e.

(5)

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14. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

(b) Using the chain rule, or otherwise, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

where k and n are constants to be found.

(2)

Initially, the radius of the balloon is 3 cm.

(c) Using the values of k and n found in part (b), solve the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

to obtain a formula for r in terms of t.

(6)

(d) Hence find the radius of the balloon when $t = 175$, giving your answer to 3 significant figures.

(1)

(e) Find the rate of increase of the radius of the balloon when $t = 175$. Give your answer to 3 significant figures.

(2)

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Q14

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