Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International A Level
In Further Pure F1 (WFM01)

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General Introduction

This paper was accessible and there were plenty of opportunities for a typical E grade student to gain some marks across all the questions. There were some testing questions involving mathematical induction, complex numbers, the application of finite series summations and matrices that allowed the paper to discriminate well between the higher grades.

In summary, Q1(a), Q2(a), Q3(a), Q6(a), Q6(c), Q6(d) and Q7(i) were a good source of marks for the average student, mainly testing standard ideas and techniques and Q1(b), Q3(b), Q4, Q5, Q6(b) and Q7(ii) were discriminating at the higher grades. Q8 proved to be the most challenging question on the paper.

Report on Individual Questions

Question 1

This question proved accessible with the majority of students scoring full marks.

In part (a), most students differentiated $f(x)$ correctly and applied the Newton-Raphson process correctly to give a second approximation for α as 1.565 to 3 decimal places. A noticeable number of students incorrectly rounded their final answer to 1.56 and so lost the final accuracy mark. Some students differentiated $-\frac{5}{3\sqrt{x}}$ incorrectly to give either $-\frac{5}{6}x^{-\frac{3}{2}}$, $\frac{5}{3}x^{-2}$ or $\frac{5}{3}x^{-\frac{3}{2}}$ and a few students differentiated the constant term -6 to give -6. In some cases, a lack of working did mean that it was sometimes difficult for examiners to determine whether the Newton-Raphson process was applied correctly. A small number of students wasted their time by performing a second iteration of the Newton-Rapson process.

Part (b) was less well answered than part (a). Those students who drew a diagram containing the relevant information usually proceeded to show a correct method. Many students used similar triangles to form a correct equation in α and proceeded to solve it correctly. Some students, however, formed an equation in α with one of their fractions the wrong way round, while others used an odd number of negative lengths in their working. Some students obtained an answer outside the interval $[1.5, 1.6]$, with many of them not realising that their answer should have been sufficient evidence that something was amiss. Occasionally, students went back to first principles and found the equation of the line joining the points $(1.5, -0.61...)$ and $(1.6, 0.36...)$, before proceeding to find where their line crossed the *x*-axis.

Question 2

This question proved accessible with many students scoring full marks.

In part (a), almost all students wrote down the complex conjugate root $2 - 3i$. Most students used the conjugate pair to write down and multiply out $(z-(2+3i))(z-(2-3i))$ in order to identify the quadratic factor $z^2 - 4z + 13$. Some students achieved this quadratic factor using the sum and product of roots method. Most students used algebraic long division to establish the other quadratic factor, although some students used a method of comparing coefficients. Those students who had progressed this far either applied the quadratic formula or completed the square to find the two roots of their $z^2 - 2z + 17 = 0$. A number of sign errors or manipulation errors were seen in this part.

In part (b), it was pleasing to see that many students used a ruler and an appropriate scale to plot all four of their complex roots on an Argand diagram. It was also pleasing that there were many Argand diagrams with roots plotted in the correct positions relative to each other and that the roots were symmetrical to each other about the real axis. A minority drew either small or freehand Argand diagrams and it was sometimes difficult for examiners to decipher the information contained therein.

Question 3

This question was accessible with most students scoring at least 6 of the 8 marks available.

In part (a), almost all students expanded the expression $r^2(r+1)$ and substituted the standard formulae for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ into $\sum_{r=1}^{n}(r^3 + r^2)$. Students who directly factorised $\frac{1}{12}n(n+1)$ were generally more successful in obtaining the correct answer. A few students expanded to give a correct $\frac{1}{4}n^4 + \frac{5}{6}n^3 + \frac{3}{4}n^2 + \frac{1}{6}n$, but some of these then struggled to obtain the correct fully factorised answer.

Part (b) was found to be more demanding in comparison to part (a). Many students evaluated $\sum_{r=25}^{45} r^2(r+1)$ by applying $\mathrm{f}(25) - \mathrm{f}(4)$ to give 111020, although a few made the error of applying $\mathrm{f}(25) - \mathrm{f}(5)$, where $\mathrm{f}(n) = \frac{1}{12}n(n+1)(n+2)(3n+1)$. Many students applied 140543 – their 111020 to find a value for $\sum_{r=1}^{k} 3^r$. At this stage some students failed to recognise that $\sum_{r=1}^{k} 3^r$ can be expressed as the sum of a geometric series with k terms, while others wrote down an incorrect formula for this sum. The correct answer $k = 9$ was usually achieved by students using logarithms to solve the equation $\frac{3\left(1 - 3^k\right)}{1 - 3} = 29523$. Some students listed powers of 3 and used a method of trial and improvement to correctly deduce $k = 9$. A small number of students arrived at a non-integer value for k, which should have been sufficient evidence that something was amiss. These students usually quoted their non-integer value as their final answer or rounded this value to the nearest integer.

Question 4

This question was accessible with most students scoring at least 6 of the 8 marks available. The majority of students recalled that the sum and product of roots in a quadratic equation $ax^2 + bx + c = 0$ are $-\frac{b}{a}$ and $\frac{c}{a}$ respectively. There were the occasional algebraic, manipulation and bracketing errors seen in some students' solutions. Some students found and applied $\alpha, \beta = \frac{-1 \pm \sqrt{14}\,\mathrm{i}}{3}$ in this question. These students lost a considerable number of marks because they did not obey the instruction 'Without solving the (quadratic) equation' which was stated in this question.

In part (a), most students substituted their values for $\alpha + \beta$ and $\alpha\beta$ into the identity $\alpha^2 + \beta^2 \equiv (\alpha + \beta)^2 - 2\alpha\beta$ and many correctly found $\alpha^2 + \beta^2$ as $-\frac{26}{9}$.

In part (b), many students correctly showed that $\alpha^3+\beta^3 = \frac{82}{27}$ by substituting $\alpha+\beta = -\frac{2}{3}$ and $\alpha\beta = \frac{5}{3}$ into either $\alpha^3+\beta^3 \equiv (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$, $\alpha^3+\beta^3 \equiv (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ or $\alpha^3+\beta^3 \equiv (\alpha^2+\beta^2)(\alpha+\beta)-\alpha\beta(\alpha+\beta)$. Students who wrote $\alpha^3+\beta^3 \equiv (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \frac{82}{27}$, with no evidence of substituting $\alpha+\beta = -\frac{2}{3}$ and $\alpha\beta = \frac{5}{3}$ into their correct identity scored no marks in this part.

The complete method in part (c) was understand by most students, but a considerable number failed to score full marks because they made manipulation or substitution errors. Many students correctly simplified the sum of the given roots to give $\alpha+\beta + \frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$ but a minority made the slip of substituting their $\alpha^2+\beta^2$ for $\alpha^3+\beta^3$ or substituting their $\alpha^2+\beta^2$ for $(\alpha\beta)^2$. Many students correctly simplified the product of the given roots to give $\alpha\beta + \frac{\alpha^2+\beta^2}{\alpha\beta} + \frac{1}{\alpha\beta}$ but a minority made the slip of substituting their $\alpha+\beta$ for $\alpha^2+\beta^2$. At this stage most students proceeded to use a correct method to form the quadratic equation described in the question. The three main errors in establishing the required quadratic equation were: applying the incorrect method of $x^2 + (\text{sum})x + (\text{product}) = 0$; the omission of "$=0$"; and the failure to give integer coefficients.

Question 5

This question discriminated well between students of all abilities. A significant number of students struggled to answer part (i), with others who did answer this part making algebraic and manipulation errors.

In part (i), there were several correct methods seen by examiners for solving the printed equation, many of which are detailed in the mark scheme. A few students who tried to "rationalise" $\frac{2z+3}{z+5-2\mathrm{i}}$ by writing $\frac{(2z+3)(z+5+2\mathrm{i})}{(z+5-2\mathrm{i})(z+5+2\mathrm{i})}$ scored no marks in this part. Many students started their solution by multiplying both sides of the printed equation by $(z+5-2\mathrm{i})$ to give $2z+3 = (1+\mathrm{i})(z+5-\mathrm{i})$. Some students gave up at this point or produced work that did not gain any more credit. Some students replaced z by $a+b\mathrm{i}$ and proceeded to equate the real and imaginary parts of both sides of their equation, while others expanded $(1+\mathrm{i})(z+5-\mathrm{i})$ and used a complete method to make z the subject of their equation.

In part (ii), many students multiplied out $(3+\lambda\mathrm{i})(2+\mathrm{i})$ and applied Pythagoras' Theorem, with many doing so correctly to give $(15)^2 = (6-\lambda)^2 + (3+2\lambda)^2$. A few students incorrectly applied Pythagoras' Theorem to give either $15 = (6-\lambda)^2 + (3+2\lambda)^2$ or $(15)^2 = (6-\lambda)^2 - (3+2\lambda)^2$. Some students made errors when attempting to solve the resulting quadratic equation to find the value of λ. A small minority of students proceeded to reject the negative root.

Question 6

This question was accessible to most students with many gaining full marks in parts (a), (c) and (d).

In part (a), almost all students wrote down the correct focus $S(8, 0)$. The points $(4, 0)$ and $(6, 0)$ were the most common incorrect answers seen in part (a).

Most students who were successful in part (b) drew a diagram depicting the parabola *C*, the point $P(2, 8)$ and the directrix with equation $x = -8$ to deduce $PT = 10$. A few students used the focus-directrix property of a parabola to write down $PT = PS$ and used Pythagoras' Theorem to find the length PT. The most common incorrect answer seen in part (b) was $PT = 16$.

In part (c), students used a variety of methods to find $\frac{dy}{dx}$. Most students wrote $y = \sqrt{32}\,x^{\frac{1}{2}}$ and differentiated this to give $\frac{dy}{dx} = \frac{1}{2}\sqrt{32}\,x^{-\frac{1}{2}}$ or $2\sqrt{2}\,x^{-\frac{1}{2}}$ while other students used implicit differentiation. A few students deduced the correct parametric equations $x = 8t^2$, $y = 16t$ for *C* and applied the chain rule of differentiation. Most students found a correct numerical value for the gradient of the tangent and proceeded to find the correct given tangent equation.

In part (d), most students substituted the tangent equation $y = 2x + 4$ into the equation of the hyperbola $xy = 4$ and formed a quadratic equation in one variable. A few students expressed the hyperbola as $x = 2t$, $y = \frac{2}{t}$ and substituted these parametric equations into $y = 2x + 4$ and formed a quadratic equation in *t*. Most students used a correct method for solving their quadratic equation and progressed to find the coordinates of both intersection points. Common errors included using an incorrect method for solving their quadratic equation and not leaving the *y* coordinates in their simplest form.

Question 7

This question discriminated well between students of all abilities. A significant number of students struggled to make any creditable progress in part (ii).

In part (i), most students applied a full method to correctly find the matrix $\mathbf{A}^{-1}$, although some made errors in their arithmetic when finding the determinant of $\mathbf{A}$ and others did not find the correct adjoint of $\mathbf{A}$. Most students correctly found the matrix $\mathbf{A}^2$, with some students making sign errors and a few squaring the individual elements of $\mathbf{A}$. Many students rewrote the printed equation $\mathbf{A}^2 + 3\mathbf{A}^{-1} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$ to give $\begin{pmatrix} 36-3k & 2k \\ -6 & -3k+16 \end{pmatrix} + \frac{3}{3k - 24}\begin{pmatrix} -4 & -k \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$, but a few only rewrote the left hand side of the printed equation and made no connection with the matrix on the right hand side.

Most students used the equation $-6 + \frac{9}{3k-24} = -3$ to find $k = 9$. Some students found two possible values of k, usually by solving the equation $36 - 3k - \frac{12}{3k-24} = 5$ to give $k = 9, \frac{28}{3}$. Only the best students tested both of these values on their original matrix equation before determining that $k = 9$.

In part (ii), many students struggled to relate the matrix $\mathbf{M}$ to a stretch followed by a rotation. Some students applied an enlargement rather than a stretch while a few applied the stretch and rotation in the wrong order. Therefore, common mistakes in this part included finding p as $\sqrt{\det \mathbf{M}}$ and attempting to solve the incorrect matrix equation $\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$. Those students who were successful in part (ii) usually found a correct $\det \mathbf{M}$ and deduced $p = 2$. They also wrote down $\cos\theta = -\frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$ followed by $\theta = 120$. Only a minority solved the correct matrix equation $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$ to find values for p and θ.

Question 8

This question discriminated well across higher ability students with part (ii) more successfully answered than part (i).

In part (i), some students failed to show $u_1 = \frac{3}{2} - \frac{7}{2} + 5 = 3$. Some students lost the first mark by stating without proof that $u_1 = 3$ and used this result to show both $u_2 = 3 + 3(1) - 2 = 4$ and $u_2 = \frac{3}{2}(2)^2 - \frac{7}{2}(2) + 5 = 4$. Many students attempted to substitute $u_k = \frac{3}{2}k^2 - \frac{7}{2}k + 5$ into $u_{k+1} = u_k + 3k - 2$. Some students lost marks by moving from either $u_{k+1} = \frac{3}{2}k^2 - \frac{7}{2}k + 5 + 3k - 2$ or $u_{k+1} = \frac{3}{2}k^2 - \frac{1}{2}k + 3$ directly to $u_{k+1} = \frac{3}{2}(k+1)^2 - \frac{7}{2}(k+1) + 5$, with no intermediate stages in their working.

In part (ii), many students successfully showed that $\mathrm{f}(n) = 3^{2n+3} + 40n - 27$ was divisible by 64 for $n = 1$. There were varying approaches with finding $\mathrm{f}(k+1) - \mathrm{f}(k)$ or directly finding $\mathrm{f}(k+1)$ being the most popular. There were also other valid methods that met with varying degrees of success, such as an attempt to find $\mathrm{f}(k+1)$ in terms of k and M, where $\mathrm{f}(k) = 64M = 3^{2k+3} + 40k - 27$, or an attempt to find $\mathrm{f}(k+1) - m\mathrm{f}(k)$ with a suitable value for m. Although many students wrote down a correct expression for either $\mathrm{f}(k+1) - \mathrm{f}(k)$ or

$\mathrm{f}(k+1)$, some did not manipulate their expression to a correct result for $\mathrm{f}(k+1)$ of either $9(3^{2k+3}+40k-27) - 320k+256$ or $9\mathrm{f}(k) - 64(5k-4)$ or the equivalent.

In part (i) and part (ii), some students did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following parts: assuming the general result is true for $n=k$; then showing the general result is true for $n=k+1$; showing the general result is true for $n=1$; and finally concluding that the general result is true for all positive integers.

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